Role of symmetries in the sciences versus the arts – A contrast

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In recent decades, symmetries have been assuming increasingly greater importance in the sciences. In some fields like particle physics they have come to occupy the centre stage, while in many other sciences too, they play a valuable role. Meanwhile, since time immemorial, symmetries have been part of the aesthetic perception of man and are an integral part of many art forms. In this article we attempt to assess and contrast their roles in the sciences and the arts. While this is really a vast subject, given the diversity of the different arts, we will be content here with some broad generalizations, using illustrative examples.

As a prelude to writing this article, I decided to attempt a small exercise to see how popular the concept of symmetry had really become with present-day physicists. The exercise was to do a sample survey of how often the word 'symmetry' appears, as compared to those long-time favourites 'momentum' and 'energy', in the titles and abstracts of recent publications in, say, particle physics. Internet made such a survey possible, using as sample all the high energy physics papers submitted this year to the electronic bulletin board maintained by the Los Alamos Laboratory. Among all the papers submitted in 1997 till July, I found that the word symmetry appeared 209 times as compared to momentum which was mentioned only 143 times! The word 'energy' was still the winner with 456 counts, but who could have been anticipated 50 years ago that symmetry would appear more often than momentum and half as often as energy in titles and abstracts of papers!

Of course one sample does not a thesis make, and in any case this was meant only to be an amusing exercise, not a real index of the importance of symmetry. But people working in particle physics will testify that in their field, symmetries have grown in importance to the point where they occupy the centre stage. All the basic forces of nature at the fundamental level, namely, the nuclear, the electromagnetic, the weak and the gravitational forces are now described by the so-called Gauge theories, whose defining characteristic is a deep and profound form of symmetry. While particle physics is perhaps an extreme example in this regard, symmetries have played an important role in other branches of physics too, as for instance in the far-reaching use of crystal-structure symmetries in solid-state physics, in the 'spontaneous breaking' of symmetry in ferromagnets and superconductors or in the use of isotopic spin symmetry

to relate different nuclear isotopes, and of rotational symmetry to explain aspects of atomic and molecular spectra. In the biological, chemical and engineering sciences symmetries, while less ubiquitous, still yield important results.

Meanwhile, notwithstanding its emergence as a major weapon in many branches of science, the notion of symmetry really originates from the artistic sensibility of man. From the earliest times, when it began as a largely decorative component of visual arts, symmetry has throughout been playing a major role in the arts. Sometimes embraced and at other times contemptuously dismissed by *avant-garde* abstractionists, symmetry, through its presence or deliberate absence, has been a force not just in painting and sculpture, but also in music and poetry. It remains one of the few major features that is common to both the arts and the sciences. But the nature of its role is quite different in these two fields. We will attempt in this article to bring out this contrast.

What do scientists mean by symmetry?

Let us begin with science, and the technical meaning of the term symmetry as employed in science. In the case of some scientific terms borrowed from the English language, their technical meaning can be hilariously different from their lay meaning. (For instance, some energy levels of atoms are called 'degenerate' even though these atoms lead, to the best of our knowledge, a blameless existence!) Fortunately, this is not so in the case of 'symmetry', whose technical meaning is just a generalization of its day-to-day meaning, suitably abstracted and made precise through the use of mathematical language.

Imagine showing the two drawings in Figure 1 to any lay person, not necessarily a scientist, or an artist.

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He will agree that Figure 1 *a* is very symmetrical while Figure 1 *b* is not. This is the familiar day-to-day meaning of symmetry. In order to use it in science, all we need to do is to identify and make precise those features of objects which give rise to this intuitive notion of symmetry, classify and quantify different types of symmetry and so on. This can be illustrated by a few simple examples. Consider some letters of the English alphabet. The letter A, for instance, enjoys a reflection or mirror symmetry. That is to say, imagine a vertical line drawn across the middle of A. If we place a mirror on this verticle line and reflect the right half of A, we will see in the mirror precisely the left half of A, and vice versa. Similarly, the letter B and C also enjoy a reflection symmetry, not with respect to a vertical line but with respect to a horizontal line through the middle. The letter O has a 'higher' symmetry. It is symmetrical under reflections about both a vertical line and a horizontal line drawn through its middle.

