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RESEARCH ARTICLES

The systematics of fundamental particles and unstable nuclear systems using the concept of continuity and discreteness

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In this review, a new method is proposed based on Cantor's Theory of Cardinality to analyse the experimental data on unstable nuclear systems which includes fundamental particles and their flavours, β-decay including spin and parity, α-decay and the systematics of the energy levels of light nuclei. The method also derives a formula for the binding energies of unstable nuclei. All this is based on the theory of discreteness and continuity.

THE earliest review on nuclear physics was written nearly sixty years ago by H. A. Bethe and his associates[1] to explain various nuclear phenomena within the framework of quantum theory. The review which was published in a series of three articles incorporated a thorough and comprehensive study of the developments in the field of nuclear physics at that time. The review covered almost every aspect of the nucleus ranging from α-, β-, γ-radioactivity, nuclear reactions, nuclear forces and many-body effects, etc.

Bethe's work was epoch-making and influenced all other research subsequent to his paper. For many years it was considered that the nuclear forces obtained by various experiments were sufficient to understand all nuclear phenomena but unfortunately the strong many-body nature of the problem shifted the fundamental approach to the study of nuclear models and has been the approach of all later workers. Though enormous developments have taken place in nuclear sciences, such as the discovery of magic numbers, excited states of nuclei up to high energies, discovery of parity, to mention only a few that fall in this category, an overall picture of nuclear systems which includes fundamental particles is still incomplete and will probably remain as such.

Even though there has been a continuous progress in dealing with nuclear systems and fundamental particles, there still is a lack of a coherent formulation which would permit an analysis of all observed phenomena in a fundamental way. In place of a single unifying theory, there are scattered islands of knowledge in a sea of seemingly uncorrelated observations. So far, attempts have been made to formulate several theories and models to understand various physical phenomena, which concentrate only on small sections of the observed behaviour. Experiments are often performed, guided by existing theories to mainly confirm certain features predicted by them, rather than making independent measurements. This process introduces some bias on the observations themselves, and as a result, features which can be explained in a simpler way, may be lost. It is therefore necessary to analyse and interpret the vast amount of experimental data which in itself carries a lot of information, e.g. the half-lives of all decaying systems and this can lead to new interpretations based on greater generalities.

Duality in mass-time and Cantor theory

In this review, a new phenomenological approach is used to unify various types of experimental measurements to understand the fundamental particle and nuclear aspects of physics. Nuclear data of decaying systems carry information about fundamental interactions through their emitted radiations like α-, β-particles. Using only the masses and life-times of decaying systems, it is shown that the observed data can be fitted to a unified scheme based on some very fundamental considerations. This is done by correlating masses and life-times corresponding to different kinds of nuclear and particle systems through an equation of the form[2]

$$n/2^n = \hbar/MT = \Gamma/M, \tag{1}$$

where $\hbar$ is the Planck's constant divided by 2π. M and T in the case of fundamental particles are the masses and mean life-times of decaying particles. In the cases of β-emitting radio-nuclides, M and T denote the neutron mass and half-life of the decaying nucleus. M is taken to be the neutron mass because the β-decay process in a nucleus can be effectively reduced to the decay of a neutron or a proton which have almost the same mass. For α-emitting radio-nuclides, M and T are chosen as the binding energy and half-life of the nucleus because the whole nucleus is involved in the process. Using the experimental values of M and T (refs 3, 4), the quantity n is calculated. Various properties of this crucial quantity are explained later.

The choice of the particular equation (1) comes about from a deeper basis[5] provided by the Cantor's Continuum Theory which suggests discreteness and continuity through the concept of the denumerability and non-denumerability by comparison of the cardinalities of various sets, especially the infinite ones[6]. One of the principles of Cantor's theory says that for a set having n elements, there always corresponds another set of 2^n elements with a higher cardinality. In principle, the difference in cardinalities can give rise to 'relative' discreteness and continuity. The adjective 'relative' for discreteness and continuity, arises especially in the context of physical quantities. In reality, we cannot regard a physical quantity as absolutely discrete. The extent of elements of discreteness and continuity depends on the topology of the dimensions in the space–time domain of observation. Any discrete quantity can always be transformed into a continuous one by the process of interpolation[7]. Hence, in principle, there would always be some degree of continuity involved in the discreteness. The concept of wave-particle duality which says that for each (discrete) particle there is associated a (continuous) wave, also supports this argument. In other words, we shall be using here the notion of discreteness–continuity in preference to particle–wave concept because of its more universal behaviour. It is seen that using a wider interpretation in terms of discreteness–continuity motivated from the theorems of Cantor, one is able to cover a lot of data under one scheme. It is interesting to note that Cantor himself believed that cardinality could explain much of physics.

Following the argument of the above paragraph, the width (Γ) or the spread of a line can be associated with more continuity (thus the higher cardinality) than that of the energy (E) of the levels. In other words, the life-time of a system can be associated with a higher cardinality than that of its mass M. In this way, the ratio Γ/M ($\hbar/MT$) which suggests a duality in mass and life-time can be related to $n/2^n$ as shown in eq. (1).

The quantity n in eq. (1) can be taken as having integral values[2]. This can be easily seen to be the case from Table 1 which gives the deviation of the precise value of n from their nearest integral values for elementary particles. The use of integral values of n makes our analysis more transparent to study and interpret the data. The integralization of n in eq. (1) suggests some sort of a discreteness in the Γ/M ratios, a result which is also proved within the more conventional theory, the 'Quark model' (for details see ref. 8). The differences in the values of n and n_{exact} could arise from the experimental errors of the widths.

The straight line plot of $\log(\hbar/MT)$ versus n for elementary particles and α- and β-emitters shown in Figure 1, clearly shows[2] the sectorwise distribution of the integer n for different types of interactions with the presence of forbidden regions in-between. Thus the range of values of n turns out to be an indicator of strength of interaction in the decay process. The systems decaying through strong interactions correspond to values of $n < 10$. The systems which decay through electromagnetic interactions have values of n lying between 10 and 40, while those decaying through weak forces have values of $n > 40$. As a result, most of the resonance particles lie in the region of $n < 10$, while only a few lie in the region of higher n. The α-, β-emitting radionuclides have values of $n > 40$ because of their weak interacting nature.

We now consider the quantity

$$p_0 = -\log(n/2^n), \tag{2}$$

where n is an integer. Table 2 gives the values of n when p is very close to integers.

Table 1. Mass and widths of elementary particles and n values ordered according to flavours

Particle identification	Mass (MeV)	Width (MeV)	n_{exact}	n	D_{n1} $(n - n_{exact})$
$\pi^{\pm}$	1.40E+03	2.53E–13	58.15	58	0.15
π^0	1.35E+03	7.85E–05	28.88	29	0.12
η	5.47E+03	1.20E–02	23.34	23	0.34
ρ (770)	7.70E+03	1.51E+03	4.52	5	0.48
ω (782)	7.82E+03	8.43E+01	9.82	10	0.18
η (958)	9.58E+03	2.01E+00	16.23	16	0.23
f_0 (980)	9.80E+03	2.20E+03	4.23	4	0.23
a_0 (980)	9.82E+03	1.76E+03	4.71	5	0.29
ϕ (1020)	1.02E+04	4.43E+01	11.34	11	0.34
h_1 (1170)	1.17E+04	3.60E+03	3.50	4	0.50
b_1 (1235)	1.23E+04	1.42E+03	5.59	6	0.41
a_1 (1260)	1.23E+04	4.00E+03	3.36	3	0.36
f_2 (1270)	1.28E+04	1.85E+03	5.14	5	0.14
f_1 (1285)	1.28E+04	2.40E+02	8.88	9	0.12
η (1295)	1.30E+04	5.30E+02	7.51	8	0.49
f_0 (1300)	1.25E+04	2.75E+03	4.27	4	0.27
π (1300)	1.30E+04	4.00E+03	3.50	4	0.50
a_2 (1320)	1.32E+04	1.07E+03	6.26	6	0.26
f_1 (1420)	1.43E+04	5.20E+02	7.72	8	0.28
ω (1420)	1.42E+04	1.74E+03	5.47	5	0.47
η (1440)	1.42E+04	6.00E+02	7.46	7	0.46
ρ (1450)	1.47E+04	3.10E+03	4.36	4	0.36
f_1 (1510)	1.51E+04	3.50E+02	8.52	9	0.48
f_2 (1525)	1.53E+04	7.60E+02	7.16	7	0.16
f_0 (1590)	1.58E+04	1.80E+03	5.62	6	0.38
ω (1600)	1.66E+04	2.80E+03	4.84	5	0.16
ω_3 (1670)	1.67E+04	1.73E+03	5.80	6	0.20
π_2 (1670)	1.67E+04	2.40E+03	5.16	5	0.16
ϕ (1680)	1.68E+04	1.50E+03	6.08	6	0.08
ρ_3 (1690)	1.69E+04	2.15E+03	5.40	5	0.40
ρ (1700)	1.70E+04	2.35E+03	5.24	5	0.24
f_1 (1710)	1.71E+04	1.40E+03	6.25	6	0.25
ϕ_3 (1850)	1.85E+04	8.70E+02	7.27	7	0.27
f_2 (2010)	2.01E+04	2.02E+03	5.86	6	0.14
f_4 (2050)	2.04E+04	2.08E+03	5.83	6	0.17
f_2 (2300)	2.30E+04	1.49E+03	6.68	7	0.32
f_2 (2340)	2.34E+04	3.19E+03	5.26	5	0.26
$K^{\pm}$	4.94E+03	5.33E–13	58.91	59	0.09
$K_S{}^0$	4.98E+03	7.38E–11	51.62	52	0.38
$K_L{}^0$	4.98E+03	1.27E–13	61.04	61	0.04
K^* (892)$^{\pm}$	8.92E+03	4.98E+02	6.95	7	0.05
K^* (892)^0	8.96E+03	5.05E+02	6.94	7	0.06
K_1 (1270)	1.27E+04	9.00E+02	6.52	7	0.48
K_1 (1400)	1.40E+04	1.74E+03	5.45	5	0.45
K^*(1410)	1.41E+04	2.27E+03	4.93	5	0.07
$K_0{}^*$ (1430)	1.43E+04	2.87E+03	4.47	4	0.47
$K_2{}^*$ (1430)$^{\pm}$	1.43E+04	9.84E+02	6.56	7	0.44
$K_2{}^*$ (1430)^0	1.43E+04	1.09E+03	6.38	6	0.38
K^* (1680)	1.71E+04	3.23E+03	4.60	5	0.40
K_2 (1770)	1.77E+04	1.86E+03	5.78	6	0.22
$K_3{}^*$ (1780)	1.77E+04	1.64E+03	6.01	6	0.01
K_2 (1820)	1.82E+04	2.76E+03	5.05	5	0.05
$K_4{}^*$ (2045)	2.05E+04	1.98E+03	5.93	6	0.07
$D^{\pm}$	1.87E+04	6.24E–09	46.99	47	0.01
D^0	1.86E+04	1.59E–08	45.60	46	0.40
D^* (2007)^0	2.01E+04	2.10E+01	13.67	14	0.33
D^* (2010)$^{\pm}$	2.01E+04	1.31E+00	18.07	18	0.07
D_1 (2420)^0	2.42E+04	1.80E+02	10.45	10	0.45
$D_2{}^*$ (2460)^0	2.46E+04	2.10E+02	10.22	10	0.22
$D_2{}^*$ (2460)$^{\pm}$	2.46E+04	2.30E+02	10.06	10	0.06
$D_s{}^{\pm}$	1.97E+04	1.41E-08	45.86	46	0.14

Contd...

