

Aerodynamic performance of SUNYA and OSHO airfoils

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The inverse design problem of an airfoil where one gets an airfoil for given velocity distribution has engaged the best brains. In revisiting this problem we discovered two new airfoils called SUNYA and OSHO. Aerodynamic performance of these airfoils is given using the existing codes.

In 1977 we gave an analytical airfoil representation¹ describing a symmetrical airfoil in terms of Wagner functions. This representation is given by

$$y' = f'(x) = \sum_{n=0}^{\infty} a_n h_n(x) - a_0, \quad (1)$$

where $f(x)$ describes the airfoil thickness distribution and dash denotes the differentiation w.r.t. x ; $h_n(x)$ are Wagner functions defined as follows:

$$h_n(x) = \frac{1}{\pi} \frac{T_{n+1}(1-2x) + T_n(1-2x)}{\sqrt{x-x^2}}, \quad (2)$$

where $T_n(1-2x)$ is the n th Chebychev polynomial. Properties of Wagner functions are given elsewhere². Figure 1 gives the airfoil geometry and Figure 2 gives the first five Wagner functions. It should however be noted that this expansion need not be a universal one.

Integrating (1) and ensuring the closure of the airfoil at the leading and trailing edges one gets the following representation of the airfoil:

$$f(\theta) = \frac{a_0}{\pi} (\theta + \sin \theta) - a_0 \sin^2 \frac{\theta}{2} + \frac{1}{\pi} \sum_{n=1}^{\infty} \left(\frac{\sin n+1\theta}{n+1} + \frac{\sin n\theta}{n} \right) a_n, \quad (3)$$

where

$$x = \sin^2 (\theta/2). \quad (4)$$

At that time (1977), we had not realized that this representation has many important consequences for the airfoil design. Recently we³ showed that this expansion contains two new symmetrical airfoils called SUNYA and OSHO as follows.

$$y_{\text{sunya}} = \frac{a_0}{\pi} [2 \sin^{-1} \sqrt{x} + 2\sqrt{x(1-x)} - \pi x], \quad (5)$$

$$y_{\text{osho}} = \frac{a_0}{\pi} [2 \sin^{-1} \sqrt{x} + 2\sqrt{x(1-x)} - \pi x] + \frac{a_1}{\pi} \sqrt{x} (1-x)^{3/2}, \quad (6)$$

where a_0 and a_1 are constants describing the properties of these airfoils. It can be easily verified that OSHO airfoil is the superposition of SUNYA and JOUKOWSKI

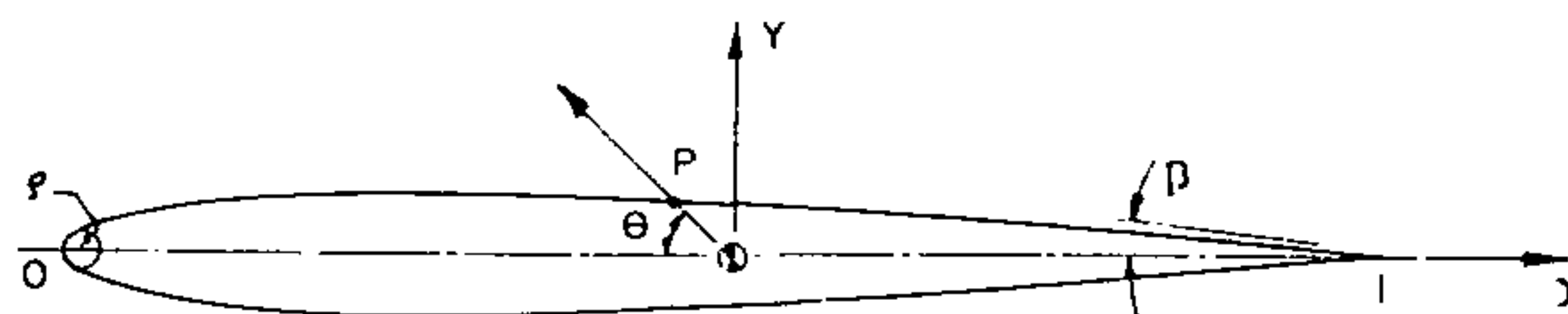


Figure 1. Aerofoil geometry.