The above were examples of reflection symmetry. There can be other forms of symmetry: under rotations, displacements, etc. Thus, a square is symmetrical with respect to (i.e. unchanged by) a rotation about its centre by 90 degrees. It is, however, not symmetrical if we rotate it by some arbitrary angle, say 37 degrees. The circle has more rotational symmetry than the square. It is unchanged by a rotation by any angle. To give an example of displacement ('translational') symmetry, consider a large corrugated sheet of asbestos – the kind that is used as roofing. As we know, it has a wavy surface (Figure 2). It has symmetry under displacement by one wavelength in the *x*-direction and by any arbitrary amount in the other flat (*y*) direction. That is, if you were an ant sitting at the top of one the ridges of that corrugated surface, and you moved over to any point on the top of any other ridge, the surface would look unchanged to you.

Abstracting from these examples, a system is considered to have a symmetry if it is unchanged upon subjecting it to a 'transformation', such as reflection,
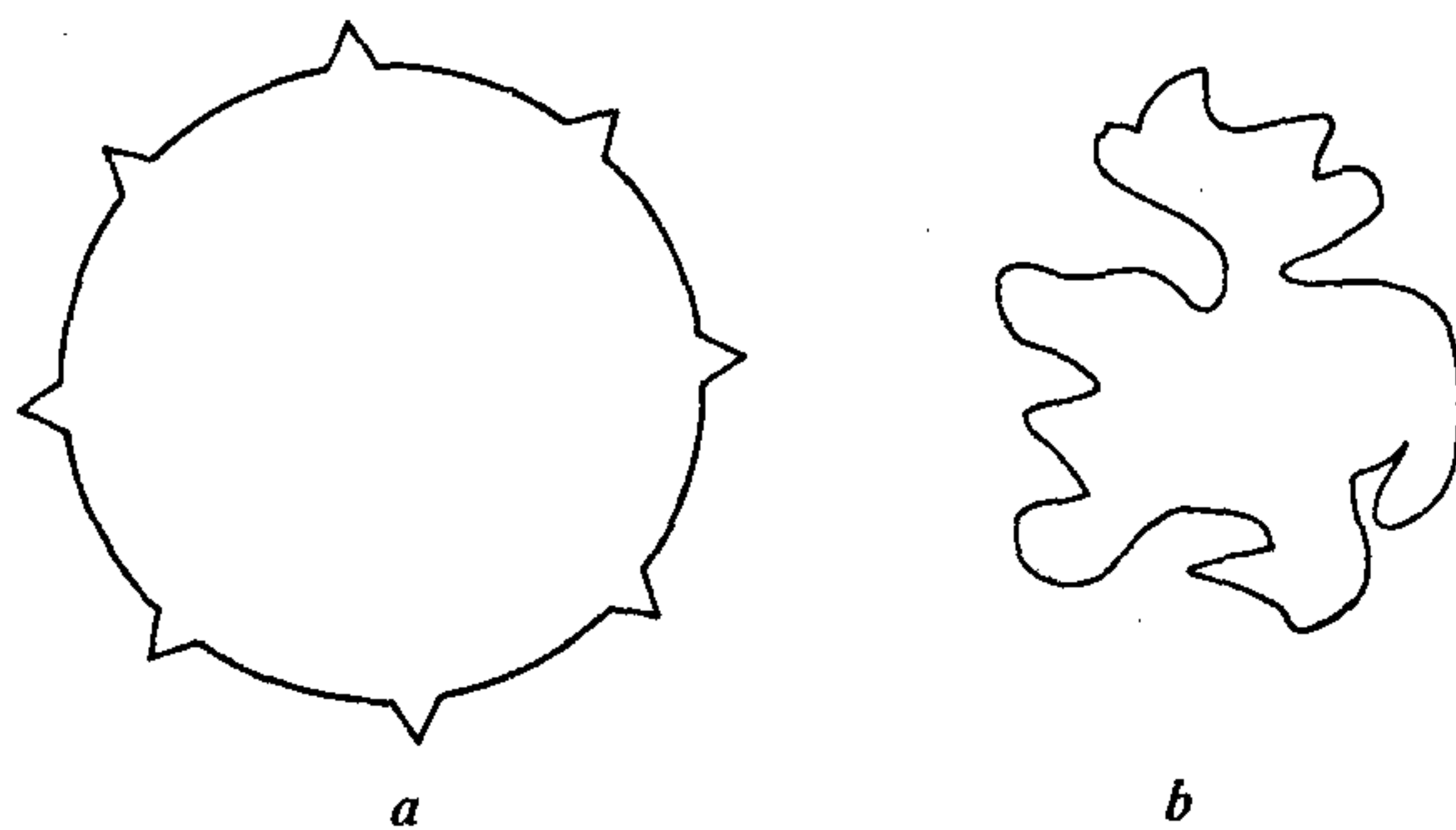
rotation or displacement. This notion can be put on a very precise mathematical footing, and the subject of Group Theory in mathematics deals with all this at great depth.

