Hall angle in high-T_c cuprates: Anomalous temperature dependence and anisotropic scattering

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The anisotropy of the scattering rates, $1/\tau_{tr} \propto T$ and $1/\tau_H \propto T^2$, implied effectively by the anomalous temperature dependence of the normal-state in-plane Hall angle, $\cot \theta_H \propto T^2$, observed in the high-$T_c$ layered cuprates is reasoned out to be a natural consequence of the semiclassical Boltzmann transport equation with crossed electric (E) and magnetic (H) fields. It is argued that while the scattering rate $1/\tau_{tr}$ describes the longitudinal relaxation of the dipolar E-perturbations to the circular zero-field reference distribution function which is known to correspond to a non-Fermi liquid with $1/\tau_{tr} \propto T$, the scattering rate $1/\tau_H$ describes the transverse relaxation of the H-perturbations to the E-induced shifted reference distribution which is Fermi-liquid-like giving $1/\tau_H \propto T^2$. Incorporation of impurity scattering gives $\cot \theta_H = aT^2 + b$ in agreement with the observed temperature dependence.

THE temperature dependence of the normal-state in-plane Hall angle $\theta_H(T)$ observed in the layered cuprate superconductors is known[1-7] to be anomalous in that $\cot \theta_H(T) = a + bT^2$ while the in-plane resistivity $\rho_{ab}(T)$ is T-linear and the Hall coefficient $R_H \propto 1/T$. This is in clear violation of the Kohler scaling rule[6,7], known to be valid generally for conventional metals, that assumes isotropic scattering rates, i.e. $\tau_{tr} = \tau_H$, defined with respect to the Hall geometry. Here τ_{tr} is the usual longitudinal relaxation time for transport parallel to the applied in-plane electric field (**E**) while τ_H is the transverse relaxation time for transport perpendicular to **E** and the out-of-plane magnetic field (**H**). Further, it has been found that (i) the anomaly is generic to high-T_c layered cuprates, (ii) it is pronounced for optimally doped cuprates with T-linear $\rho_{ab}(T)$, and (iii) it gets weaker for over-doped samples. Also, no such anomaly is seen in the case of an out-of-plane Hall current[3].

Semi-phenomenological approaches to resolving the Hall-angle puzzle have invoked anisotropic scattering rate, $\tau_H \neq \tau_{tr}$, following the proposal due originally to Anderson[8] that these two distinct scattering rates arise naturally from the spin-charge separation of carriers in the normal state of the layered cuprates. In this note we have argued out, in generality, the observed anisotropy, $\tau_H \neq \tau_{tr}$, basing on a re-interpretation of the structure of the Boltzmann transport equation for crossed **E**- and **H**-fields without any microscopic particularity. The basic idea is that while the longitudinal response to an electric field **E** involves relaxation of a dipolar deformation to the circular reference phase–space distribution, given to be a non-Fermi liquid with T-linear $\rho_{ab}(T)$ implying $\tau_{tr}^{-1} \propto T$, the incremental transverse (Hall) response to the add-on cross field (**H**) involves relaxation to a reference state which is now a rotated dipolar sub-distribution. The latter may be viewed as a dilute Fermi-system *per se*, but one with an *excluded* circular phase–space, that makes it a normal-Fermi liquid with the usual relaxation rate $\tau_H^{-1} \propto T^2$.

Our argument can be appreciated best by reference to the structure of the Boltzmann equation for the velocity distribution function under the perturbing crossed **E**- and **H**-fields. As is well known, the Hall effect is a *multiplicative* response where the **E**-field produces a dipolar deformation of the unperturbed circular distribution function $f_{0,0}(\mathbf{v})$, and the **H**-field response consists in the rotation of this dipolar deformation by an angle, the Hall angle θ_H.

Let $f_{0,0}(\mathbf{v})$ be the distribution function for $\mathbf{E} = 0 = \mathbf{H}$ and $\delta f_{E,0}(\mathbf{v})$ be the dipolar deformation generated by the in-plane electric field **E**. Then (in obvious notation[6])

$$\delta f_{E,0}(\mathbf{v}) = -\tau_{tr} e\mathbf{E} \cdot \mathbf{v} \frac{\partial f_{0,0}}{\partial \varepsilon} . \qquad (1)$$