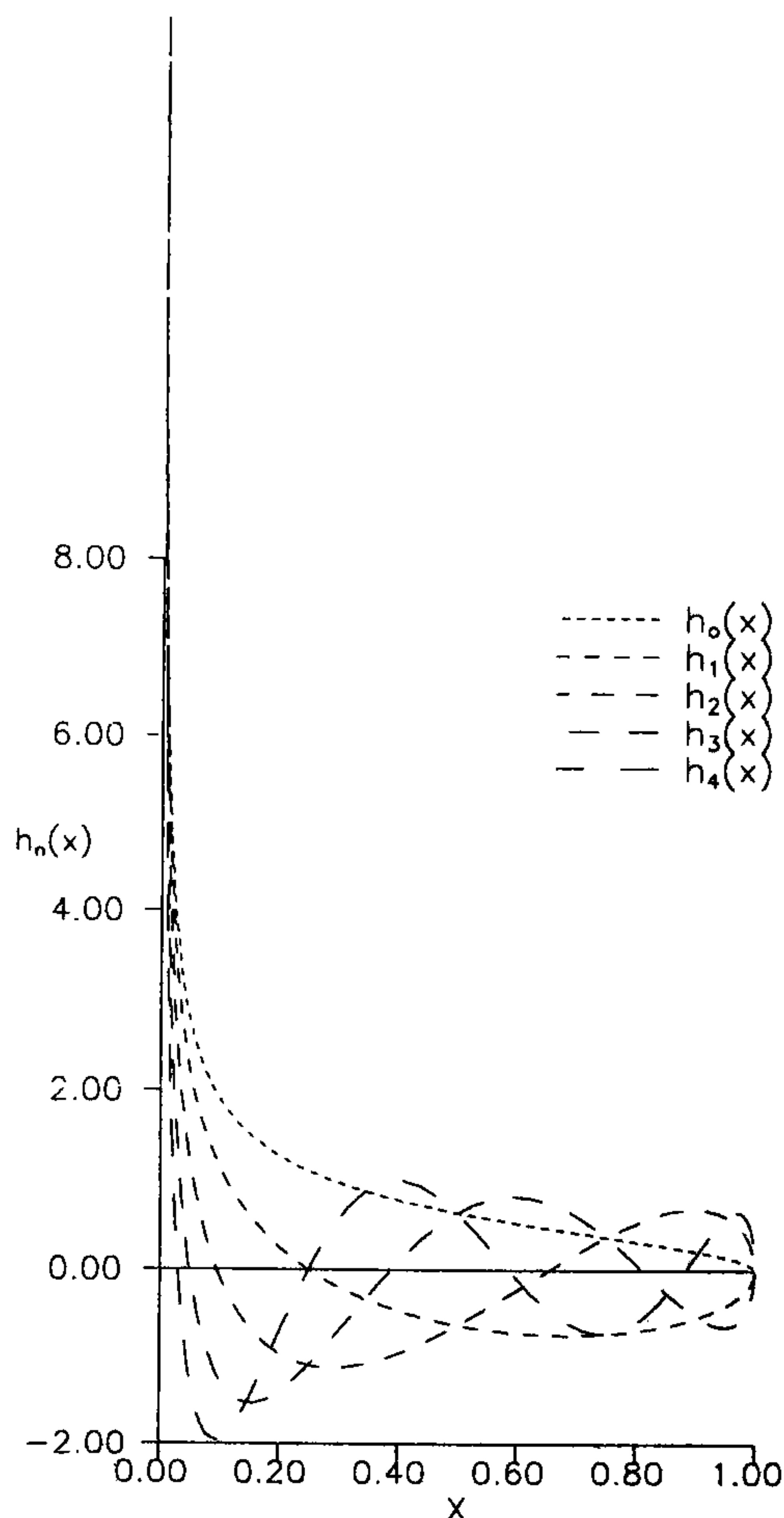


Figure 2. Wagner functions $h_n(x)$ for $n=0$ to $n=4$.

airfoils. It is interesting to compare these airfoils with other classical analytical airfoils given below

$$y_{\text{joukowski}} = \frac{4a_0}{\pi} \sqrt{x} (1-x)^{3/2}, \quad (7)$$

$$y_{\text{kar.trefftz}} = a_0(1-x)^{3/2} \sqrt{x} + a_1(1-x)x, \quad (8)$$

$$y_{\text{piercy}} = a_0 \sqrt{x} (1-x)[1 + a_1(1-x)]^{1/2}. \quad (9)$$

It is recalled that the three latter airfoils are obtained by conformal transformation methods.