Why are symmetries important in science?

The important thing, from the point of view of real science, is that such symmetries are present not just in the mental constructs of mathematicians or in artificial man-made objects like letters of the alphabet and corrugated asbestos sheets, but in real natural systems of great interest to science. Thus, many solid substances are crystalline, i.e. their atoms are arranged in a regular 3-dimensional pattern, giving the solid a symmetry under displacement by specific amounts. Figure 3, for instance, shows a lattice of atoms with hexagonal symmetry. The widely used gas ammonia (NH_3) has in its basic molecule a reflection symmetry. The nitrogen atom in the molecule sits in one of two possible locations which are mirror images of each other, the 'mirror' being on the plane containing the three hydrogen atoms. The molecule of benzene has a very symmetrical hexagonal structure. The symmetries exhibited by elementary particles are of a more abstract kind, but very real nevertheless.

In short, symmetries are very much present in the natural world that science deals with. What is it, however, that makes them so important? Not the visual beauty of symmetry but rather its enormous usefulness in making scientific predictions. The systems that science has to deal with are so complicated that to calculate or predict all aspects of their behaviour accurately is exceedingly difficult, even if one knew the underlying basic laws that govern them. Faced with this situation, the simplification brought about by presence of symmetries helps extract at least some facets of their behaviour with comparatively less labour. Such simplification can come about either because of the symmetry of the object of our interest, or because of the symmetry of its environment. To get some feel for the latter situation, let us go back to ants on the corrugated asbestos sheet of Figure 2. Supposing there are two scientist ants living on that sheet (shown by two dots in Figure 2) planning to wrestle with each other for some institutional supremacy. As part of training for this fight, they may have to learn to deal with the ups and downs and slopes of the terrain under their feet. But if one of those ants is aware of the symmetries of the surface he does not need to practice all over the entire terrain. Familiarity with a one-wavelength domain will do. As the wrestling ants stumble from one ridge to the next, the smart ant will know that the terrain will merely repeat itself and, as they say nowadays, information is power!

Of course ants living on a corrugated roof may not be of great interest to everyone. But replace the

Figure 1. Two figures. Any layman will concur that while Figure (a) is symmetrical, Figure (b) has no obvious symmetry.

corrugated sheet by a crystal lattice and the ants by electrons and you have a very similar situation in miniature. Again you can get, for essentially the same reason, much information about electronic behaviour in that solid just by invoking the symmetries of the lattice, even though one has to shift to quantum theory in going from ants to electrons. Similarly, the mere fact that a crystal has a periodic array of atoms is enough to prove that when X-rays or neutrons bounce off them, they will form nice patterns which can be predicted. The fact that the ammonia molecule has a reflection symmetry (mentioned earlier) implies that its energy levels will come in closely spaced pairs. The fact that photons have zero rest mass follows directly from the so-called gauge symmetry of quantum electrodynamics. Even the symmetries of empty space by itself, under displacements and rotations, leads to the all-important conservation of momentum and angular momentum. And so on.

