Stochastic Partial Differential Equations: A Modelling, White Noise Functional Approach. H. Holden, B. Oksendal, J. Uboe and T. Zhang. Birkhaüser Verlag AG, Postfach 133, CH-4010, Basel, Switzerland. 1996. 231 pp. Price: DM 118, SFr 98.

Partial differential equations arise in modelling various physical problems when the functions representing certain physical quantities depend on two or more independent variables. These equations are usually derived using certain physical laws. The governing laws lead to a certain functional form of the equations but these equations contain several parameters. Such modelling usually requires that the parameters involved are completely known. This is an ideal situation which may not be realizable in practice. Due to various uncertainties in the environment, measurement errors, lack of information, etc., it is very difficult to find out the 'true' values of the parameters. In such situations the system cannot be determined completely. A standard way of circumventing this difficulty has been to replace the 'true' values of these parameters by some kind of average and hope that the behaviour of this averaged model is 'close' to that of the original one. This works in some cases while in several cases the behaviour of the averaged model may be fundamentally different from that of the original one. There may be some systems which are governed by probabilistic law of motion. The issue, whether randomness really exists in this physical world or it manifests itself due to lack of information, is of philosophical nature; it is certainly true that probabilistic (or stochastic) models quite often lead to a better understanding of many physical phenomena. Stochastic models are all pervasive today in almost all branches of science.

Stochastic analysis is the main tool in treating the problems of stochastic partial differential equations (SPDE for short). There are two fundamental objects in stochastic analysis: Brownian motion and white noise. Brownian motion is a stochastic process having continuous sample paths and with independent increments which are normally distributed. Due to the (functional) central limit theorem Brownian motion represents the limiting behaviour of a large class of stochastic processes. Hence, it plays a central role in stochastic analysis. Roughly speaking, white noise is a stochastic process which is independent at different times and is identically distributed with zero mean and infinite variance. Due to this distinct feature it is treated as a good model for a random noise having large fluctuation. It is formally treated as the time derivative of a Brownian motion. Since almost all Brownian paths are nowhere differentiable, this formal definition of white noise runs into technical difficulty from a mathematical viewpoint. Nevertheless, this is popularly used in the literature of physical and engineering sciences and has led to the development of several important stochastic models. However, due to its lack of rigour it is desirable to develop a mathematical theory which would not only make the analysis of the existing stochastic models sound but would lead to new results as well. To this end Ito[1] developed the theory of stochastic integrals with respect to Brownian motion. He bypassed the notion of white noise completely. Though Ito's stochastic calculus is one of the most useful branches of mathematics with far-reaching applications, it has certain limitations too. In Ito's integral one requires that the integrand is nonanticipating with respect to the Brownian motion. Thus Ito's calculus is not applicable to anticipating SPDE. One way to extend the theory of stochastic integrals to the anticipative case would be to involve the white noise as a part of the integrand. One of the first requirements in this direction would be the development of a rigorous theory of white noise. Gelfand and his collaborators[2] made an important contribution towards this. Since almost all Brownian paths are continuous, it can be regarded as generalized stochastic process. Then the (generalized) derivative of Brownian motion exists and may be treated as white noise. This definition of white noise has played some positive role in stochastic analysis but there are some limitations in so far as its application to SPDE is concerned. Generalized functions are defined as continuous linear functionals on certain test function spaces; thus there is a difficulty in defining the product of generalized functions. Balakrishnan introduced a new theory of white noise based on finitely additive measures which has been extensively developed by Kallianpur and Karandikar[3]. In the seventies Hida[4] introduced a theory of white noise. Since white noise is an analogue of independent and identically distributed random variables, it can be thought of as a coordinate system on a certain infinite dimensional space. Indeed, let $\mathcal{S}$ be the space of infinitely differentiable functions rapidly decreasing at infinity and let $\mathcal{S}'$ denote the space of tempered distributions. Let $\mathcal{B}$ denote the Borel σ-field on $\mathcal{S}'$. Then there exists a probability measure μ on $(\mathcal{S}', \mathcal{B})$ such that