In reference 3 we have shown how to use SUNYA and OSHO airfoils for analysis and design of any arbitrary symmetrical airfoil. The properties of these airfoils are given below.

Properties of SUNYA airfoil

- SUNYA airfoil is described by the equation (5) and is a single parameter airfoil, the parameter being a_0 .

The leading edge radius ρ_s is given by the equation

$$\sqrt{\frac{\rho_s}{2}} = 2 \frac{a_0}{\pi}. \quad (10)$$

The trailing edge angle β (positive with respect to negative x -axis) is given by the equation

$$\tan \beta = -a_0, \quad a_0 > 0. \quad (11)$$

- The maximum thickness position x_{max} is given by

$$x_{\text{max}} = \sin^2(\theta_m/2) = 0.2844,$$

$$\text{where } \theta_m = 2 \tan^{-1}(2/\pi). \quad (12)$$

- The maximum thickness is given by

$$t_{\text{max}} = 4(a_0/\pi) \tan^{-1}(2/\pi). \quad (13)$$

All the symbols and variables are defined in Figure 1.

Properties of OSHO airfoil

- OSHO airfoil is described by the equation (6).

Its leading edge radius ρ_{osho} is given by

$$\sqrt{\frac{\rho_{\text{osho}}}{2}} = \frac{2}{\pi} [a_0 + a_1]. \quad (14)$$

- The trailing edge angle β is given by the equation

$$\tan \beta = -a_0, \quad a_0 > 0. \quad (15)$$

- Maximum thickness position x_{max} is given by

$$x_{\text{max}} = \sin^2(\theta_m/2),$$

where

$$\frac{[2(1 + \cos \theta_m) - \pi \sin \theta_m]}{[\cos \theta_m + \cos 2\theta_m]} = -\frac{2a_1}{a_0}. \quad (16)$$

- The maximum thickness is given by

$$\begin{aligned} \frac{t_{\text{max}}}{2} = & \frac{a_0}{\pi} [\theta_m + \sin \theta_m] - a_0 \sin^2 \frac{\theta_m}{2} \\ & + \frac{a_1}{\pi} [\sin \theta_m + 0.5 \sin 2\theta_m]. \end{aligned} \quad (17)$$

Table 1. 15% SUNYA, OSHO and JOUKOWSKI airfoil coordinates

X/C	SUNYA	OSHO	JOUK
0.00000	0.00000	0.00000	0.00000
0.00099	0.00810	0.00809	0.00724
0.00394	0.01578	0.01575	0.01442
0.00886	0.02302	0.02299	0.02145
0.01571	0.02981	0.02978	0.02827
0.02447	0.03614	0.03611	0.03481
0.04759	0.04737	0.04736	0.04683
0.06185	0.05226	0.05227	0.05219
0.09549	0.06060	0.06063	0.06139
0.11474	0.06404	0.06407	0.06516
0.13552	0.06699	0.06704	0.06833
0.15773	0.06947	0.06952	0.07090
0.18129	0.07148	0.07153	0.07284
0.20611	0.07303	0.07307	0.07416
0.23209	0.07412	0.07415	0.07487
0.25912	0.07477	0.07479	0.07497
0.28711	0.07500	0.07500	0.07448
0.31594	0.07482	0.07480	0.07344
0.34549	0.07424	0.07419	0.07188
0.37566	0.07329	0.07322	0.06983
0.40631	0.07198	0.07188	0.06734
0.43733	0.07034	0.07021	0.06446
0.46860	0.06839	0.06823	0.06124
0.50000	0.06615	0.06596	0.05744
0.53140	0.06365	0.06343	0.05400
0.56267	0.06092	0.06067	0.05010
0.59369	0.05797	0.05770	0.04609
0.62434	0.05485	0.05456	0.04201
0.65451	0.05158	0.05127	0.03794
0.68406	0.04819	0.04786	0.03392
0.71289	0.04471	0.04436	0.03000
0.74088	0.04116	0.04081	0.02622
0.79389	0.03400	0.03366	0.01925
0.81871	0.03045	0.03012	0.01613
0.84227	0.02697	0.02665	0.01328
0.86448	0.02357	0.02327	0.01071
0.88526	0.02029	0.02001	0.00845
0.90451	0.01716	0.01691	0.00648
0.93815	0.01147	0.01128	0.00344
0.95241	0.00896	0.00880	0.00234
0.96489	0.00671	0.00659	0.00149
0.97553	0.00475	0.00465	0.00087
0.98429	0.00309	0.00303	0.00045
0.99114	0.00177	0.00173	0.00019
0.99606	0.00080	0.00078	0.00006
0.99901	0.00020	0.00020	0.00001
1.00000	0.00000	0.00000	0.00000