Apart from this invaluable utilitarian side of symmetries, there is also an aesthetic side to their appeal to scientists. This aesthetic beauty is of a theoretical and mathematical kind. The process of obtaining these predictions from the presence of symmetries is not a laborious nit-picking task. Rather, very compact and elegant ideas are involved, which is another reason why scientists like symmetries.

This article is not the place to establish the detailed connection between the presence of symmetries and their consequences, in the various examples mentioned above and the many others like them. That will require the setting up of much theoretical machinery and is the stuff of textbooks. Most readers of *Current Science* will in any case be familiar with details of how symmetry helps in their field. For the others we have tried to give some feel for what scientists mean by symmetry. Hopefully we have also made it plausible, at a very qualitative level, that since symmetries imply some sort of a pattern in the structure of a system, they should also lead to some pattern in the behaviour of that system. Consequently, for both utilitarian and aesthetic reasons, more the symmetries the better as far as science is concerned.

Symmetry in the arts: Is more better?

Now let us turn to the arts. Do we welcome symmetries unilaterally in all forms of art, whether it be music, poetry, painting, or sculpture? Is the cry 'The more symmetries the better' echoed in the world of arts too? Let me begin this part of the discussion with a somewhat extreme example. In Figures 4 and 5 I show two segments of musical score. From a purely visual point of view, Figure 5 is very haphazard and untidy. It looks as if a fly has stepped out of an inkpot and wandered all over the page. This score has nothing very symmetrical about it. In contrast, look at Figure 4. It is visually very symmetrical. In the physicist's sense, it has all kinds of symmetries. You could even hold it upside down and musically it is unchanged! Yet there is no question as to which, musically, the greater piece. In fact there is no comparison between the two! Figure 5 is from *Hammerklavier Sonata* in B-Flat major by
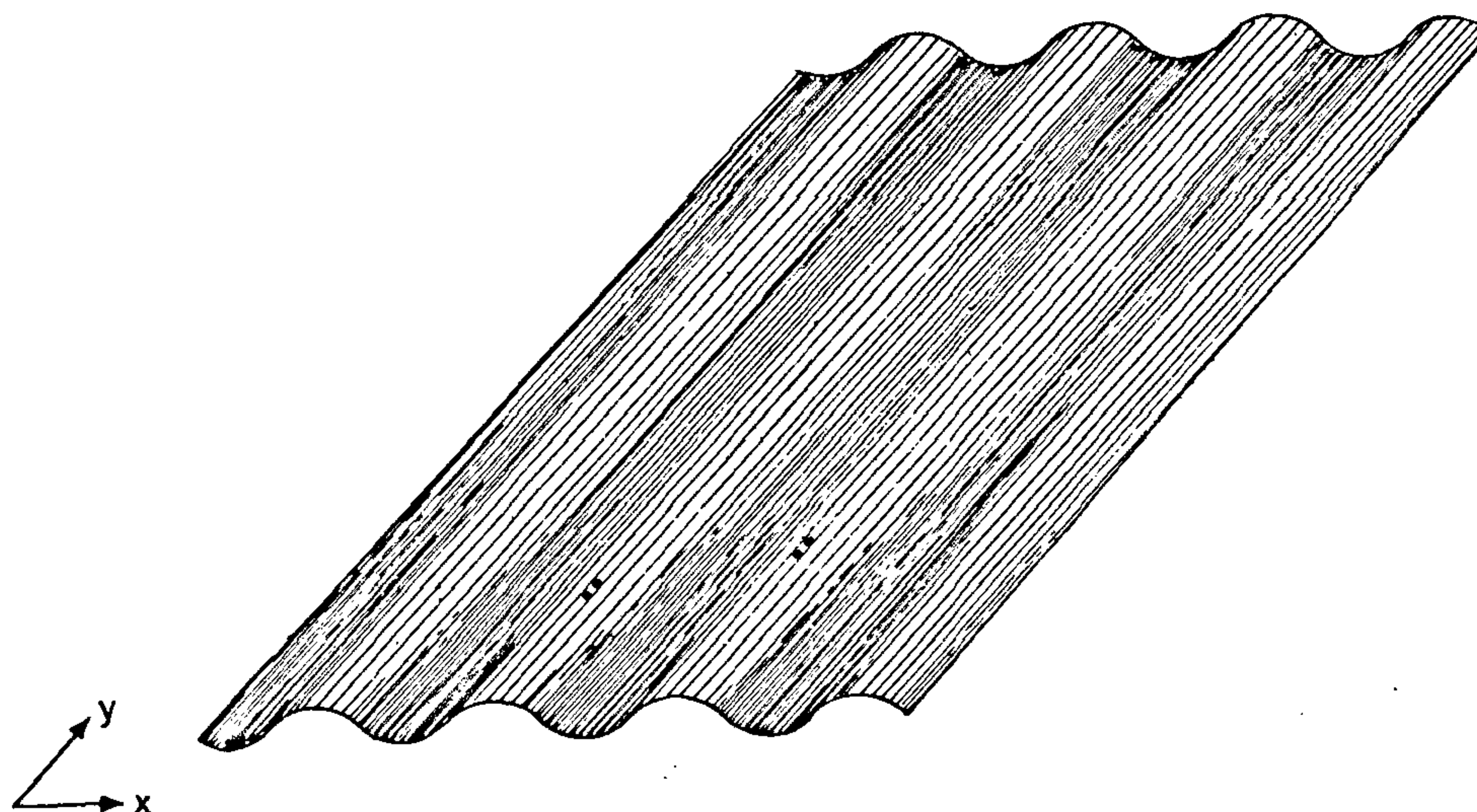