$$E[\exp (i \langle ., \theta\rangle)] = \exp (-(1/2) \| \theta\|^2),$$

for all $\theta \in \mathcal{S}$, where $\|.\|$ denotes the L^2-norm. The space $(\mathcal{S}', \mathcal{B}, \mu)$ serves as the basic probability space. The coordinate map $w : \mathcal{S} \times \mathcal{S}' \to \mathbb{R}$ given by

$$w(\phi) = w(\phi, \omega) = \langle \omega, \phi\rangle, \quad \omega \in \mathcal{S}', \phi \in \mathcal{S},$$

$\langle , \rangle$ denoting the L^2 inner product, defines the white noise. One can then define the white noise process using the shift operator. Brownian motion in this framework becomes the integral of the white noise. This definition of Brownian motion generalizes the usual one. Since Brownian motion appears as the integral of the white noise, this approach provides an intrinsic way to define stochastic integral without the nonanticipating condition. During the last two decades this white noise theory has evolved as an infinite dimensional distribution theory. Its space $(\mathcal{S})$ of test functions is the infinite dimensional analogue of the Schwartz space $\mathcal{S}$ and the dual space $(\mathcal{S}')$ the infinite dimensional analogue of the space $\mathcal{S}'$ of tempered distributions. The rapidly increasing domain of applications of this theory includes stochastic (ordinary and partial) differential equations, stochastic variational equations, infinite dimensional harmonic analysis, Dirichlet forms, quantum probability, etc. The book under review deals with the white noise theory and its applications to SPDE. The authors have presented a self-contained theory of white noise. Its starting point is Hida's theory but it goes much further in this direction covering the major developments in the past two decades. There are three central ideas in the theory: Wiener–Ito chaos expansion, Wick product and Hermite transform. Wiener–Ito chaos expansion enables us to represent every square integrable function on the basic probability space as a series in terms of certain stochastic Hermite functions. Wick product substantially broadens the applicability of the theory. The Wick product of two elements F, G in ($\mathcal{S}'$) is expressed in terms of a (formal) chaos expansion with coefficients which are inner products of the coefficients in F and G. This product generalizes the usual product and satisfies all the ordinary algebraic rules of multiplication. Thus one can carry out the calculations in much the same way as with usual products. Problems arise, however, when limit operations are involved. To get rid of this difficulty, one defines the Hermite transform which converts Wick product into ordinary (complex) product. This transformation converts an element in ($\mathcal{S}'$) into an element in the space of complex numbers. It plays the most crucial role in its applications to SPDE. The general approach to solve SPDE is as follows. Interpreting all products in the SPDE as Wick product take Hermite transform to convert the SPDE into a deterministic PDE with complex coefficients. If the transformed equation has a 'nice' solution, then by taking inverse Hermite transform one obtains a solution of the original SPDE. This general methodology has been applied to solve several SPDE of practical importance: stochastic Poisson equation, stochastic transport equation, stochastic Burger equation, etc. The methodology seems to be very powerful in handling linear SPDE and certain quasilinear SPDE, but it seems to have its limitations in dealing with fully nonlinear SPDE.

Apparently the book is based on the research work carried out by the authors to investigate the flow of fluid in reservoirs undergoing random fluctuations. Their endeavour has certainly added a fresh lustre to the theory of SPDE. It is therefore heartening to note, in conclusion, that through their work the authors have re-established the fact that the two forms of human activities, namely the following of the events in the physical world and imagination keep on crossing each other's path. The history of mathematics is full of such instances. This book adds, albeit in a small way, to these cases.