Table 1. *(contd...)*

Particle identification	Mass (MeV)	Width (MeV)	n_{exact}	n	D_{nt} ($n - n_{exact}$)
$D_s^{*\pm}$	2.11E+04	4.50E+01	12.51	13	0.49
D_{s1} (2536)$^{\pm}$	2.54E+04	2.30E+01	13.90	14	0.10
$B^{\pm}$	5.28E+04	4.28E–09	49.10	49	0.10
B^0	5.28E+04	4.39E–09	49.06	49	0.06
B_s^0	5.38E+04	4.92E–09	48.92	49	0.08
η_c (1S)	2.98E+04	1.03E+02	11.72	12	0.28
J/ψ (1S)	3.10E+04	8.80E–01	19.37	19	0.37
χ^{c0} 1P	3.42E+04	1.40E+02	11.44	11	0.44
χ^{c1} 1P	3.51E+04	8.80E+00	15.95	16	0.05
χ^{c2} 1P	3.56E+04	2.00E+01	14.66	15	0.34
ψ (2S)	3.69E+04	2.77E+00	17.85	18	0.15
ψ (3770)	3.77E+04	2.36E+02	10.74	11	0.26
ψ (4040)	4.04E+04	5.20E+02	9.52	10	0.48
ψ (4160)	4.16E+04	7.80E+02	8.88	9	0.12
ψ (4415)	4.42E+04	4.30E+02	10.00	10	0.00
γ (1S)	9.46E+04	5.25E–01	21.91	22	0.09
γ (2S)	1.00E+05	4.40E–01	22.27	22	0.27
γ (3S)	1.04E+05	2.63E–01	23.11	23	0.11
γ (4S)	1.06E+05	2.38E+02	12.42	12	0.42
γ (10860)	1.09E+05	1.10E+03	9.93	10	0.07
γ (11020)	1.10E+05	7.90E+02	10.51	11	0.49
n	9.40E+03	7.43E–24	96.62	97	0.38
N(1440)P_{11}	1.45E+04	3.50E+03	4.07	4	0.07
N (1520) D_{13}	1.52E+04	1.23E+03	6.28	6	0.28
N (1535) S_{11}	1.54E+04	1.75E+03	5.62	6	0.38
N (1650)S_{11}	1.66E+04	1.68E+03	5.85	6	0.15
N (1675) D_{15}	1.68E+04	1.60E+03	5.96	6	0.04
N (1680) F_{15}	1.68E+04	1.30E+03	6.35	6	0.35
N (1700) D_{13}	1.70E+04	1.00E+03	6.86	7	0.14
N (1710) P_{11}	1.71E+04	1.50E+03	6.12	6	0.12
N (1720) P_{13}	1.72E+04	1.50E+03	6.13	6	0.13
N (2190)G_{17}	2.15E+04	4.50E+03	4.38	4	0.38
N (2220) H_{19}	2.25E+04	4.35E+03	4.54	5	0.46
N (2250) G_{19}	2.24E+04	3.80E+03	4.82	5	0.18
N (2600) $I_{1,11}$	2.65E+04	6.50E+03	4.03	4	0.03
Δ (1232) P_{33}	1.23E+04	1.20E+03	5.92	6	0.08
Δ (1600)P_{33}	1.63E+04	3.50E+03	4.32	4	0.32
Δ (1620) S_{31}	1.65E+04	1.50E+03	6.04	6	0.04
Δ (1700) D_{33}	1.72E+04	3.00E+03	4.76	5	0.24
Δ (1900) S_{31}	1.90E+04	1.90E+03	5.87	6	0.13
Δ (1905) F_{35}	1.90E+04	3.60E+03	4.59	5	0.41
Δ (1910) P_{31}	1.90E+04	2.30E+03	5.49	5	0.49
Δ (1920) P_{33}	1.94E+04	2.25E+03	5.58	6	0.42
Δ (1930) D_{35}	1.95E+04	3.50E+03	4.70	5	0.30
Δ (1950) F_{3-}	1.95E+04	3.20E+03	4.89	5	0.11
Δ (2420) $H_{3.11}$	2.40E+04	4.00E+03	4.86	5	0.14
Λ	1.12E+04	2.50E–11	54.42	54	0.42
Λ (1405) S_{01}	1.41E+04	5.00E+02	7.76	8	0.24
Λ (1520) D_{03}	1.52E+04	1.56E+02	9.91	10	0.09
Λ (1600) P_{01}	1.63E+04	1.50E+03	6.03	6	0.03
Λ (1670) S_{01}	1.67E+04	3.75E+02	8.57	9	0.43
Λ (1690) D_{03}	1.69E+04	6.00E+02	7.77	8	0.23
Λ (1800) S_{01}	1.79E+04	3.00E+03	4.84	5	0.16
Λ (1810) P_{01}	1.80E+04	1.50E+03	6.21	6	0.21
Λ (1820) F_{05}	1.82E+04	8.00E+02	7.39	7	0.39
Λ (1830) D_{05}	1.82E+04	8.50E+02	7.28	7	0.28
Λ (1890) P_{03}	1.88E+04	1.30E+03	6.56	7	0.44
Λ (2100) G_{07}	2.10E+04	1.75E+03	6.21	6	0.21
Λ (2110) F_{05}	2.12E+04	2.00E+03	5.97	6	0.03
Λ (2350) H_{09}	2.36E+04	1.75E+03	6.43	6	0.43

Contd...

Table 1. *(contd...)*

Particle identification	Mass (MeV)	Width (MeV)	n_{exact}	n	D_{n1} ($n - n_{exact}$)
Σ^+	1.19E+04	8.25E–11	52.75	53	0.25
Σ^0	1.19E+04	8.91E–02	21.45	21	0.45
Σ^-	1.20E+04	4.46E–11	53.67	54	0.33
Σ (1385) P_{13}^+	1.38E+04	3.58E+02	8.32	8	0.32
Σ (1385) P_{13}^0	1.38E+04	3.60E+02	8.31	8	0.31
Σ (1385) P_{13}^-	1.39E+04	3.94E+02	8.16	8	0.16
Σ (1660) P_{11}	1.66E+04	1.20E+03	6.48	6	0.48
Σ (1670) D_{13}	1.68E+04	6.00E+02	7.75	8	0.25
Σ (1750) S_{11}	1.77E+04	1.10E+03	6.75	7	0.25
Σ (1775) D_{15}	1.78E+04	1.20E+03	6.60	7	0.40
Σ (1915) F_{15}	1.92E+04	1.20E+03	6.74	7	0.26
Σ (1940) D_{13}	1.93E+04	2.25E+03	5.57	6	0.43
Σ (2030) F_{17}	2.03E+04	1.75E+03	6.15	6	0.15
Σ (2250)	2.25E+04	1.05E+03	7.27	7	0.27
Ξ^0	1.31E+04	2.27E–11	54.81	55	0.19
Ξ^-	1.32E+04	4.02E–11	53.97	54	0.03
Ξ (1530) P_{13}^0	1.53E+04	9.10E+01	10.82	11	0.18
Ξ (1530) P_{13}^-	1.54E+04	9.90E+01	10.69	11	0.31
Ξ (1690)	1.69E+04	5.00E+02	8.09	8	0.09
Ξ (1820) D_{13}	1.82E+04	2.40E+02	9.49	9	0.49
Ξ (1950)	1.95E+04	6.00E+02	8.02	8	0.02
Ξ (2030)	2.03E+04	2.00E+02	9.97	10	0.03
Ω^-	1.67E+04	8.02E–11	53.30	53	0.30
Ω (2250)$^-$	2.25E+04	5.50E+02	8.42	8	0.42
Λ_{+c}	2.29E+04	3.30E–08	44.81	45	0.19
Λ (2625)$^+$	2.63E+04	3.20E+01	13.42	13	0.42
Ξc^+	2.47E+04	1.88E–08	45.76	46	0.24
Ξc^0	2.47E+04	5.20E+01	12.53	13	0.47
$\Lambda^0 b$	5.64E+04	6.16E–09	48.66	49	0.34
μ	1.06E+03	3.00E–15	64.29	64	0.29
τ	1.78E+04	2.23E–08	45.02	45	0.02
W	8.02E+05	2.08E+04	8.32	8	0.32
Z	9.12E+05	2.49E+04	8.23	8	0.23

Table 2. Values of n_0 and p

$n =$	6	10	14	21	28	35	42	49	52	59	66
$p_0 =$	1.028	2.010	3.068	4.999	6.982	8.992	11.020	13.060	13.938	15.990	18.048
$p =$	1	2	3	5	7	9	11	13	14	16	18
$p_0 - p =$	0.028	0.010	0.068	–0.001	–0.018	–0.008	0.020	0.060	–0.062	–0.010	0.048

It is seen that for the elementary particle region of $n < 65$, the quantity p_0 turns out to be close to prime integers for every n being a multiple of 7. However, the first two cases of $n = 6$, 10 corresponding to the resonance particles are exception to this rule. But this rule does not hold strictly for the α- and β-emitters with higher values of n. By taking into consideration only those values of n for which p_0-values are close to integers, amounts to a higher degeneracy which corresponds to diluting the mass effects, thus resulting in quantization of life-times as seen by Mac Gregor's work[9]. Any sequence of numbers n_{exact} which has a fixed difference can be shown to be related to the nearest integer n and prime number through the relation.

$$R = (n_{exact} - n)/(p - p_0).$$

R has a maximum at $n = 6$, 10, 14, 21, 28, etc. which correspond to p nearly equal to 1, 2, 3, 5, 7, etc. respectively. Figure 2 shows that the numbers n_{exact} obtained experimentally from eq. (1) follow this pattern, showing the close relation between fundamental particles and primes. The groupings around the primes also correspond to the flavour of the particles. This is shown in a later paper.

Now we shall study separately different sectors, viz. fundamental particles, α- and β-emitting radionuclides in the context of the present formalism motivated by Cantor's theory.