Here τ_{tr}^{-1} denotes the relaxation rate of deformed distribution $f_{E,0}(\mathbf{v})$ towards the reference circular distribution $f_{0,0}(\mathbf{v})$. Now, consider the effect of the crossed out-of-plane-field **H**. This field acts on the dipolar deformation $\delta f_{E,0}(\mathbf{v})$ and generates a rotationally re-deformed distribution $\delta f_{E,H}(\mathbf{v})$. (The action of **H** on the original circular reference distribution $f_{0,0}(\mathbf{v})$ generates (quantum-mechanically) only a diamagnetic response which is not relevant here.) The question now is how $\delta f_{E,H}(\mathbf{v})$ is to relax. The first point to note is that $\delta f_{E,H}(\mathbf{v})$ has to relax to the *dipolar* reference distribution $\delta f_{E,0}(\mathbf{v})$. This reference state characterized by the dipolar deformation $\delta f_{E,0}(\mathbf{v})$ may now be treated as a fermionic sub-system. The latter is certainly dilute (for low-enough E-field). Also, the rotational relaxation $\delta f_{E,H}(\mathbf{v}) \to \delta f_{E,0}(\mathbf{v})$ is not affected by the background $\delta f_{0,0}(\mathbf{v})$ which merely provides an *excluded phase-space*. Thus, the relaxation $\delta f_{E,H}(\mathbf{v}) \to \delta f_{E,0}(\mathbf{v})$ is essentially characteristic of a normal Fermi liquid. We must, therefore, have (up to orders linear in **E** & **H**)

$$\delta f_{E,H}(\mathbf{v}) = \delta f_{E,0}(\mathbf{v}) + H\tau_H \frac{e}{c} \mathbf{v} \times \hat{H} \frac{\partial v}{\hbar \partial \mathbf{k}} \frac{\partial}{\partial \mathbf{v}} \delta f_{E,0}(\mathbf{v}) \qquad (2)$$

with $\hat{H}$ a unit vector along **H**, and $\tau_H^{-1} \propto T^2$ as for a normal Fermi liquid without impurity scattering.

From eqs (1) and (2), we at once have for the Hall angle θ_H:

$$\cot \theta_H \propto \frac{1}{\tau_H} . \qquad (3)$$

Now, for an impure normal Fermi-liquid $\tau_H^{-1} = \alpha T^2 + \beta$, where β denotes the effect of impurity scattering. Hence,

$$\cot\theta_H = \alpha T^2 + b \qquad (4)$$

as indeed observed by Chien *et al.*[1] in Zn-doped YBCO single crystals, and subsequently reported more widely[2–7].

The above argument is consistent with the absence of anomaly when the Hall current is out-of-plane (i.e. along the *c*-axis) with the crossed-fields in-plane. This is because the *c*-axis transport has been shown to be controlled intrinsically by the in-plane transport in the normal state, and hence there is a single relaxation time $\tau_{tr} \propto T^{-1}$ consistent with the T-linear in-plane resistivity[9–11].

We would like to conclude with the following comments. The main point of our argument is that while the system may be a strongly correlated one with non-Fermi liquid-characteristics, e.g. the T-linear in-plane resistivity, a small deformation (in the sense of distribution function) of the system when probed appropriately may behave differently, and in particular as a normal Fermi-liquid. This notion is, of course, somewhat familiar in terms of the idea of the electron- or the hole-pockets of a complex Fermi-surface representing sub-sets of carriers with different characteristics, e.g. effective masses, etc. In our case of the Hall angle, the electric field prepares the small deformation (sub-set of carriers) and the magnetic-field probes it. It should be possible to extend this argument to other, possible multipolar, deformations.

Note added in proof: In response to the clarification sought by the referee in his report, received at the proof stage, I clarify once again that in this work we do not introduce explicitly any specific modification of the collision term subsumed in the treatment of B. G. Kotliar, A. Sengupta and C. M. Varma (*Phys. Rev.*, 1996, **B53**, 3573–3577) as a dissipative force proportional to magnetic field and acting sideways, arising from a singular skew-scattering mechanism. Instead, we have decomposed the Hall response as a two-step process, and have distinguished the two relaxation rates in terms of the natures of the respective deformations of the distribution functions involved and the reference distributions to which these deformations relax. This crucial point is totally missed in the usual Boltzmann transport equation that balances the effect of the force $[\mathbf{E}+\mathbf{V}\times\mathbf{H}]$ against the collision integral.

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Direct electrochemistry of heme undecapeptide in aqueous surfactant solutions: The effect of hydrophobicity and axial ligation on redox potential of heme

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Direct electrochemistry of heme undecapeptide (microperoxidase) in micellar solution has been achieved for the first time. Results show a smooth increase in the $E_{1/2}$ with increase in surfactant concentration. The diversity of redox potentials of the heme in various redox hemeproteins is an important topic of investigation in recent years. The origin of this diversity can either be variation in axial ligation or hydrophobicity of the heme pocket in different hemeproteins. We report here that association of heme undecapeptide (microperoxidase) with aqueous detergent micelles can indeed modulate the redox potential of the heme to a sizable extent. Micellar environment mimics the hydrophobicity of the protein cavity. Thus, our results demonstrate on how the hydrophobicity can tune the redox potentials of hemeproteins.

ELECTROCHEMICAL techniques are recognized[1] as powerful means to characterize electron transfer properties of chemical and biological systems. Several workers have reported[2] electrochemical studies on metalloproteins using mediators such as ferrocenes and methylviologen. Direct electron transfer between the protein and the electrode has recently attracted extensive interest and several studies on 'direct' electrochemistry of metalloproteins have been reported[2]. Thus, direct application of cyclic voltammetry to the study of redox processes of various metalloproteins is achieved in presence of suitable pro-