1. Ito, K., *Proc. Imp. Acad. Tokyo*, 1944, **20**, 519–524.
2. Gelfand, M. and Vilenkin, N. Y., *Generalized Functions*, Academic Press, New York (English Translation), 1964, vol. 4.
3. Kallianpur, G. and Karandikar, R. L., *White Noise Theory of Prediction, Filtering and Smoothing*, Gordon and Breach, Basel, 1988.
4. Hida, T., *Brownian Motion*, Springer, New York, 1980,

MRINAL K. GHOSH

Department of Mathematics,
Indian Institute of Science,
Bangalore 560 012, India

Annual Review of Nuclear and Particle Science 1996. Chris Quigg Assoc. Vera Lüth and Peter Paul, eds. Annual Reviews Inc., 4139 El Camino Way, Palo Alto, CA 94303-0139, USA. Vol. 46. 658 pp. Price: US$ 67, Elsewhere $72.

. . . we have to keep running to stay where we are, says the white rabbit to Alice.... In an effort to keep track of the principal developments in nuclear and particle sciences, one looks forward to sources of good review articles, that can critically examine the current advances. The *Annual Review* sequence continues to be an important source. In an era, when the active physicist has no time or inclination to stop and write a critical overview of developments, Chris Quigg has accomplished a very difficult task in both the choice of topics to be reviewed and the authors to carry out the assignment.

I enjoyed the delightful history, the march of particle physics over the last five decades brought out in the prefatory chapter by Sam Treiman. From QED that was firmly established as a precision theory in the 50s to the Standard Model as the comprehensive theory of the day and then beyond is quite a breathtaking journey. This includes three stages; "a heroic era of rampart experimental discovery and theoretical casting about, dating from the immediate post-war years to the late 1960s; a triumphant middle period in which the Standard Model is formulated, passes critical tests and establishes as a serious dynamical theory by about the middle of 1980s; and 'the what next era' of grand unification, supersymmetry, superstring...". There is a brief excursion into the affairs of JASON, which is referred to by Sam Treiman as a technical consultant group composed of university scientists interacting with the agencies of the US government, chiefly on security matters. There was a considerable controversy in those days on the propriety of the involvement of eminent scientists in the covert war efforts in Vietnam and elsewhere. Nevertheless, Treiman makes a case that it is quite purposeful for the innovative scientists, with tremendous skills for complex problem solving, to constitute themselves as an unconstrained think tank for the society. Should there not be JASON here, as India gropes into the 21st century carrying with it a reservoir of enquiring tradition and a halo of a glorious past on the one hand and the glitter of modern science on the other that is yet to be absorbed in our native psyche?

The main event in the nuclear and particle science for 1996 is clearly the sighting of the top quark, believed to be the last of the species of quarks. As a fundamental elementary constituent it is unexpectedly heavy. An overview of the two experiments detailing the top quark measurements and a perspective and prospects for the future is given by Stephen J. Wimpenny and Brian L. Winer.

The standard model that describes all basic forces, save gravity, is made up of two components: the electroweak part and the QCD; and the emphasis in the *Annual Review* is clearly on the QCD part. QCD is tested in three different regimes; (1) perturbative probes using deep inelastic scattering; (2) quasi nonrelativistic effects in the spectra of heavy quarkonia; and (3) the quark gluon plasma phase revealed in the high energy heavy ion collisions. The status of developments in each of these three modes is found in this volume of reviews. The technical developments in the task of next to leading order QCD computations are straightforward in principle, but rather complicated in practice. The progress in this makes use of the lessons we have learned from the String Theory that effectively organizes several independent Feynman diagrams into just a few, making yet another use of the principle of duality. This represents one of the progresses in one loop computation according to Bern, Dixon and Kosower. The rate of production of various charmonium states at Tevatron collider was several orders of magnitude larger than the best theories a few years ago. This had called for a revision of notions to be incorporated in the non-relativistic QCD and the review by Braaten *et al.* summarizes the theoretical developments and draws up the consequences for the heavier quarkonia that remains to be analysed. But the real puzzle of the hadronic world is that we have a beautiful theory in QCD, however, with the basic entities of which remaining hidden in the phase in which it is immediately encountered. It is believed that there is another phase in which these basic ingredients are explicit and hence the theory must be more directly relevant. The quark–gluon plasma phase is expected to be realized in the high energy heavy ion collisions and we must be able to see a glimpse of the matter as it existed in the first tiny fraction of a second after the big bang. The unambiguous signal that recognizes such a phase has been sought keenly for more than a decade of experiments at AGS, Brookhaven. So far we seem to have drawn a blank. What are the prospects at RHIC (Relativistic Heavy Ion Collider) to be commissioned soon and at LHC (Large Hadron Collider) that will be ready in the first decade of the next century? Kahana *et al.* and Harris and Müller point out that a much larger canvas in the parameter space (density and temperature) will be probed in the future, so that we may hope for a clearer picture.