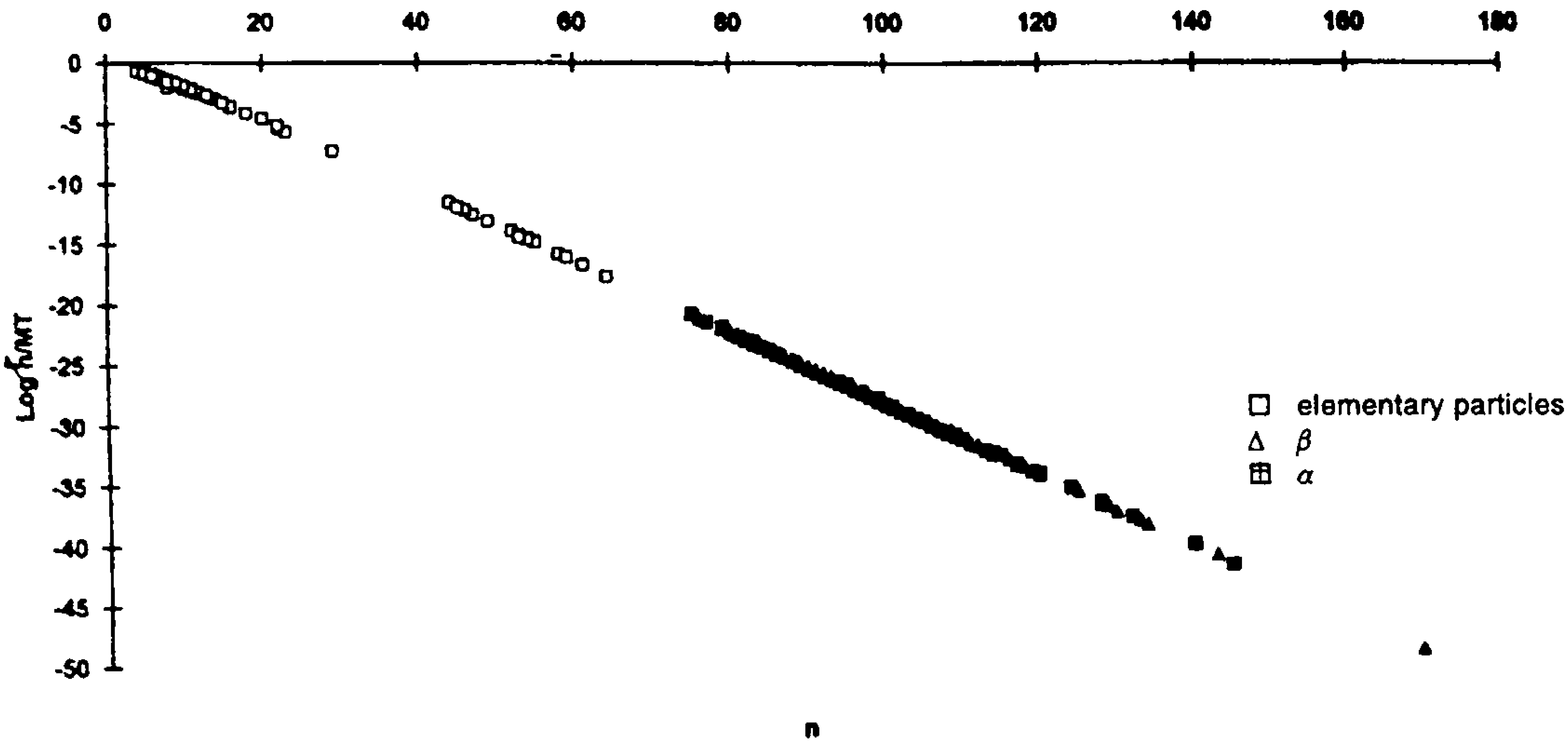

Figure 1. Log $\hbar/MT$ vs n for elementary particles, β and α emitters.

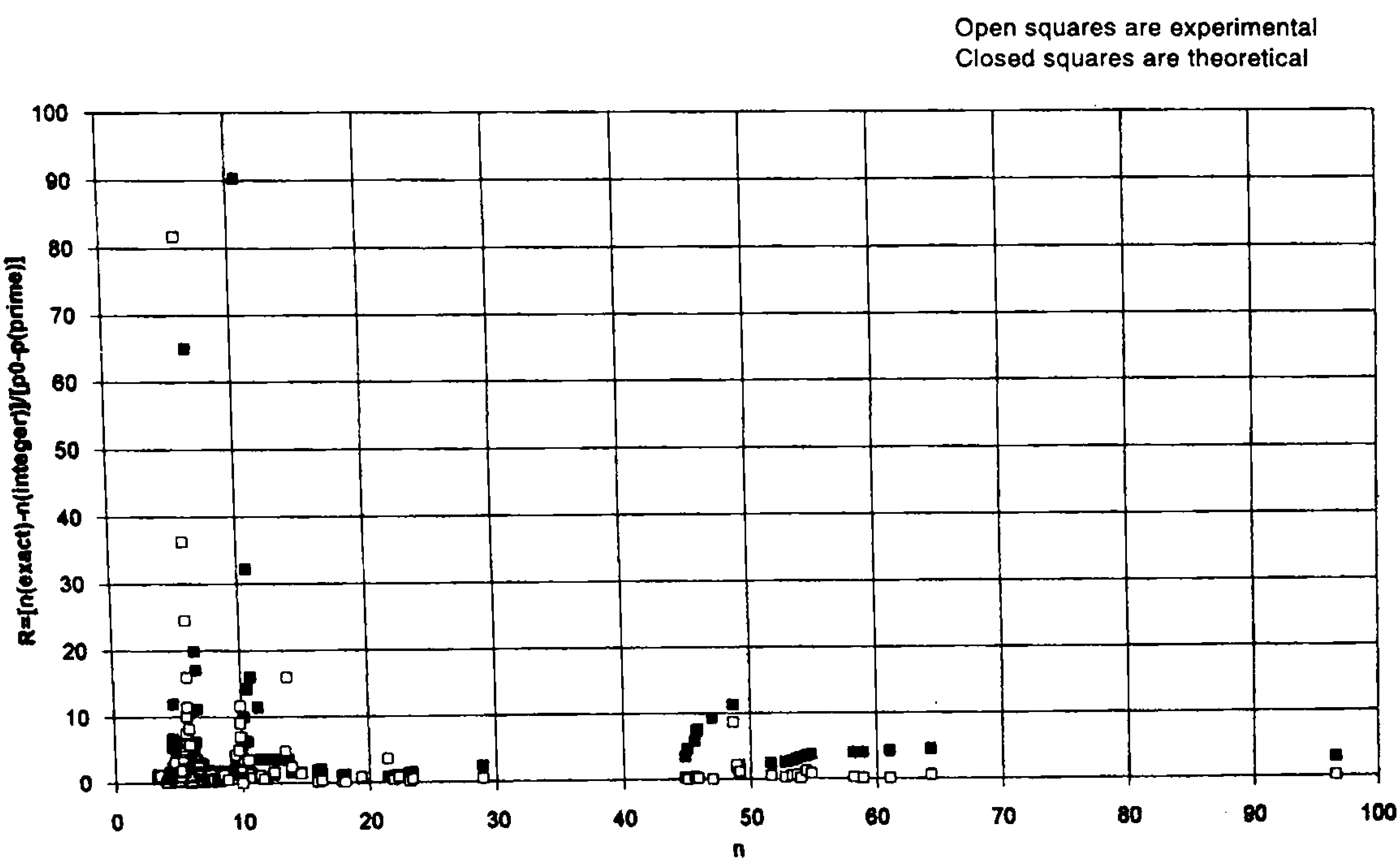

Figure 2. Ratio of the departures of n from integral values and p from nearest primes against n for experimental and theoretical values.

Elementary particles and the n-theory

The plot of n-values against various elementary particles shows a non-uniform distribution. It is seen that in the region of n between 24 and 44, no particle except the π^0 is found. An examination of the decay modes of various particles classified in terms of n shows that the majority of resonances that decay through strong interactions have values of $n < 10$. Those decaying through electromagnetic interactions have values of n lying between 10 and 30, and those that decay through weak interactions have values of $n > 40$.

If a toponium (tt bar) is formed having mass around $2\,m_t \sim 350$ GeV, and it decays through hidden strong interactions in a manner similar to bb bar and cc bar states, then its life-time range is predicted to be 2×10^{-25} to 2.5×10^{-21} s. The limits of n-values for the top quark which decays through weak interaction can be set out as $40 < n$ (top) < 64 considering the maximum value of n to be 64 for elementary particles. Thus, the life-time of the top quark with mass 174 GeV (ref. 10) can be predicted to lie between 10^{-16} s and 10^{-9} s.

The analysis of elementary particles in terms of the masses and lifetimes brings out many new general features in the spectrum. The masses of the unstable particles detected so far range from 0.1 GeV to about 15 GeV (excluding gauge bosons $W^{\pm}$, Z^0) wherein the most populated region corresponds to 0.4–2.5 GeV. In contrast to the short mass domain, the lifetime domain for unstable particles extends widely from 10^{-24} s to 10^3 s, with several distinct peaks and regions of total absence of particles. The most prominent peak corresponds to the resonance region of 10^{-24} s to 10^{-20} s, with subsidiary peaks at 10^{-13}, 10^{-12}, 10^{-10}, 10^{-9} s with a paucity of particles between 10^{-20} and 10^{-13} s. In the entire region 2×10^{-16} to 10^3 s, only one particle, the neutron is found.

The identification of flavours of elementary particles can be achieved by studying the plot of log T vs n (see Figure 3). This plot shows some parallel lines that correspond to a constant value of M which in turn indicates signature of the presence of subhadronic particles. Particles having similar substructure lie on one straight line associated with a constant value of mass corresponding to that of constituent particles taken from

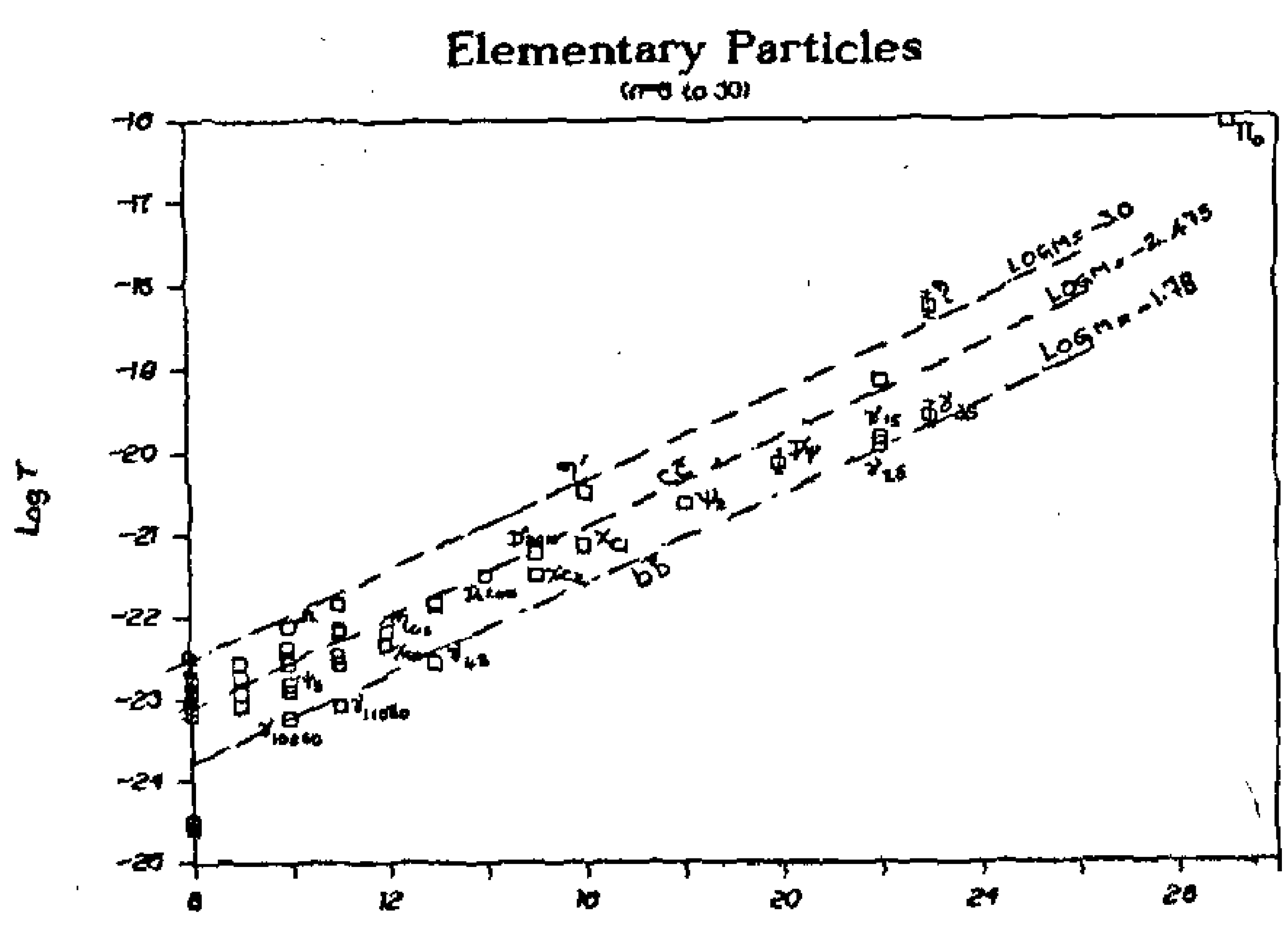

Figure 3. Plot of log T vs n for elementary particles showing the effect of mass.