Figure 2. A corrugated asbestos sheet of the type used to make roofs of sheds. It is wavy if we move in one direction (along the x-axis) and flat if we move in the perpendicular y-direction. The two dots near one another represent a pair of ants. They are shown standing at two different locations on that surface.

Beethoven (Opus 106), one of the greatest works for the solo piano. Figure 4 is totally trite as a piece of music despite all its symmetries. I had 'composed' it myself just to make this point.

This example was meant to illustrate, although admittedly in a flippant and exaggerated way, the fact that symmetry is not the sole criterion, or even a major criterion in distinguishing good art from bad. If one went about trying to create a work of art by maximizing its symmetry content, it would lead to ludicrous results. I do not, however, mean to imply that symmetries have nothing to do with the arts or that they have found no place in the arts. In fact, symmetries have appeared in art forms from time immemorial, long before they began playing such a major role in science and mathematics. Further, one of the early entrance points of symmetry in science and engineering was through a medium that is a blend of art and science, namely, architecture where symmetry had a role to play not just in the aesthetics of the building but also in its structural stability and engineering feasibility. Nevertheless, as both the sciences and the various arts – music, painting, sculpture, archi-

tecture, poetry and literature evolved over the centuries, the status of symmetries in these two branches of human activity has come to be quite different.

To analyse and understand the role of symmetries in the arts, and to see how it varies with the level of sophistication of the particular type of art, it may be useful to first consider the utilitarian aspect, as we did for the sciences. Art *does* have its utilitarian side at its simpler and more primitive level, where it aims to please, 'to soothe the savage breast', to be appealing and attractive in the superficial sense.

