Chandrasekhar's contributions to general relativity

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Professor S. Chandrasekhar's main contributions to general relativity are summarized in the context of other developments in the field. The article is addressed to scientists who are not experts in this area.

ALMOST from its inception, general relativity was regarded as a triumph of the human intellect, the most beautiful physical theory ever created. However, for almost fifty years, since its discovery, the theory was ignored by most physicists and astronomers. 'It appeared', as Max Born put it, 'a great work of art, to be enjoyed and admired from a distance.' This attitude underwent a profound change in the sixties due to major advances, both theoretical and observational. Since then, the subject has moved toward centre-stage both in fundamental physics and in a number of areas in astrophysics. (For Chandra's own writings on this subject, see refs 1–6.) Indeed, by now, issues concerning strong gravitational fields are among the central ones both in the very small – the Planck regime – and the very large – galactic centers, quasars, and the early universe. While a number of people have contributed in important ways to bring about this dramatic shift, it is clear that, in the astrophysical realm Chandra played the decisive role: more than any other single researcher, it was he who brought the beautiful creation of Einstein's to its natural home, astronomy.*

In this article, I will sketch these contributions. However, I should emphasize that I will not cover all of Chandra's work in this area nor will the discussion be detailed or technically sophisticated. For the first limitation, I will not even apologize; I think it would be nearly impossible for anyone to provide an exhaustive account of all of Chandra's contributions even to this one area. For the second, I have a good reason. The Indian Academy of Sciences will soon bring out a memorial volume in which Thibault Damour will write on Chandra's work on post-Newtonian methods; John Friedman, on Chandra's papers on a stability; and Roger Penrose, on Chandra's contributions to black-hole physics and astrophysics. All of them are better qualified than me to discuss this material at a technical level. Therefore, I will try to make this article complementary to theirs by putting Chandra's contributions in the context of other developments in general relativity and providing some anecdotes that, I hope, will shed further light on what motivated Chandra and on the work style he followed.

Chandra became interested in general relativity in the early sixties. By then he had essentially completed his research on hydromagnetic and hydrodynamic stability and was looking for a new field. His usual way when he changed fields was to think about possibilities, study the literature sufficiently to gain a personal view of the field, perhaps talk to some experts and then come to a decision based on a combination of interest and compatibility with his temperament and strengths. In the case of general relativity, I think, he took longer than the usual few months. In 1961, he taught an advanced course on the subject, which, incidentally, was probably the first time general relativity was taught at the University of Chicago[5]! Then, in 1962, he went to Warsaw to attend the third international conference on general relativity and gravitation[†]. This was a most interesting meeting in many ways. Established figures like Paul Dirac, Richard Feynman, and Leopold Infeld were all there and so were the 'young turks' like Ted Newman and Roger Penrose. There was much discussion of unifying general relativity and quantum mechanics, use of new spinorial methods in the classical theory and of reality and the meaning of gravity waves. It was at this meeting that Feynman unveiled his work on the perturbative quantization of general relativity and it was around this time that Hermann Bondi established the reality of gravitational radiation in full general relativity.

During the conference Chandra played the role of an observer. However, coming from the background he did, he decided to work in a direction that was different from the 'mainstream' topics mentioned above and explored instead the role of general relativity in astronomy.

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*Even this phrase was coined by Chandra in his address to the International Astronomical Union[2]. *General theory of relativity is a theory of gravitation; and like the Newtonian theory of gravitation which it refines and broadens, its natural home is astronomy.*

[†] There is an interesting story here. Since Chandra had already accomplished so much in so many different areas, the National Science Foundation wanted to support Chandra's travel to Warsaw. But unfortunately he did not fit in any of the 'standard categories' they then had. 'Are you giving an invited talk?' they asked him. 'No', he answered. Are you making a presentation? 'No', again. 'Are you an expert?' 'No'. He explained that he was in fact going there because he was thinking of *entering* the field. Finally they gave up, stopped asking these official questions and found the appropriate excuse to provide the travel funds. Considering how much Chandra contributed to general relativity for the subsequent thirty-three years, this was probably one of the best investments NSF ever made!