conventional Quark model[11]. This implies that information about flavour substructure of elementary particles is hidden in the mass–life-time relation. Some other important properties can be noted by relating n with different flavours of elementary particles. It is then observed that while for light elementary particles (unflavoured ones), there exist a large number of resonances for each particle, for heavy particles like charmed strange baryons, bottom and charmed mesons, very few resonances have been discovered so far. This calls for efforts to detect some more new resonances for the heavy particles. In contrast, there are particles mostly quarkonia, e.g. cc bar and bb bar for which only short-lived resonances are present with no corresponding longer-lived particles in the current experimental scenario. This indicates the unstable flavour antiflavour substructure of such particles.

n-Symmetries for elementary particles

We shall now study some relations between the n-values (Γ/M ratios) for the particles which are associated through (unitary) SU(3) isospin symmetry. Some striking regularities in the differences in their n-values are found[12] for pairs of particles having similar set of isospin quantum numbers. The identification of such pairs can be easily facilitated by the following table which gives the set of mesonic and baryonic supermultiplets for SU(3) octets and singlets (Table 3).

The n-values for isospin singlets and neutral member of the isospin triplets belonging to SU(3) octets are found to be related to each other through some periodicity. The following equations (i) show that (π^0, η) for pseudoscalar mesonic octet, (ρ^0, ω) for vector mesonic octet and (Σ^0, Λ) for $1/2^+$ baryonic octet are examples of such pairs. These pairs of states with different values of full isospin (I), but the same values of its third component I_3 (=0) show the following regularities:

$$\text{(i) } n_{\pi 0} - n_\eta = 2^2 + 1;\ n_{\rho 0} - n_\omega = 2^2 + 1;$$
$$n_{f1'} - n_{a1} = 2^2 + 1;\ n_\Lambda - n_{\Sigma 0} = 2^5 + 1.$$

Table 3. Meson and baryon supermultiplets

Triplet	Octets singlet	Doublet	Singlets
	Meson supermultiplets		
π	η	K	η'
ρ	ϕ	K	ω
a	f'	K^*	f
	Baryon supermultiplets		
Triplet	Octets singlet	Doublet	Singlets
Σ	Λ	N, Ξ	Λ'

Table 4. Several sets corresponding to various n

	A	B	C	D	E
$\alpha =$	n	$n/3$	$n/7$	$n/15$	$n/31$
$k =$	2^{n-1}	$3*2^{(n-6)/3}$	$7*2^{(n-21)/7}$	$15*2^{(n-60)/15}$	$31*2^{(n-155)/31}$
	$n \geq 1$	$n \geq 6$	$n \geq 21$	$n \geq 60$	$n \geq 155$

On the other hand, the singlets belonging to SU(3) octets and their orthogonal singlets with same values of I and I_3 (=0), show a slightly different kind of periodicity in n-values. Examples of such pairs for the mesons are (ϕ, ω), (η, η'), (f_2, f_2'). For the baryons, the excited states $D_{03}\Lambda(1690)$ and $D_{03}\Lambda(1520)$ represent such a pair because the ground state SU(3) flavour singlet Λ is forbidden by Fermi statistics.

(ii) $n_\phi - n_\omega = 2^1$ and their higher excitations
$n_{\phi 3} - n_{\omega 3} = 2^1$
$n_\eta - n_{\eta'} = 2^3$; $n_{f2'} - n_{f2} = 2^1$; $n_{\Lambda(1520)} - n_{\Lambda(1690)} = 2^1$.

Two symmetries described above can be combined together to give a more general result for association of neutral particles belonging to SU(3) symmetry with same values of I_3 through the relation

$$|\Delta n| = 2^q + |\Delta I|, \tag{3}$$

where Δn and ΔI represent the differences in their integer n and isospin values respectively. It is to be noted that this relation holds for only integral values of ΔI. Some interesting properties of the integer are pointed out later.

The regularities found above in the n-difference point out to the quantization of the Γ/M ratios for the isospin levels which can be written in the mathematical form as[12]

$$[MT/\hbar]_{n+\alpha} = k[MT/\hbar]_n, \tag{4}$$

where subscripts denote the corresponding n-values for $MT/\hbar$. Note that n and α are integers, and α is the index used for the isospin. This equation can be simplified by using eq. (1) to

$$k = (n/n + \alpha)2^\alpha. \tag{5}$$

The quantization of the isospin levels suggests that k should be an integer which is true when $n + \alpha = 2^q$, for every $q \leq \alpha$. The second condition for k to be an integer can be obtained in the special case of n being a multiple of α. This gives rise to several sets as shown in Table 4.

The third row in the table which gives the limits of n for each case, rules out the condition of $n = 0$ for the stable systems. Thus $n \to \infty$ is the only limit for such systems. It is interesting to find that the region corresponding to the sets B and C matches fairly well with the resonance region limits of $6 \leq n \leq 24$, while the limits $2*21 \leq n \leq 60$ derived from sets C and D agree well with the more stable region of $44 \leq n \leq 64$. Thus the assumption of quantization of isospin levels yields as seen from the above results, which are in conformity with the observed ones. This equation applies only to particles having isospin, but it does indeed give the required limits for various types of interactions with intermediate n for non-isospin particles for which this quantization rule would have no meaning. These are some of the aspects which should be looked into.

As stated earlier, the integer α is the isospin index or, in other words, it is a measure of the spacing of isospin levels. $I = 0$ is a single level while $I = 1/2$ is a doubly degenerate level. Similarly $I = 1$ has three-fold degeneracy and so on. The integer α has different values for different isospin doublets like nucleon, kaon, etc. and also for isospin triplets like sigmas, pions, etc. This suggests that integer α apart from being an isospin index can also be distinguished by other parameters such as flavours unknown at present.

Based on the general principles of the tendency for a system to take lowest energy shift intervals, the value of α must be lowest and always positive. In this respect, n represents the lowest ground state in terms of the isospin for each $I = 0$ particle. Now α will be the same for all members of an isospin multiplet. On the other hand, the integer quantity q would change for different values of the third component (I_3) of the isospin, which distinguishes various members of the same isospin multiplet. In particular for a triplet, q increases by one unit as we move from ($I_3 = 0$) neutral member to the ($|I_3| = 1$) charged one. The second case (ii) of same I, I_3 correspond to same values of α's for each n, so that their difference is equal to 2^m, m being an integer. On the other hand, for case (i) which relates states having different I's with α being different, a different kind of symmetry exists.

Proton lifetime

We shall now use the above discussion for relating the n-values for the case of different members of the isospin multiplet which differ from each other only through the third component I_3. Since I is same, all members have similar value of α. But the value of q changes in units of one. This implies that the n-difference is equal to 2^q. But the exciting part in this scenario is that the quantity q can be related to one of the n-values, and hence the other n-value can be predicted. This we shall explain by taking up various isospin multiplets available in the experimental data.

1) We first consider the isospin triplet of baryon ($\Sigma^\pm$, Σ^0).

For $\Sigma^{\pm}$ with $|I_3| = 1$ and $n = 54$, the postulate of choosing the minimum value of α gives $q = 6$, and thus $\alpha = 64–54 = 10$. This implies that for Σ^0 ($I_3 = 0$) for which $\Delta I_3 = -1$ q goes to $q-1 = 5$, while the α-value remains the same since we are considering the same multiplet. Thus, we get $\alpha = 2^5 - 10 = 22$. This value of n for Σ^0 matches exactly with that obtained using experimental values. Note that the difference in their n-values is $n_{\Sigma^{\pm}} - n_{\Sigma^0} = 2^5$.

2) Now, we take the example of isospin triplet of psuedoscalar mesons ($\pi^{\pm}, \pi^0$). For $\pi^{\pm}$ $|I_3| = 1$ and $n = 58$ which implies that minimum value of α is possible only for $q = 6$, which is equal to 6 (64–58). With this value, the n-value for π^0 with $I_3 = 0$ and $q = 6-1 = 5$, is equal to 26 (32–6). This value of n corresponds to a life-time of 1.26×10^{-17} s for π^0 in comparison to its experimental value of $8.4 \pm 0.6 \times 10^{-17}$ (with $n = 29$).

3) Encouraged by the above cases which give fairly good agreement with the experimental results, we venture further into the calculation of the value of n for the proton by using the corresponding value for n of its other doublet member, the neutron. Since $I_3 = 1/2$ for proton and $I_3 = -1/2$ for the neutron, so that $\Delta I_3 = 1$ as we go from neutron to proton. For the case of neutron, $n = 97$, thus $\alpha = 2^7 - 97 = 31$. As a result, for the corresponding case of the proton for which $\alpha = 31$; $q = 7 + 1$, n is $2^{7+1} - 31 = 225$. With this value of n for the proton, we get the life-time of the proton as 5.33×10^{33} years, which does not contradict the present experimental limit of proton lifetime $> 10^{31}$ to 5×10^{32} years (refs 3, 12).

By analysing eq. (3) in context of three cases associating

i) neutral particles of triplet and singlet groups belonging to an SU(3) octet
ii) neutral particles of SU(3) octet and singlets
iii) various members of isospin triplets (with different values of I_3), it is found that the integer q takes values which are only prime numbers, e.g. $q = 1$ for (ϕ, ω), (f_1', a_1); $q = 2$ for (π^0, η), (ρ^0, ω), (f_2, f_2'); $q = 3$ for (η, η'); $q = 5$ for ($\pi^{\pm}, \pi^0$), ($\Sigma^{\pm}, \Sigma^0$) (Σ^0, Λ) and $q = 7$ for (p, n).