QCD is also probed in the spectra of mesons and the non-leptonic decays of charm and beauty carrying particles. The central issue remains the confirmation of the glueball (flavourless, quarkless state) and the hybrid states that may be accessible in the low energy machines. There is a wealth of data that are being accumulated and there is much physics to be understood. Chiral perturbation theory is recognized as a valuable tool in the analysis. While the articles on these topics in the *Review* are rich with information, the reader, I am afraid, is bewildered if he is looking for a direction.

There is no review on either the theoretical developments or experimental expectations on supersymmetry. Nor is there any mention of the recent excitement on the new symmetries observed in string theory. Is there an unwritten convention that what is beyond the Standard Model is also beyond the *Annual Review of Nuclear and Particle Science*?

R. RAMACHANDRAN

The Institute of Mathematical Sciences, Taramani, Chennai 600 013, India

---

Experimental Techniques in Bioelectrochemistry. V. Brabec, D. Walz and G. Milazzo, eds. Birkhaüser Verlag AG, Postfach 133, CH-4010, Basel, Switzerland. 1996. Vol. 3. 558 pp. Price: not known.

Bioelectrochemistry represents one of the earliest interfaces between biology and chemistry and it is now acquiring a fashionable label. This volume is the third in a series initiated by the late G. Milazzo who has contributed significantly to the recognition of this area.

The book consists of nine chapters contributed by various experts in their respective areas. As often happens with such an enterprise, the quality is variable both in terms of style and substance. The material contained in the book can roughly be divided into two classes. The first represents basically electrochemical techniques which can also be applied to biological systems and the second represents techniques resulting from a synthesis of electrochemical and biological principles. More than half the book is devoted to the former class of techniques. Unfortunately, this part suffers from comparison with the excellent books on electrochemical techniques which are currently available. This is particularly true of the chapter on the electrochemical impedance technique. The authors have mostly referred to older texts and research papers from the 60s and 70s. Rapid advances in instrumentation and software for data analysis have taken place since then and it is a pity that no reference is made to these in terms of recent books and papers. This chapter also suffers from the fact that the authors do not describe any examples of application of electrochemical impedance to biological problems. This will leave the reader clueless about what to do with this technique.

About 200 of the 500 odd pages are devoted to two chapters on voltammetry. The authors have included a lot of avoidable maths which sometimes also suffers from typographical errors. Again there are no significant examples of applications in biology. It is nice to see a chapter on biosensors which is a rapidly growing area of application which provides an interface between electrochemistry and biology. Though the chapter is well-documented and supported by more than 400 references, only a handful are later than 1990 and none after 1992. One can imagine there was a long wait between the writing and publication of the book in 1996. The chapter on *in vivo* use of microelectrodes is interesting and contains some useful experimental details. The authors of this chapter have devoted space to theory of voltammetry which is a repetition of concepts already presented. This space could have been better utilized for a detailed discussion of some applications. The review of spectroelectrochemical techniques is good and is accompanied by numerous examples, some taken from the authors' work. The book ends with a very readable chapter on the patch–clamp technique. It contains a good mix of experimental details and applications to entice one to look at the technique more seriously.

The book can be recommended as a good starting point for a biologist who is interested in using electrochemical techniques. Though some of the chapters contain a heavy dose of maths describing mass transport and electron transfer processes, the beginner can easily skip these sections without being handicapped. One hopes it will help to make bioelectrochemical techniques more popular in experimental biology.

A. Q. CONTRACTOR

Indian Institute of Technology, Powai, Mumbai 400 076, India