As he explained in his 1978 Oppenheimer memorial lecture[5], Chandra felt that 'Einstein's theory is incredibly rich' but that not enough had been done to extract its physical predictions, i.e. its real content. Indeed, Chandra's lectures and some of the writings of his later years contain remarks on Einstein which seem critical. It is not that he did not appreciate the depth and aesthetic beauty of Einstein's theory – he emphasized it time and again. Rather, Chandra felt that, in a certain sense, Einstein did not 'believe in the theory sufficiently deeply'. For, if he had, he would have tried energetically to work out its physical consequences in the strong field regime where the 'soul' of the theory lies. Many of the fascinating results would then have been discovered decades earlier and general relativity would have entered the 'mainstream' much sooner. This, I believe, was the regret Chandra was expressing when he appeared to be critical of Einstein.

In his very first work on general relativity, therefore, Chandra attacked a problem that had direct astronomical implications. He considered spherical self-gravitating objects and studied the problem of their stability against *radial* perturbations. Thus, one begins with an equilibrium configuration – a static, spherical solution to Einstein's equations with a perfect fluid source – and perturbs it in a spherically symmetric manner. In the first approximation, one linearizes the coupled set of equations around the equilibrium configuration. The key question is whether the perturbations correspond just to small oscillations or do they grow unboundedly in time. In the former case the system is stable, and in the latter, it is unstable. This problem was studied in the Newtonian theory and it was known that if the adiabatic index γ of the perfect fluid source is greater than 4/3 – which is likely to be the case in realistic cases – the system is *stable*. Chandra found that this is no longer the case once general relativity is brought in: there is a qualitative difference! In particular, this relativistic instability implies that there is an upper limit on how dense the stars can be. In the hands of Jim Bardeen, Jim Hartle and others, this result grew in to what is today *the best theoretical argument for the existence of black holes.* It also has important implications for white dwarfs. This work was completed in 1964 and still stands as a landmark in relativistic astrophysics. It is rare indeed for the very first paper in a new area to have such a deep impact on the subject.

Mathematically, the assumption of spherical symmetry made the problem effectively 'one dimensional' and, hence, a complete solution was possible. However, the assumption is very restrictive from a physical viewpoint. It ruled out rotation of the source and, more importantly, the possibility of emission of gravity waves. Newtonian theory does not allow gravity waves; their presence is a purely relativistic effect. And to probe this qualitatively new aspect of general relativity, one must consider non-spherical scenarios. Chandra's next series of papers was devoted to the development of a formalism to analyse these issues and especially the question of whether the dissipative effects caused by the back reaction give rise to secular instabilities which, of course, would be absent in the Newtonian theory.

Up until the early sixties, however, this area was riddled with conceptual puzzles and uncertainties. For it was not clear then that full, nonlinear general relativity allows gravitational radiation. In the weak field linearized approximation, the problem was solved by Einstein soon after the discovery of the theory. In this approximation, there were indeed gravity waves. However, in the full theory, the issue of disentangling gauge from dynamics is non-trivial and there was some confusion as to whether, in the full theory, the waves were purely coordinate effects which could be 'transformed away'*. In retrospect, the confusion seems surprising. But there were some subtleties and, as Chandra often emphasized to students, the issue was not clear-cut at the time. In particular, explicit solutions were known – for example the so-called *C*-metric discovered by Levi-Civita – which seemed to admit gravity waves and yet, looked at from a different perspective, seemed to be static. (Much later, when the global structure of this space–time was analysed carefully, it became clear that it is not static and *does* admit gravity waves.) Furthermore, people were right in not fully trusting the conclusions of the linearized theory. For example, it is now known that (in a spatially compact context which is of interest in cosmology) there are solutions to the linearized equations which have no analogue at all in the full theory. These issues were getting cleared up just as Chandra entered the field and by the mid-sixties, thanks to the analysis by Bondi, Rainer Sachs, Penrose and Newmann, the reality of gravity waves was firmly established.

The techniques used in these calculations are mathematically rigorous and elegant but cannot be applied directly to extract answers to the astrophysical questions in which Chandra was interested. What was lacking was a systematic approximation scheme. Chandra undertook the task of developing one and the work was carried out partly in collaboration with his students Yavuz Nutku and Paul Esposito. The scheme is called the 'post-Newtonian approximation' and its aim is to develop a perturbation theory by starting with the Newtonian answers and correcting them step by step. The smallness parameter is $(v/c)^2$ where v is the velocity of matter.

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*It is interesting to note that in 1936 Einstein himself wrote to Max Born: *Together with a young collaborator, I have arrived at the interesting result that gravitational waves do not exist, though they had been assumed to be a certainty to the first approximation.* By the time the result mentioned here was written up, however, Einstein had realized the error and there is no mention of this remark in the published (Einstein–Rosen) paper.