It is intriguing to find that the life-time of a particle with known mass can be calculated using that of the other particle which is related to the former through isospin symmetry. This is seen in examples (1)–(3) which correspond to the particles with roughly the same mass, and is well supported by cases (i) and (ii). The present approach does not distinguish between the particles and their antiparticles with respect to their n-values. This formulation holds true for all light-flavoured hadrons which contain at least two isospin flavours. The ρ-meson case (with $n < 6$) is not considered here because in this case of very short life-time, the experimental errors are too large to enable a unique determination of integral n-value[2]. It is to be noted that the present approach does not work for the systems consisting of only one isospin flavour (e.g. kaons, bottom and charmed mesons) and of course for non-isospin states. Since there are not many cases of mass multiplets available in the current experimental scenario[3], we do not yet have a more firm basis for our formulation. However, the limited number of multiplets available support the hypothesis for a particular type of system.

The regularities described above with respect to n are the reflections of the symmetries in the life-times of the particles since their masses play a passive role in the Γ/M ratios because variations in the mass domain are very small compared to those in the life-times. This suggests that the mean life-time for the decay of a particle depends on its internal structure, and hence can be calculated inclusively. This is also supported by the work of Mitra and Sharma[8]. It is well known that the decay modes of various particles are formulated by taking into consideration various internal quantum numbers, e.g., isospin, strangeness, charm, etc., and are subsequently confirmed by experiments. Therefore, these internal quantum numbers and their corresponding wave functions must play an important role in deciding how a particle decays. This calls for a detailed analysis of the present approach in the context of more conventional theories.

β-Decay and n theory

(i) For the β-emitting nuclei, since M is taken to be the constant mass of the neutron, n effectively represents the behaviour of half-life of the nucleus. In the present case, the n-values mostly range from 70 onwards owing to the weak interacting nature of β-decay (see Figure 1). But few light nuclei have n-values lying in the region of strong interactions. When Z/A is plotted against integer n-values (see Figure 4), the electron and positron emitting nuclei distinctly separate out in the region of n values smaller than the n for neutron. This can be explained by the fact that electron emission results in the increase of nuclear charge by one unit and the positron emission results in the decrease of nuclear charge by one unit. As a result, the upper branch of the plot corresponds to electron (β^-) emitters while the lower one corresponds to the positron (β^+) emitters which is just the reflection of the former curve. This plot shows the exponential behaviour of Z/A with respect to n, which can be expressed as $Z/A \sim (1 + e^{-n})/2$ for β^--emitters and $Z/A \sim (1-e^{-n})/2$ for β^+-emitters.

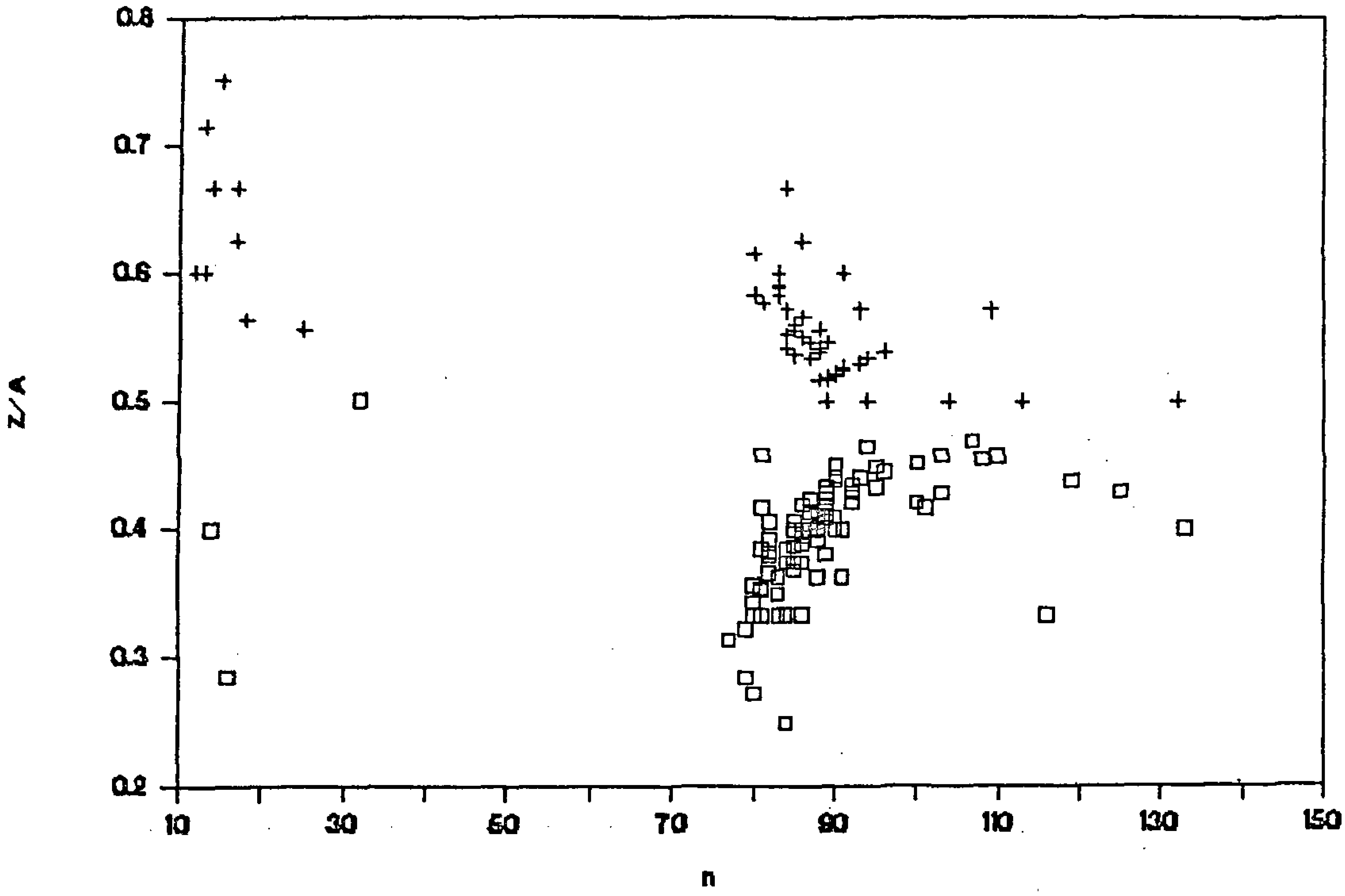

Figure 4. Z/A vs n for β-emitters showing the separation of positrons (+) and electrons (□).

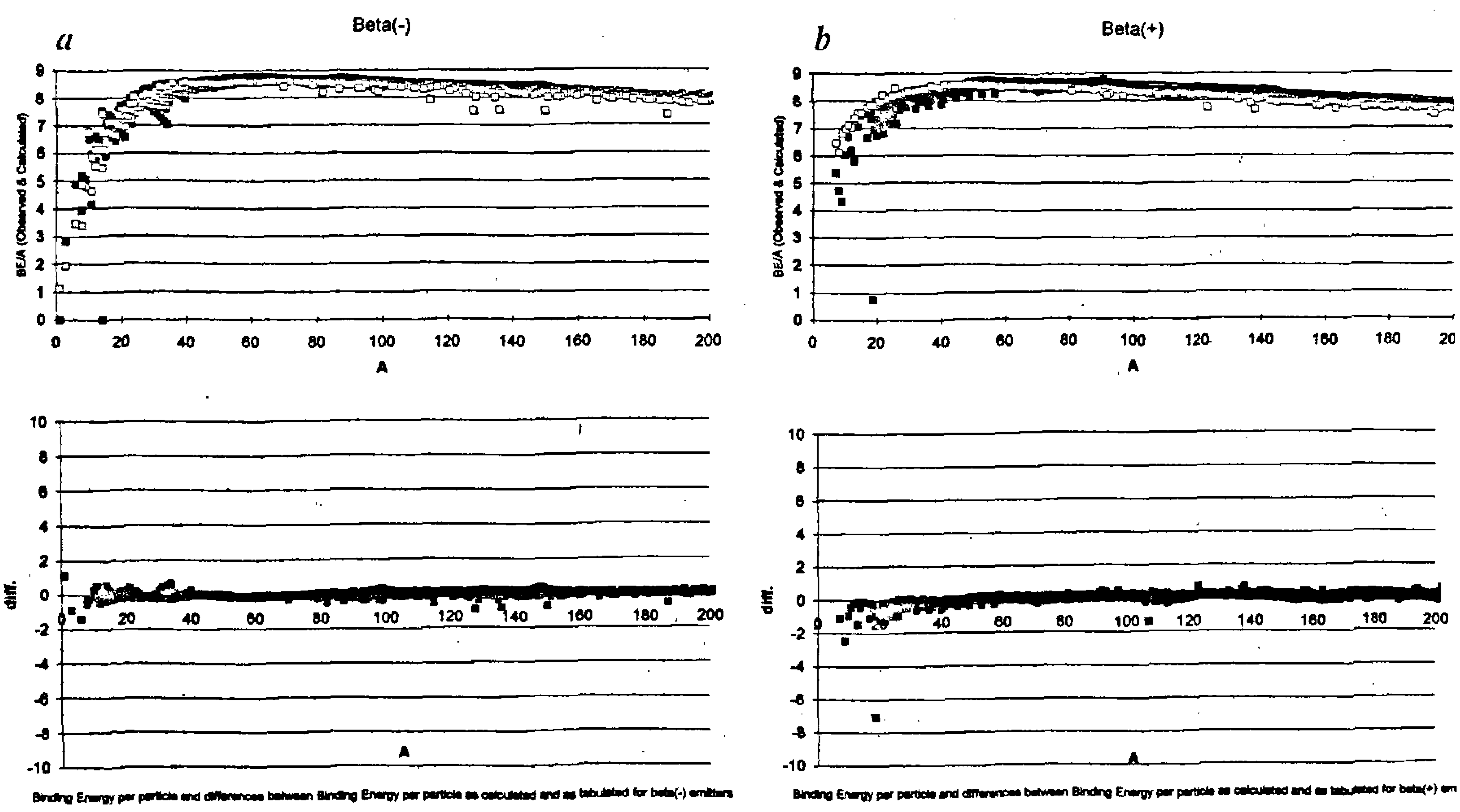

Figure 5. *a*, Binding energy per nucleon and differences between binding energy per nucleon as calculated and as tabulated for β^- emitters. Binding energy per nucleon and differences between binding energy per nucleon as calculated and as tabulated for β^+ emitters.