Thus nursery rhymes for children are designed to be very symmetrical. 'Mary had a little lamb ...', 'Twinkle twinkle little star ...' and so on are characterized by couplets with repetitive tunes, a clear rhythm, meter and endings that rhyme neatly. They enjoy a high degree of symmetry. To give a 5-year-old child a diet of Sylvia Plath or E. E. Cummings would not 'serve the purpose'. This holds not only for children but for many adults as well. If we could make a quantitative statistical analysis of all the poems written in preceding centuries, we would probably find that a substantial fraction of them were composed by courtiers in honour of kings, noblemen, feudal landlords and such. Barring exceptions, these latter worthies wanted simple, nice-sounding rhymes with the laudatory content not obscured by undue abstractions. Simple elements of symmetry gave these patrons the satisfying feeling that they were cultured connoisseurs of art. In airports and cocktail lounges,
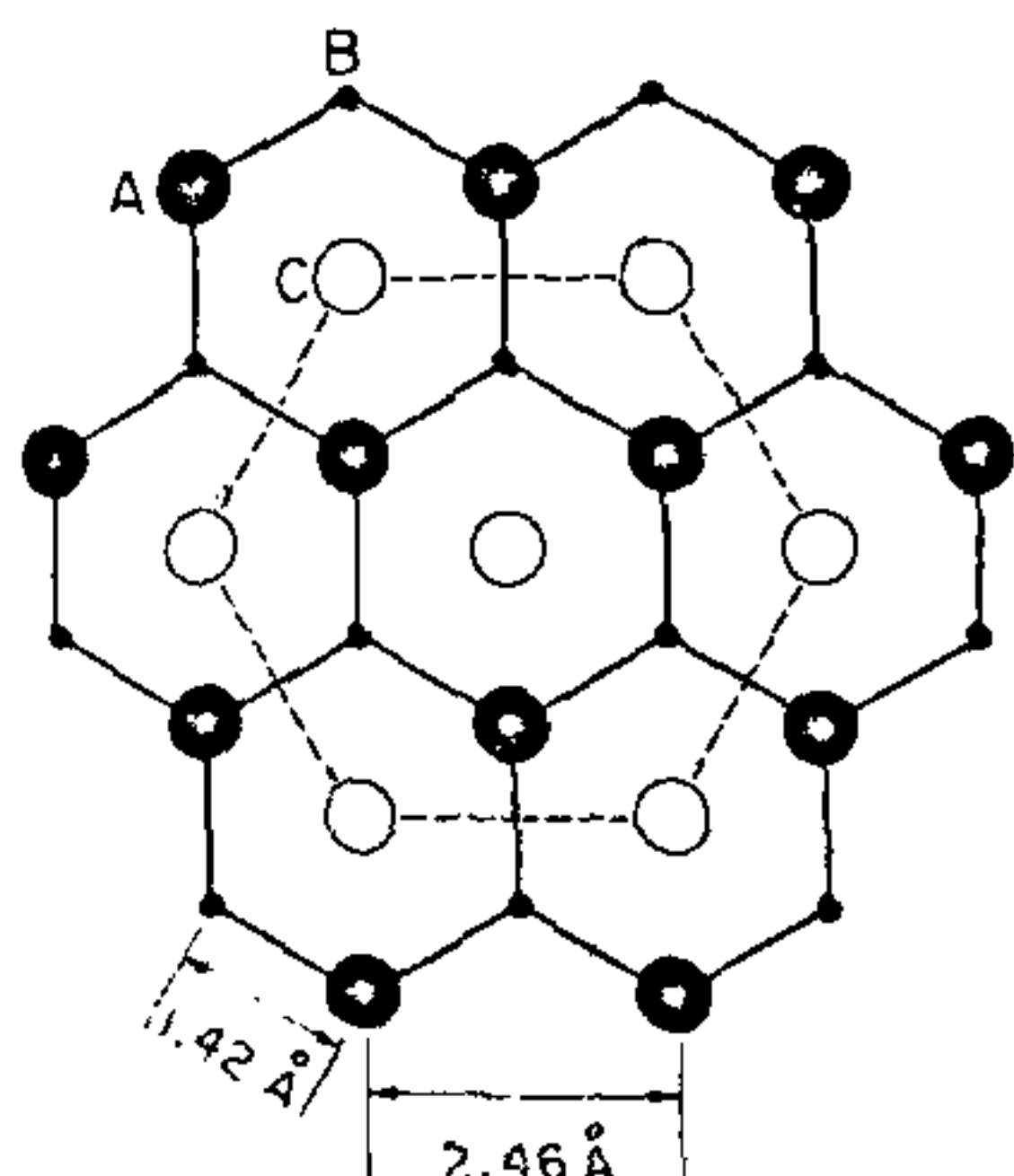