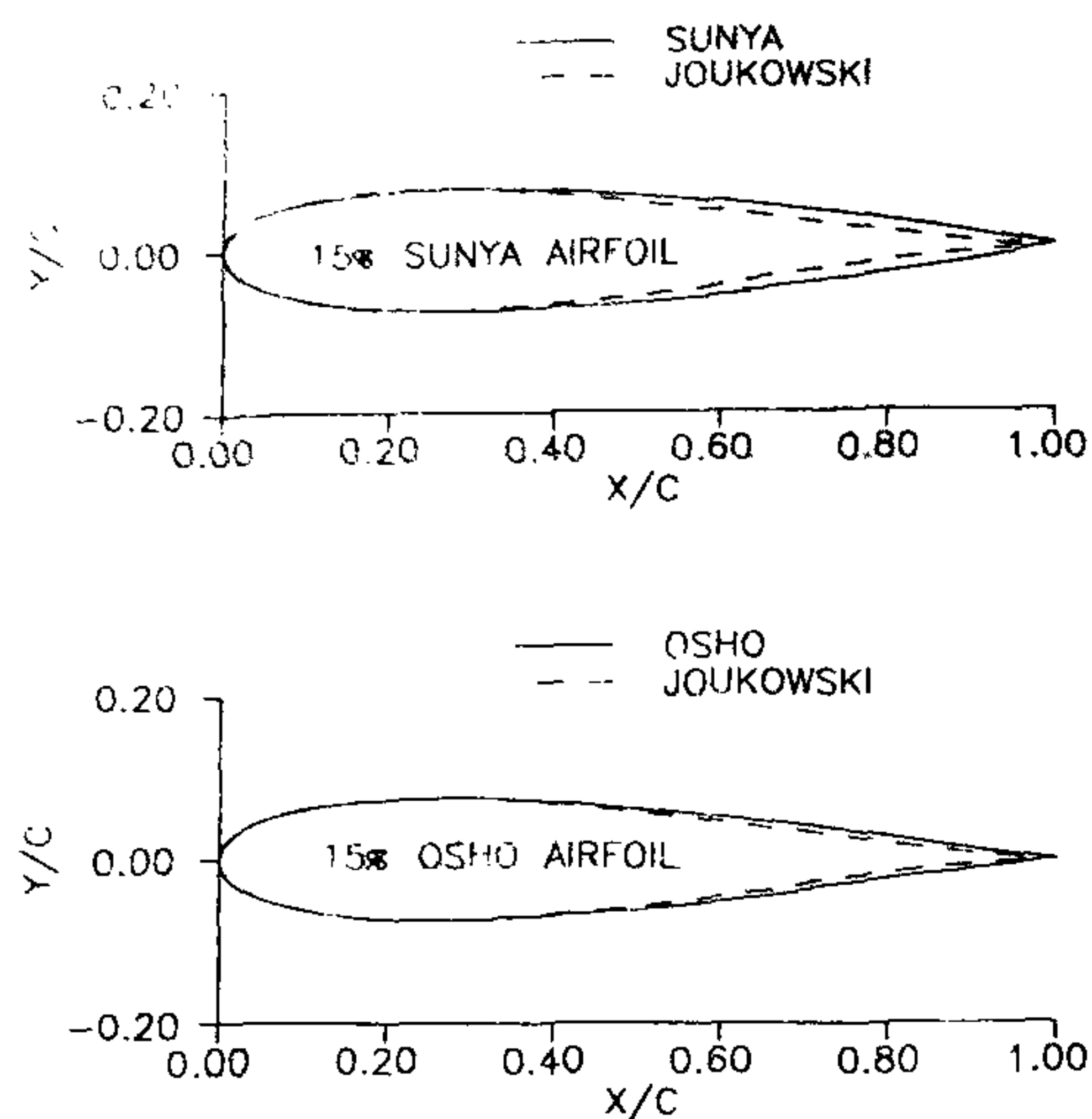


Figure 3. 15% SUNYA and OSHO airfoils.