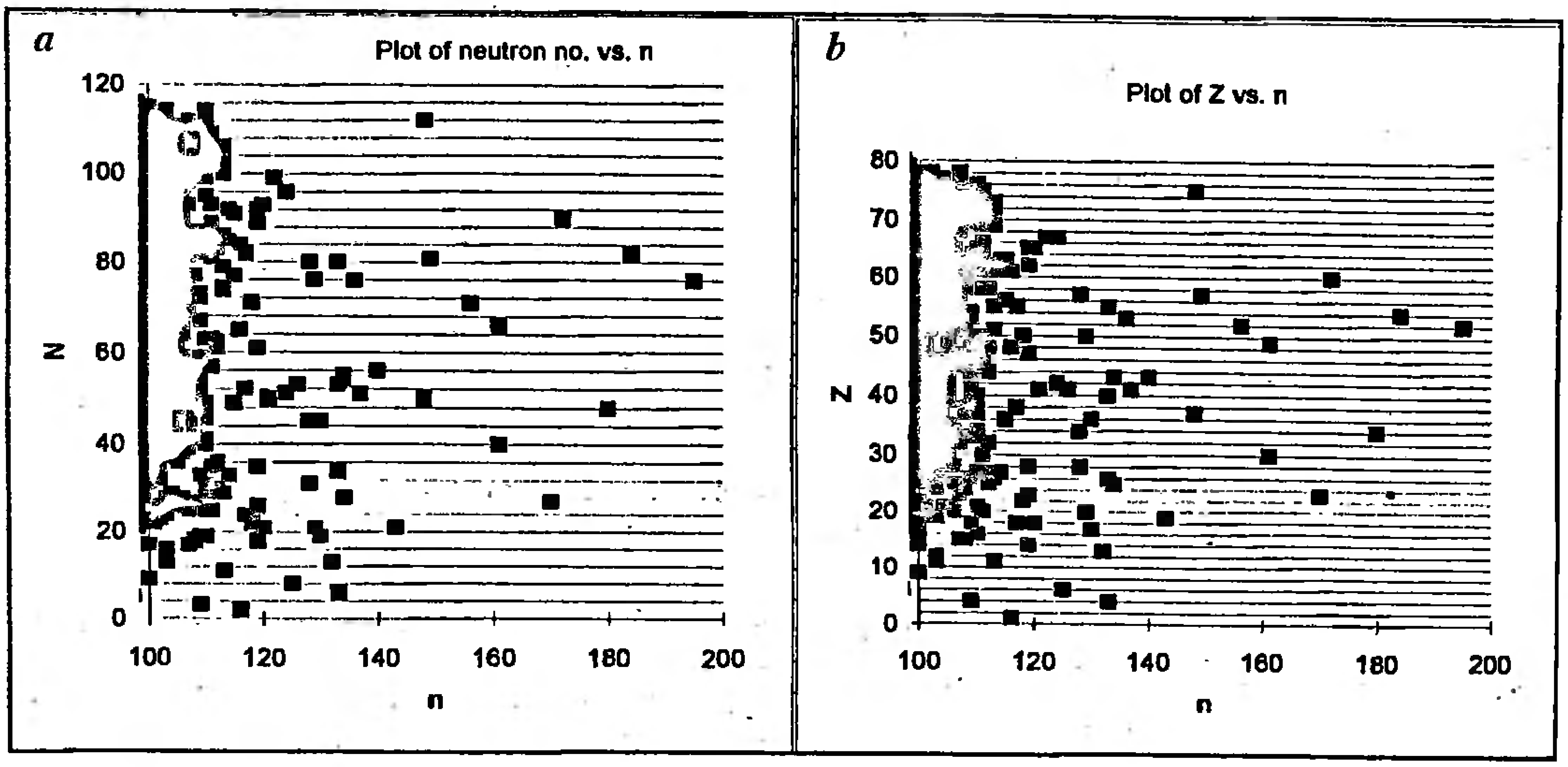

Figure 6. Plot of n vs neutron number and proton number showing the existence of magic numbers.

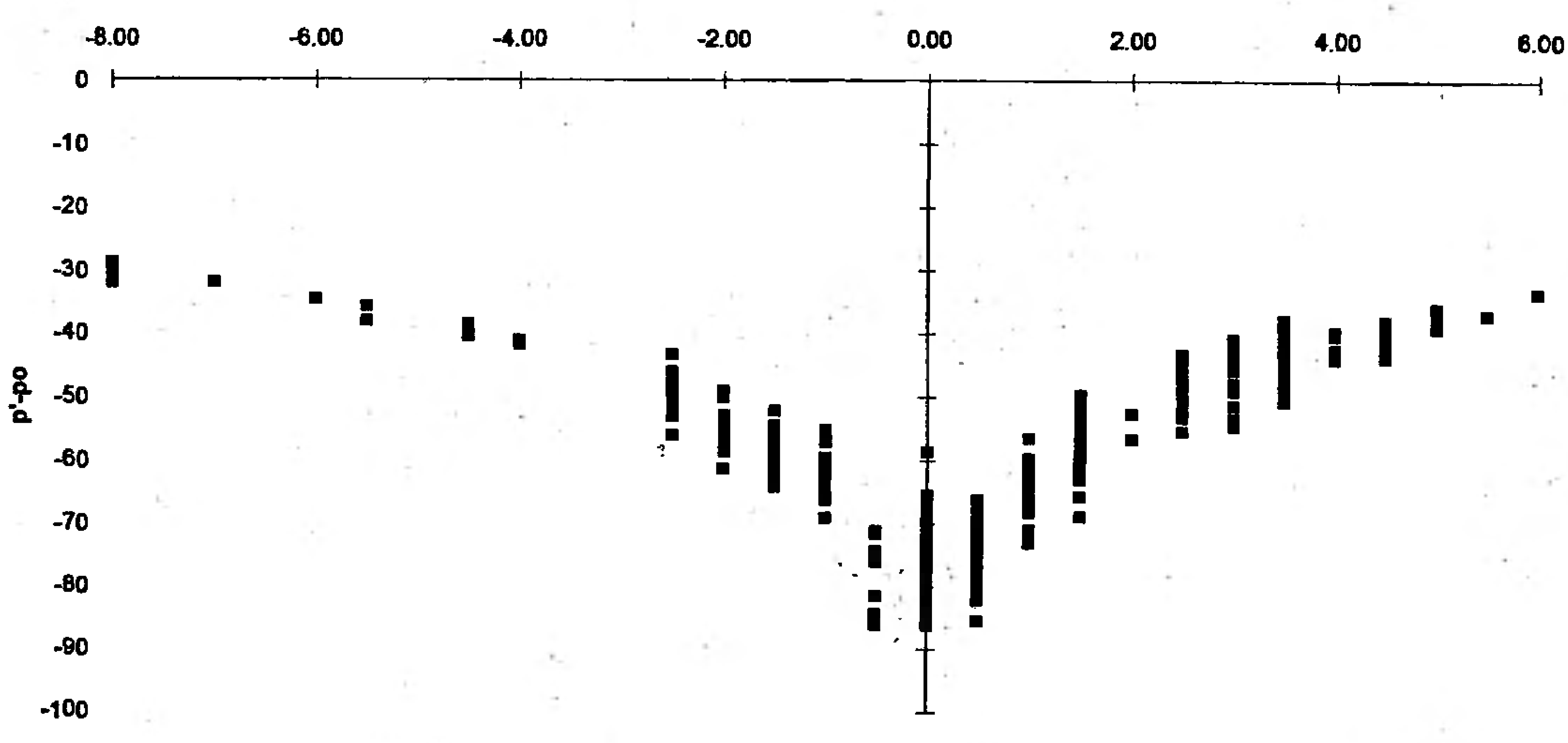

Figure 7. Plot of $p'-p_0$ vs spin.

The proportionality of the half-life of a nucleus on the Z/A seen in the above paragraph, suggests the dependence of nuclear-binding energies on the half-life through Z/A. The least square fit curve (Figures 5 *a*, *b*) of the binding energies shows that they can be expressed as ref. 13.

$$BE/A = \alpha(M_n - M_p)\,\{\log (Zn/n_0) - 0.316\,(\log Zn/n_0)^2\}, \tag{6}$$

This formula holds for all β^-, β^+ emitters as well as K-electron capture nuclei. Since β-decay of a nucleus can be effectively reduced to the decay of a neutron inside a nucleus into a proton and an electron, the difference $(M_n - M_p\ m_e) \sim (M_n - M_p)$ between their masses should play an important role in contributing to the binding energies. BE/A shows its proportionality to the half-lives through the logarithmic argument (Zn/n_0), where n_0 represents the minimum values of n for each case which is equal to 70 and 75 for β^-, β^+-emitters respectively. Here we have not taken into account the nuclei which do not decay through weak interaction. Since the n values for β-emitters range from 70 to 150, the ratio n/n_0 changes from 1 to 2. As a result, Zn/n_0 moves from Z to A as we go towards nuclei having longer half-lives. The quantity α in the expression BE/A represents a dimensionless normalizing factor of value 8.45 which is same for all cases. Figures 5 *a*, *b* give the comparison of the observed BE/A and those calculated using the formula (14). The agreement holds fairly well considering the huge number (1300 cases) of nuclei involved.

Though only unstable systems are under consideration, it is clear from Figures 6 *a*, *b* that as N and Z approach magic numbers, the value of n increases, indicating longer life-time and thus great stability.

The explanation of the β-spectrum is given by Fermi's theory of β-decay which expresses the half-life of decaying nucleus as ref. 13.

$$0.693/T_{1/2} = |M|^2 f\,(g^2 m e^2 c^4)/(2\pi^3 h^7). \quad (7)$$

Here $|M|^2$ is the nuclear matrix element which accounts for the effects of particular initial and final nuclear states. The Fermi function f takes into account the effect of the Coulomb field of the nucleus. g is the weak interaction constant. The product fT is called the comparative half-life which gives us a way to compare the β-decay probabilities in different nuclei. Since the quantity f incorporates the dependence on the atomic number and the maximum electron energy, the difference in fT values[15] must be due to differences in the nuclear matrix elements[16].

(ii) *Ground state spins of β-emitters and their half-lives.* The groundstate spins of β-emitters can be obtained from the same equation (2), i.e. $p_0 = n\log 2 - \log n$. Let us consider a situation where p_0 is not equal to $n\log 2 - \log n$ but is modified by spin and many body effects to

$$p' = n\log 2(q-1) - \log n + \log (Z/A),$$

where 2 is replaced by $2(q-1)$ and has a term containing Z/A, as Z/A plays an important part in spin and other many-body effects. We assume that $(q-1)$ is directly proportional to spin and $(q-1)$ is always negative and $(1-q)$ is always positive. As we shall see later, these positive and negative values result in space distortion of n. Figure 7 shows the effect of change in $p'-p_0$ as $(q-1)$ departs from 1 on either side for nuclides. The set of nuclides on the left corresponds to those of negative parity and vice versa. It can be noted that the two parity distributions are not identical and those with values of $(q-1)$ being negative have real values only when n is even.

The departure from log2 to log $2(q-1)$ implies expansion of n space geometry for the –ve components, while for +ve values the relative values of n are contracted. One can conjecture that it is this change in geometry of the n-space that gives rise to spin.