Figure 3. Arrangement of atoms in a hexagonal lattice.

Figure 4. A piece of music composed for the piano by the author especially for this article. Notice how symmetrical it is! The same bar of music repeats itself and the left hand plays just an inversion of the right hand notes. In addition, the whole score is designed to look the same even if held upside down. Musically of course it is terrible!

Figure 5. The last page of the score of Beethoven's *Hammerklavier* Sonata for the piano in B-Flat Major (Opus 106).

tired customers need to be fed simple soothing music with a high degree of symmetry rather the abstractions of Ustad Amir Khan's *khayals* or Beethoven's quartets.

Similarly symmetry plays an important role in the decorative arts – for instance in Kolam – the decorations drawn every morning on the thresholds of traditional South Indian homes, and in the borders of sarees and carpets. Sarees printed with Picassoesque themes, apart from being expensive to produce, would not sell anyway.

Symmetries as backdrop

I do not mean that symmetries have a role to play only in the softer decorative arts. Even great works of art can contain some elements of symmetry. Let us analyse a few examples. Let me start with the exquisite sculpture of the dancing Nataraja so familiar to us in India (Figure 6). Much of the significance of this sculpture comes from its religious and spiritual symbolism. It would be fascinating to go into those aspects of this work, but in the context of this article we are mainly interested in it as a great piece of art containing elements of symmetry. We can see that it has a very symmetrical, perfectly circular frame, with the flickering fingers of flame arranged around it at regular angular intervals. However what makes this sculpture great as a work of art is not the symmetrical circular frame but rather the central figure of Nataraja placed in it. This figure of Nataraja, although beautiful and delicately balanced, is not symmetrical in the mathematical sense. Nevertheless it is the most powerful element of the sculpture, slender

in form but dynamic in the tension of its twisting hips and the gracefully crossed balance of its arms and legs. Of course, the symmetrical backdrop does have its important artistic role. The flaming circle greatly enhances, by providing just the right type of backdrop, the appeal of the dancing figure placed at its centre.

As another example, consider the sculpture by Michelangelo in the New Sacristy in the church of San Lorenzo in Firenze (Figure 7). It graces the Tomb of Guiliano and we see senor Guiliano, presumably a Duke of some sort, in the middle. (The story goes that someone complained to Michelangelo that the face of Guiliano in the statue did not particularly resemble the original. He is supposed to have retorted, 'In a few centuries who will know?' It was of course self-evident to him that his sculptures will be admired for centuries, long after the Duke has been forgotten!) Returning to our subject, the symmetric elements of this sculpture are evident at first glance. The entire work is embedded in a symmetrical framework of arches, panels and columns. Guiliano is smack in the middle, with the two reclining figures of Night and Day located symmetrically just below. However, while the symmetry of whole arrangement is certainly pleasing to the eye, what gives the entire work its power is *not* its symmetrical panels and columns but rather its *asymmetrical* dynamic forms. On a closer look we can see that the reclining figures of Day and Night are far from identical. The graceful languor of Night is to be contrasted with the twisting torso of Day. Guiliano himself is not sitting in a very symmetrical fashion, facing front and with his arms

Figure 6. The dancing Nataraja. He needs no introduction.