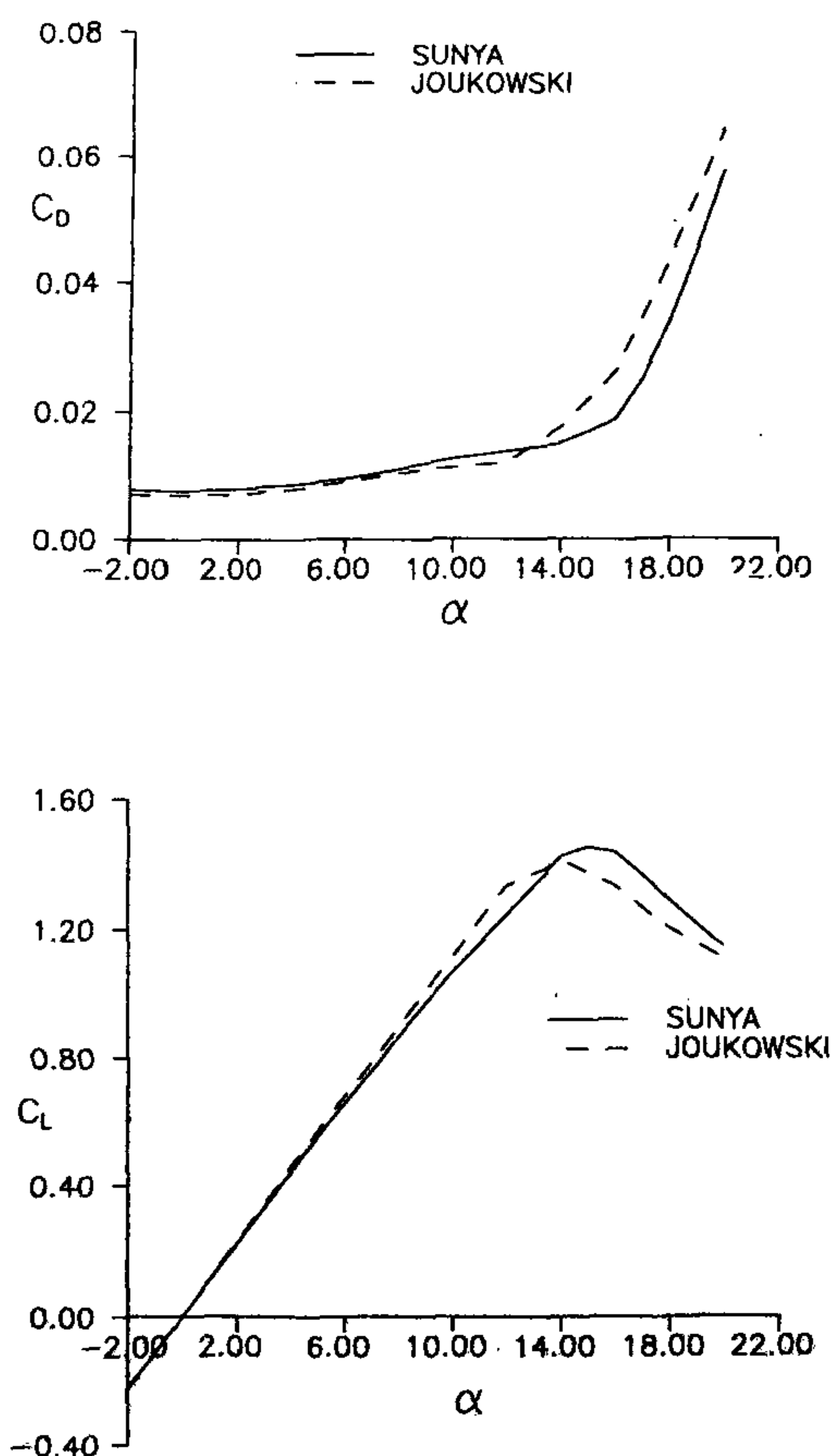


Figure 4. Aerodynamic performance of 15% SUNYA and JOUKOWSKI airfoils ($M=0.1$, $R_e=3 \times 10^6$).