α-Decay and the n-theory

The general features of α-emission which occurs mostly in heavy nuclei with mass number greater than 150, can be accounted for by a quantum mechanical theory developed in 1928 almost simultaneously by Gamow and Gurney and Condon. The central feature of this one-body model is that the α-particle is preformed inside the nucleus. In fact, there is not much reason to believe that α-particles do exist separately within the heavy nucleus; nevertheless the theory works quite well, especially for even–even nuclei. The success of the theory does not prove that α-particles are preformed but merely that they behave as if they were. The theory gives the possibility of leakage or tunneling of the α-particle through the large potential barrier of the nucleus which is absolutely impossible in the classical interpretation. The leakage probability is so small that the α-particle on the average must make 10^{38} tries before it escapes (this amounts to 10^{21} per second!). The disintegration constant of an α-emitter is given in one-body theory by the following expression[13]:

$$\lambda = f P, \quad (8)$$

where f is the frequency with which the particle presents itself at the barrier, which is given by its velocity in the nucleus divided by the radius of the nucleus. P is the probability of transmission through the barrier which is given by the following expression

$$P = \mathrm{e}^{-2G}. \quad (9)$$

The Gamow factor G is[13]

$$G = [(2mc^2)/(\hbar c)^2 Q]^{1/2} (ZZ'\ \mathrm{e}^2)/(4\pi\varepsilon_0) \\ [\pi/2 - 2(Q/B)^{1/2}], \quad (10)$$

where m and Z are the mass and charge of the α-particle and Q is its kinetic energy. B is the binding energy of the daughter nucleus and Z' is its charge. So, the final expression can be written in terms of half-life as

$$f = (0.693/T_{1/2})\, \mathrm{e}^{-2G}. \quad (11)$$

If we replace $T_{1/2}$ by the binding energies and the n using equation (1) for the α-particle

$$n/2^n = \hbar/MT_{1/2} = \hbar/V_0\, T_{1/2},$$

where V_0 is the binding energy of the parent nucleus, we get

$$f = (0.693\ V_0 n/\hbar 2^n)\,\mathrm{e}^{-2G}. \quad (12)$$

Table 5. Values of log [E_r/Γ] and Dn

ID	E_r	Γ (Exp)	$\Delta\Gamma$ (Errors)	Log (E_r/Γ)	Dn
Li-6	2.18	0.320	0.050	0.83	5
Li-6	3.56	0.025	0.001	2.15	11
Li-6	5.4	0.350	0.150	1.19	7
Li-6	6.56	0.005		3.12	14
Li-7	0.9779	0.268	0.030	0.25	1
Li-7	4.629	1.000		0.67	4
Li-7	7.475	0.006	0.000	3.1	14
Li-7	8.9	0.089	0.007	2	10
Li-7	11.13	0.093	0.008	2.08	10
Li-8	2.258	2.000		0.05	
Li-8	3.21	4.000		−0.10	1
Li-8	6.4	1.000		0.81	5
Li-8	6.51	0.040		2.21	11
Li-8	6.53	0.310	0.005	1.32	7
Be-7	4.55	0.298	0.025	1.18	7
Be-7	7.185	1.800		0.6	4
Be-7	9.9	1.200		0.92	6
Be-7	10.79	0.836		1.11	6
Be-7	11.4	0.100		2.06	10
Be-8	2.9	7.000		−0.38	1
Be-8	4.57	0.010	0.001	2.66	12
Be-8	16.63	0.190		1.94	10
Be-8	16.92	0.097	0.011	2.24	11
Be-8	17.6	0.083	0.010	2.33	11
Be-8	18.15	0.270	0.020	1.83	9
Be-8	19.05	1.160		1.22	7
Be-8	19.22	0.147		2.12	10
Be-8	20.36	5.000		0.61	7
Be-8	21.6	1.450	0.060	1.17	1
Be-10	3.366	0.000	0.000	8.88	35
Be-10	5.959	0.000		8.86	35
Be-10	6.178	0.000		9.67	37
Be-10	7.377	0.016		2.66	12
Be-10	7.548	0.006		3.1	14
Be-10	9.27	0.100		1.97	10
Be-10	9.4	0.400		1.37	7
Be-10	17.79	0.110	0.035	2.21	11
Be-10	18.47	0.500		1.57	8
B-10	0.7173	0.000	0.000	12.04	46
B-10	1.74	0.000	0.000	8.6	34
B-10	2.154	0.000	0.000	9.65	37
B-10	3.585	0.000	0.000	8.72	34
B-10	4.774	0.001		3.68	16
B-10	5.114	0.001		3.63	16
B-10	5.166	0.000		6.24	25
B-10	5.183	0.120	0.020	1.64	9
B-10	5.923	0.005		3.07	14
B-10	6.029	0.001		3.78	17
B-10	6.133	0.005		3.09	14
B-10	6.566	0.070		1.97	10
B-10	6.884	0.140		1.69	9
B-10	7	0.095	0.010	1.87	9
B-10	7.43	0.14	0.030	1.72	10
B-10	7.468	0.08		1.97	10
B-10	7.479	0.072	0.004	2.02	10
B-10	7.561	0.003	0.000	3.4	15
B-10	7.62	0.225	0.050	1.53	8
B-10	7.77	0.21	0.060	1.57	8
B-10	8.07	0.8	0.200	1	6
B-10	8.892	0.084	0.007	2.02	10
B-10	8.896	0.036	0.002	2.39	11
B-10	9.7	0.6		1.21	7
B-10	10.83	0.5		1.34	7
B-10	18.6	0.5		1.57	8
B-10	18.8	0.6		1.5	8
B-10	19.3	0.35		1.74	9
C-10	10.2	1.5		0.83	5
Al-25	2.5	0.001		3.4	15
Al-25	2.69	0.000		3.84	17
Al-25	2.74	0.001		3.32	15
Al-25	3.49	0.010		2.54	12
Al-25	3.72	0.000		4.09	18
Al-25	3.84	0.036		2.03	10
Al-25	4.05	0.01		2.61	12
Al-25	4.22	0		4.55	20
Al-25	4.59	0		3.99	21
Al-25	4.9	0.01		2.69	13
Al-25	5.06	o.010		2.7	13
Al-25	5.1	0.05		2.01	10
Al-25	5.3	0.2		1.42	8
Al-25	7.32	0.1	0.020	1.86	9
Al-25	7.78	0.34	0.050	1.36	7
P-29	4.342	0.053	0.003	1.91	10
P-29	4.765	0.016	0.001	2.49	12
P-29	4.968	0.004		3.09	14
P-29	5.53	0.425	0.050	1.11	6
P-29	5.74	0.013	0.001	2.66	12
P-29	5.968	0.01	0.002	2.8	13
P-29	6.195	0.095	0.006	1.81	9
P-29	6.329	0.073	0.005	1.94	10
P-29	6.59	0.2	0.020	1.52	8
P-29	6.836	0.005	0.000	3.14	14
P-29	6.956	0.12	0.010	1.76	9
P-29	7.024	0.1	0.008	1.85	9
P-29	7.963	0.008	0.001	2.95	14
P-29	7.513	0.007	0.003	3.03	14
P-29	7.62	0.165	0.025	1.66	9
P-29	7.74	0.002		3.59	16
P-29	7.92	0.014	0.004	2.75	13
P-29	7.97	0.125	0.025	1.8	9
P-29	8.08	0.036	0.010	2.35	11

Figure 8 shows that f is constant over a wide range of n-values, and is of the order of 10^{21} to 10^{22} s^{-1} (ref. 14).

Excited nuclear levels and elementary particles

The transition of a ground state nucleus to an excited one of higher energy occurs under the external influences when the required energy is transferred to the nucleus through interaction with an energetic particle. When excited externally, one or many nucleons (depending on the quantity of energy applied) occupy higher energy levels. Since the nucleon levels are separated by finite energy intervals, the nucleus cannot receive any quantity of energy but only strictly definite

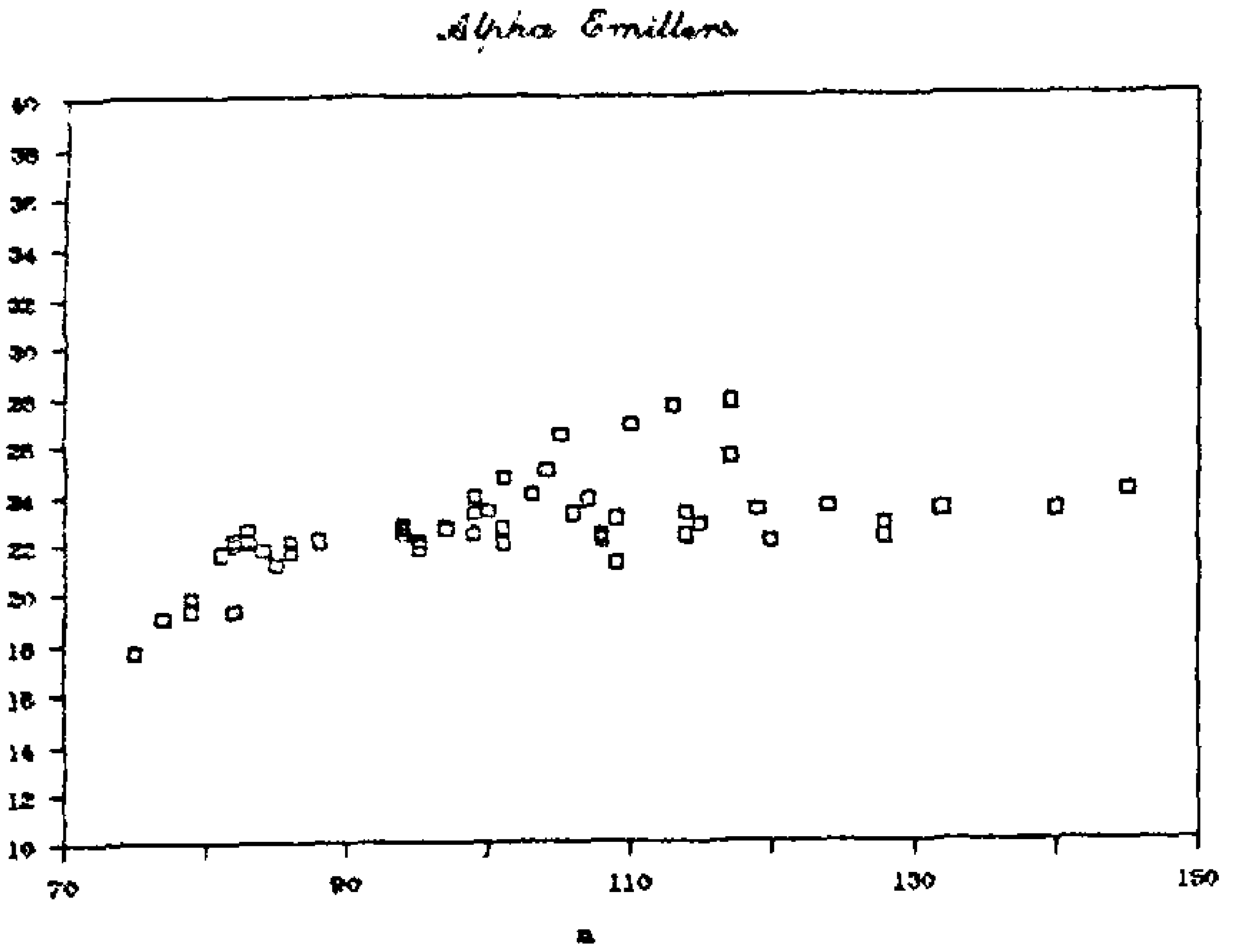

Figure 8. Plot of $\log \lambda_0$ (nuclear factor which appears in Gamow theory of α emission) vs n.