Figure 7. The Tomb of Guiliano by Michelangelo in the New Sacristy of the church of San Lorenzo in Florence, Italy.

placed nicely on either side, the way people do in group-photographs. If he were, and if Day and Night had been represented by identical statues, the symmetry content of the sculpture would be higher, but would it have been anywhere nearly as good a work of art? Rather, the role of the symmetrical elements of this sculpture is primarily to provide the backdrop which brings out, through contrast, its dynamical non-symmetrical components.

This role of symmetry as a contrasting background for displaying all the better the more exciting dynamic innovations, seems to be common to several art forms. The use of percussion in certain forms of Indian classical music is another example. When Kumar Gandharva sang his bhajans, the tabla generally maintained a simple and repetitive rhythm (symmetrical in time), forming the background against which he soared in the most unpredictable fashion. Great Indian poets like Kalidasa or Kabir composed their work in well-structured forms paying great attention to meter and rhyme. Similarly, alliteration often appears in Tamil poetry. An analysis of such forms will certainly reveal the presence of some symmetries. And no doubt these symmetries contribute to the overall impact of the work, through the resonance of their rhythmic cadences. But I would like to submit that such symmetric elements are not the decisive factors in making these works great. After all, for every Kalidasa or Kabir, there were any number of minor poets contemporary with them, who also adopted the same verse forms with the same meter and rhythm. The symmetry content of these minor poems would have been the same as that of the great poems, but that by itself did not help elevate these minor poets to levels of immortality.

To take another example consider the fugues of Bach, which are among his great contributions to western classical music. To hear Bach's Toccata and Fugue in D minor, played by E. Power Biggs on the Organ can be on overpowering experience. Now, fugues have a very distinctive symmetry to them. One does not have to search very hard to see the symmetry, which is quite explicitly visible in the musical score (Figure 8). One of the voices, say, the soprano or the first violin is given a theme. As this theme is progressing, another voice, say the cello, starts the same theme, and a little later yet another voice. Soon there are several voices, all singing the same theme, but displaced with respect to one another in time. Thus fugues carry some sort of a displacement symmetry. Of course the genius of the composer lies in choosing the notes of his fugal theme in such a way that when different voices simultaneously sing different portions of it, it nevertheless sounds not only harmonious but profound. But once again, while this particular symmetry of fugues adds to the pleasure

Figure 8. Opening bars of a Bach Fugue.

they give, and exhibits the composer's virtuosity, the greatness of Bach's fugues does not come from the symmetry alone. Many lighter forms of Western music (the so-called 'catches' or 'runs', which are sung in beerfests or in girls' boarding schools) also have the same sort of symmetry, but would not be considered great.

In short, while in the lighter art-forms symmetry plays a major role, in the more sophisticated art forms, it is only one more trick in the artists' bag of tricks. He may employ it as backdrop, as an expression of his virtuosity and so on. But it is certainly not true that the greater the symmetry in it, the greater the value of the work. Indeed, much of the art of the past century, whether it be music, poetry or painting, has turned abstract where any type of symmetry is viewed with contempt unless it is used for ironic effect!

Symmetry is, at some level of aesthetics, beautiful. It is therefore sad to have to conclude that its role in the arts is often secondary. However, such a judgement can be tempered by one final, important remark. At a very abstract and deep level there is another sort of symmetry present in every successful work of art. It is something like a reflection symmetry between what is present in the work of art and what lies somewhere in the psyche of the viewer/listener. Art achieves greatness by reflecting the innermost crevices and darkest corners of our soul. Something in the work, built in there by the artist, mirrors some elusive and inarticulable aspect of our feelings and is responsible for eliciting our response. It is perhaps stretching the definition of symmetry quite a bit to include in it this conjugacy between the work of art and its audience. But if we may be permitted the artistic license to do so, then this symmetry is one that is indeed central to all art. In that sense symmetry becomes as crucial to art as it is to the sciences.