A 15% SUNYA airfoil is obtained by equation (5), where a_0 is obtained by the equation (13) by assuming $t_{\max}=0.15$. Table 1 gives the coordinates of this airfoil. Figure 3 shows this airfoil with 15% JOUKOWSKI airfoil superposed on it. NCSU code⁴ and Rokicki code⁵ are utilized for obtaining the aerodynamic performance at prestall and stalling angles of attack respectively. Figure 4 shows the results at design condition of $M=0.1$ and $R_e=3 \times 10^6$, where M and R_e are Mach number and Reynolds number respectively. The performance of a 15% JOUKOWSKI airfoil is also shown in this figure for comparison.

A 15% OSHO airfoil is obtained by assuming the maximum thickness point to be at $x_{\max}=0.28$ and $t_{\max}=0.15$ in the equations (16) and (17) respectively and evaluating a_0 and a_1 . The coordinates of this airfoil are also given in Table 1. This airfoil is also displayed in Figure 3 with 15% JOUKOWSKI airfoil superposed on it. Again NCSU and Rokicki codes are used for obtaining the aerodynamic performance of this airfoil. Figure 5 gives the results. The performance of the 15%

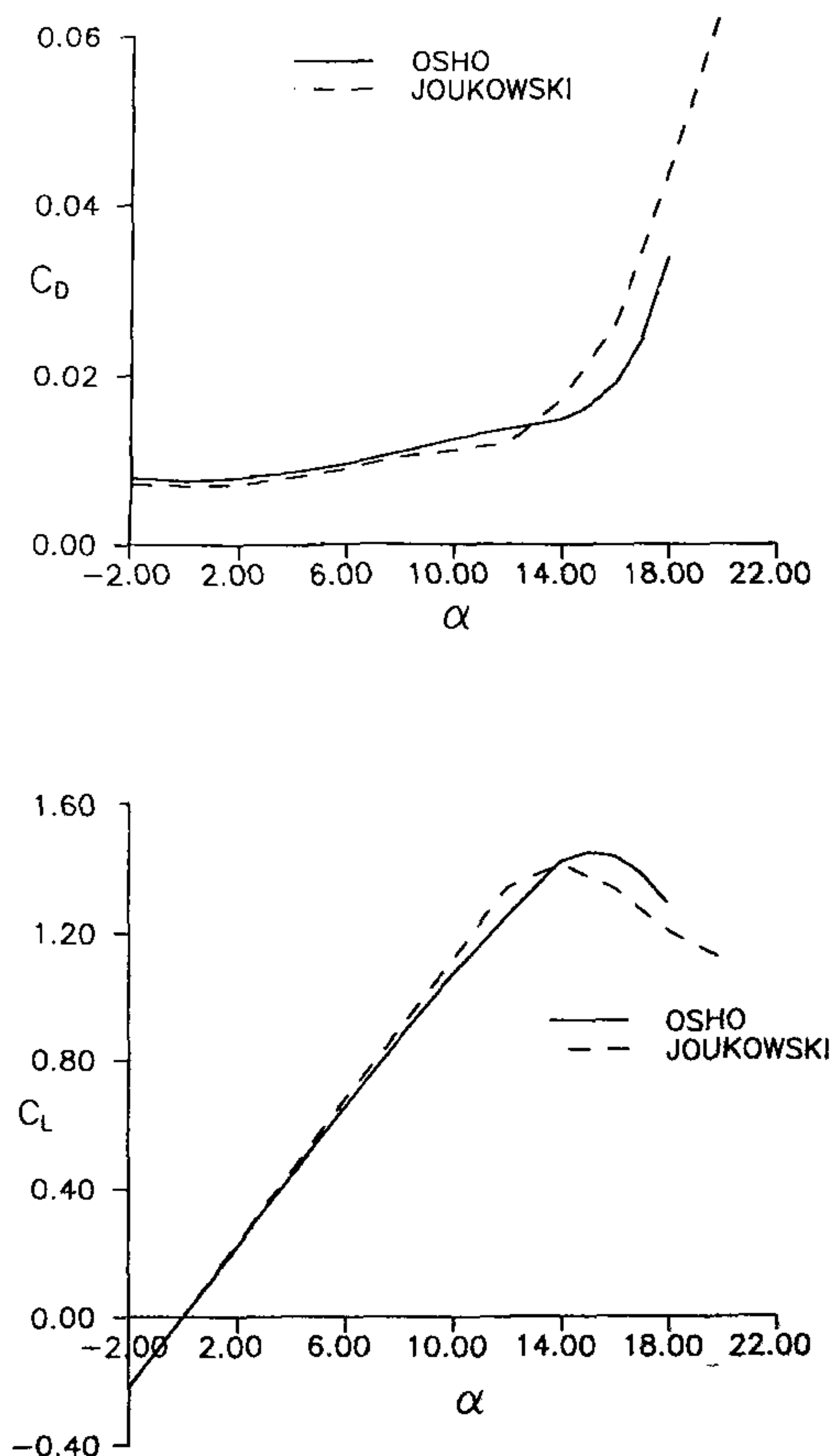


Figure 5. Aerodynamic performance of 15% OSHO and JOUKOWSKI airfoils ($M=0.1$, $R_e=3 \times 10^6$).

JOUKOWSKI airfoil is superposed also on this for comparison.

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Application of equalized molar refraction of zeolites and its correlation with the equalized electronegativity and hardness

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The equalized molar refraction values for various zeolites have been determined by using Sanderson's equalization concept and atomic refraction (R_D) values. Using Komorowski equation, $\eta = (4\pi\epsilon_0 R_D^{1/3})^{-1}$, hardness (η) and equalized electronegativity (χ) are correlated with the equalized molar refraction (R_m). The average deviation was 5%. From the equalized molar refraction various useful properties like molar polarizability, refractive index, etc. can be determined.

THE applications of equalized electronegativity concept to zeolites and other systems are well-established¹⁻⁴. Based on equalized electronegativity, the charges on oxygen and aluminium were correlated to Si/Al ratio of various zeolites⁵. The water content due to the aluminium fraction was also correlated with equalized electronegativity. We have introduced the concept of equalized chemical hardness of zeolites to correlate various physico-chemical properties⁵. Here we report the concept of equalized molar refraction. The equalized molar refraction is correlated with equalized electronegativity and equalized chemical hardness of various zeolites. Hati and Datta^{6,7} have correlated electronegativity/hardness with