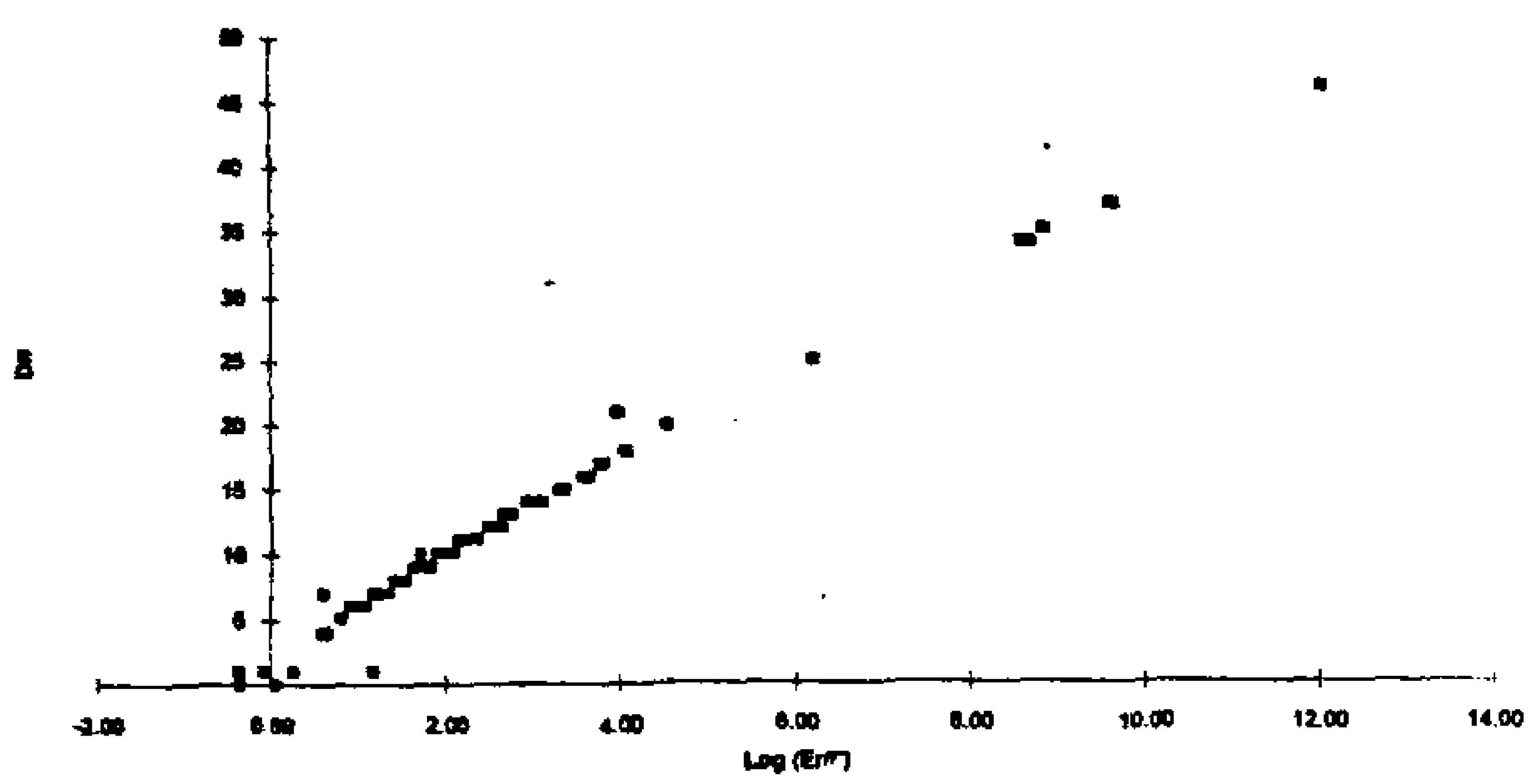

Figure 9. Plot of level width Γ vs p.

portions precisely corresponding to the energies of nucleon transitions from lower to higher states. In this section, we propose that the energy is absorbed by the nucleus in a discrete way in the form of virtual mesons, mainly the light ones. As a result, the system gets excited to a level whose width and energy depend on the mass and decay time of the meson involved in the process. The type of virtual meson depends on the external energy imparted to the nucleus.

Each excited level of a nucleus is associated with an elementary particle which is identified by comparing the ratio $\log(E_r/\Gamma)$ (ref. 17, Figure 9) for an excited level of energy E_r and width Γ with the $\log(MT/\hbar)$ for an elementary particle of mass M and lifetime T. In Figure 9, integer Dn is given by $p = \log(2^{Dn}/Dn)$. The quantity n for a resonance level is calculated from E_r/Γ by using eq. (1) which helps in the identification of the particle involved in the excitation process (see Table 5). The elementary particles involved are found to be light mesons. Figure 10 gives the number distribution of various n-values, which shows that it is the ω-meson that is mostly involved in the excitation process.

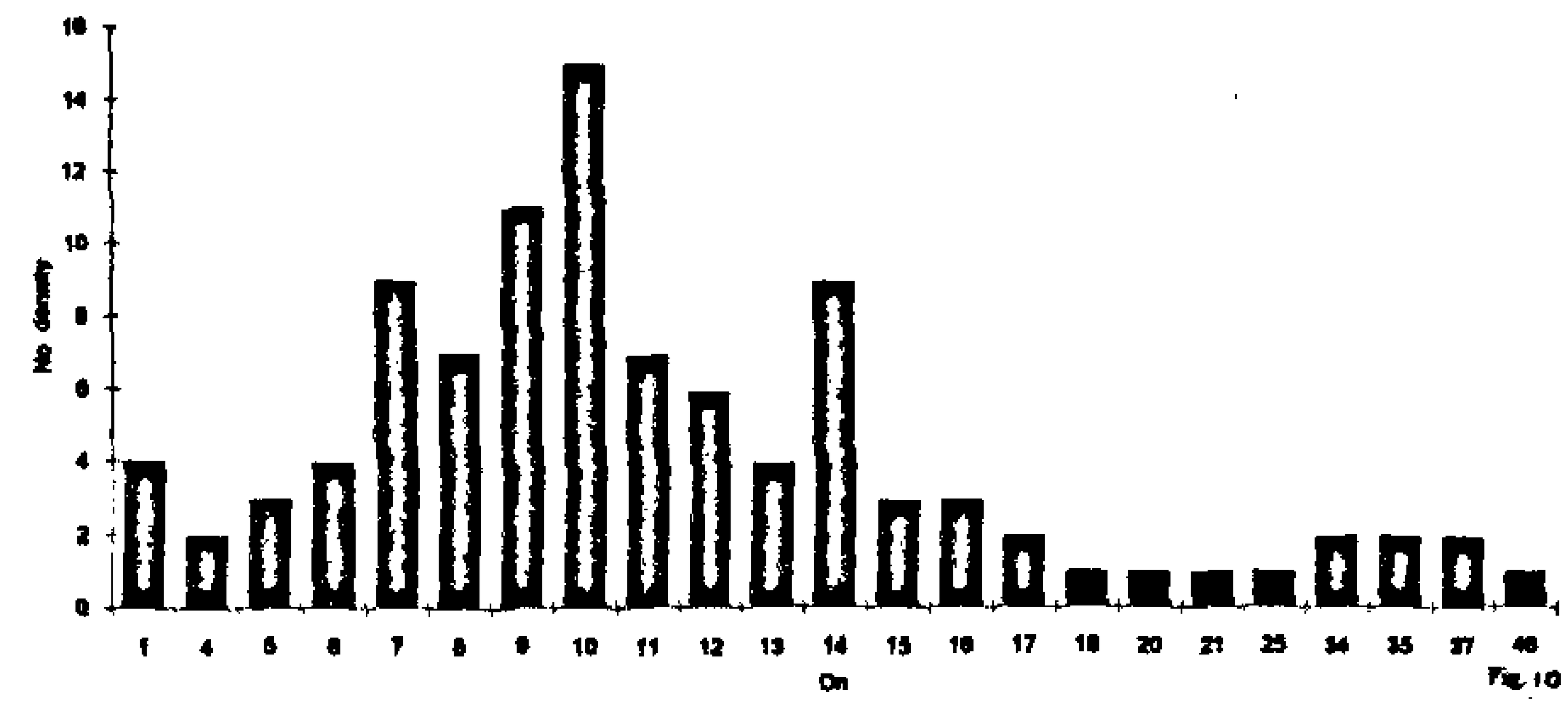

Figure 10. Histogram of Dn distribution.

Comparison of present formalism with other approaches

The present formalism which deals with duality in particle masses and lifetimes gives a discreteness in Γ/M ratios. This approach, of course, has not been attempted for the first time. Nambu, as early as in 1952 had written a paper on empirical mass spectrum of elementary particles[18] wherein he had suggested a discreteness of particle masses by taking electron mass as the basic unit. In other work by Mac Gregor[9] and others[19], the quantization of particle life-times was found in terms of the muon life-time. The present approach is unique in the sense that it incorporates two discrete quantities together in a unified framework without choosing a basis dependent on electron masses or muon lifetimes. Moreover, the present approach is more transparent which can bring out many more properties in a simple manner.

Mac Gregor's model[9] which predicts quantization in the lifetimes of the elementary particles represents a higher degree of degeneracy in the quantized levels compared to the present approach. As seen in the present text, the inclusion of particles masses along with their life-times thus suggests a duality between them bringing out hyperfine quantization for elementary particles. On the other hand, Mac Gregor's approach does not incorporate the particle masses in his formula[9]. It can be easily seen that the quantum levels in his formula correspond to a special case of present formalism. If we constrain the quantity p_0 to take only integral values, then the corresponding n-values obtained (see Table 2) have one to one correspondence with the quanta in Mac Gregor's approach[9].

Conclusion

It is seen that from the parameter n or p based on the lifetime and the energy available for the decaying system, one can arrive at several properties of fundamental particles, spin and parities of the β-emitters and the number of times the α-particle strikes the barrier of the radioactive nuclides before it is expelled. The binding energy of about 1300 nuclei can be obtained by an empirical formula based on the above parameter. It is also possible to get the systematics of the resonance levels of light nuclei, as a process in which short-lived fundamental particles play the role of a virtual particle for the transfer of energy.

The relation between this method based on cardinality and the usual quantum mechanical approach has yet to be worked out. We, however, feel that there will not be any inconsistency as waves and particles are a special form of continuity and discreteness.

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Determination of kinetic parameters and thermodynamic constant of antigen–antibody reaction by solid phase binding method

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Study of the dissociation of [^{125}I]hCG from immobilized MAb–[^{125}I]hCG complex was carried out. The dissociation data fitted into a two-step reaction mechanism of the interaction. Kinetic parameters obtained were consistent, and were used to obtain the thermodynamic constants of the interaction. The first step (epitope recognition reaction) was shown to be a high-affinity intermolecular binding reaction involving epitope–paratope interaction, while the transformation reaction (second step) is an intramolecular rearrangement of the [^{125}I]hCG–MAb complex formed in the first step. The method described has the potential to be automated.

STUDY of real time kinetics of ligand–ligate interaction has greater information potential compared to equilibrium kinetics in understanding the chemistry of the interaction. However such kinetic study was rather difficult[1–4] until the advent of BIAcore or BIOS-1 (refs 5–7). Even though these methods are extensively used, they are expensive and suffer from uncertainties of interpretation[8,9]. We have described another method for these investigations using immobilized ligand and radioactive ligate[10,11]. This method though easy and cost effective, has the potential for further simplification. We present here an improved procedure, and have utilized these results to calculate the thermodynamic parameters of the MAb–[^{125}I]hCG interaction. The feasibility of automating the method is also discussed.

Materials and methods

Human chorionic gonadotropin (hCG) purified from early pregnancy urine[12] was used throughout the studies. Monoclonal antibody (MAb) VM-4 was raised against