electric dipole polarizability of atoms and clusters.

The electronegativity (ψ) and chemical hardness (η) are defined as

$$\psi = (dE/dN)_z = -\mu \quad (1)$$

$$\eta = \frac{1}{2} (d\mu/dN)_z, \quad (2)$$

where E is the energy, N the number of electrons, μ the electronic chemical potential and z the potential due to the fixed nuclei. The equalized electronegativity is represented as,

$$\psi_{eq} = [N/\Sigma (V/\psi)], \quad (3)$$

where N is total number of atoms and V the number of atoms of a particular element or group. The details are given elsewhere⁵.

Similarly $R_{M(eq)}$, equalized molar refraction is defined as

$$R_{M(eq)} = [N/\Sigma (V/R_D)]. \quad (4)$$

$R_{M(eq)}$, the equalized molar refraction is determined⁸ from atomic refractions (R_D).

As given by Komorowski⁹ the chemical hardness can be correlated with ionic (atomic) refractions by the equation,

$$\eta = (4\pi\epsilon_0 R_D^{1/3})^{-1}, \quad (5)$$

where R_D = ionic (atomic) refraction. Based on this Komorowski equation, we have correlated the equalized molar refraction with equalized chemical hardness and equalized electronegativity separately.

The applications of the equalized electronegativity concept and chemical hardness added to our understanding of various materials and provided new method to determine the properties like charges⁵. This prompted us to understand another fundamental property like atomic refraction through equalization concept. Knowing that the chemical hardness is related to ionic (atomic) refraction, we consider the following equation, on the basis of equation (5)

$$\eta_{eq} = A + (B/R_{M(eq)}^{1/3}), \quad (6)$$

where $R_{M(eq)}$ is the equalized molar refraction.

Using the equation (6), 120 zeolites have been studied with the average per cent deviation of 3.72, and the values of the constants A and B are 13.17 and -4.96 respectively. If we consider ionic refraction values instead of atomic refraction values for exchangeable cations, then the average deviation is 5.11% and the values of the constants, A and B are 0.09 and 5.39 respectively.