The only work along these lines available then was that due to Einstein–Infeld and Hoffman, carried out in the late thirties, which had encountered difficulties (singularities) because the sources were treated as point particles. Chandra could deal with hydrodynamics and also consider rotating objects. To see the effects of gravitational radiation, however, one has to keep terms up to $(v/c)^5$, or, in the usual terminology, up to the $2\frac{1}{2}$ post-Newtonian order. Chandra did carry out the calculations up to this order and discovered a new secular instability caused by the radiation reaction effects. One literally sees the gravity waves of general relativity making a *qualitative* difference!

Chandra completed his work in this area in late seventies. Since then the post-Newtonian formalism has become a standard tool in relativistic astrophysics used in the studies of stars, star clusters and even planetary motions. In the hands of Clifford Will and others it has had an unanticipated application to physics as well: it has led to what is called parametrized post-Newtonian formalism which provides a systematic method to compare and contrast various theories of gravity. In particular, a large number of alternatives to general relativity could be ruled out by comparing the values of their post-Newtonian parameters to those observed experimentally. More recently, the post-Newtonian methods have also been used to study the 'death dance' of coalescing binaries and the gravitational radiation emitted in the process. The resulting wave forms supply critical theoretical inputs for the analysis of signals that one expects to see in the LIGO and VIRGO gravity wave detectors. In spite of these and other applications, however, some basic questions still remain. For example, it is now known that the perturbation series is not likely to converge and a trustable analysis of error-estimates is still unavailable. It is not even known if the series is asymptotic. New ideas such as Borel summability and re-summation techniques which have been used successfully in quantum field theory, may be necessary to fill such gaps. It is perhaps because of this open endedness that, when his research work was complete, Chandra did not, in his usual fashion, write a definitive monograph on this subject. He is said not to have had the 'aesthetic feeling of completeness and coherence' that he had in other areas.

In 1970, Chandra decided that he would, once and for all, study in detail the then recent results in general relativity. These involved the so-called global techniques and the underlying differential geometric methods were quite different from the more classical mathematical techniques over which Chandra had complete mastery. However, the new methods were essential to analyse such physical issues as the properties of black holes and the nature of gravitational radiation in exact general relativity. In his unique way, Chandra invited three bright young relativists, Brandon Carter, George Ellis and Robert Geroch – all in their late twenties – for a summer in Chicago and had them give lectures. Chandra followed the courses with great diligence, taking notes and calling the lecturers, some times late at night, when he had a difficulty. And he was then nearing his 60th birthday! It is this unique spirit that kept Chandra intellectually fresh forever. Soon after that, Bob Geroch and Jim Ipser joined the faculty and a relativity group was created at Chicago. From then on, there were regular relativity seminars and, as a general rule, Chandra attended them all.

However, in his own research, he continued to follow his own inner instincts and the next topic he undertook was motivated by his previous experience with Newtonian gravity: he returned to the analysis of the structure and stability of rotating stars but now in the framework of general relativity. Most of this work was carried out in collaboration with his student John Friedman. They developed a framework which parallels the one for Newtonian gravity (which itself is based largely on some earlier work of Chandra's) and analysed the issue of stability against small perturbations. Under the assumption of slow rotation, they found a qualitative similarity with Newtonian results: rotation tends to stabilize the star. The problem was also investigated by Hartle, Kip Thorne and Kumar Chitre around the same time. However, while these authors used a computer to do the complex algebra, the Chandrasekhar–Friedman work, of course, was all done by hand. These papers provided a foundation for a systematic analysis of the stability of relativistic rotating stars which culminated, several years later, in Friedman's result that, in general relativity, *all* rotating stars are unstable due to radiation reaction. It is no wonder that Chandra's intuition about the importance of radiation reaction in general relativity is widely regarded as one of the deepest insights in relativistic astrophysics.

I must pause to marvel also at Chandra's impeccable sense of timing. He came to the subject just a few years ahead of the dramatic inputs from observations. The subsequent discoveries of quasars and pulsars gave credence to the existence of massive, compact objects in the universe and general relativity suddenly became relevant to astrophysics. Quasars need engines and black holes are the only plausible candidates. Pulsars are compact, rapidly rotating bodies and hence the issue of relativistic stability became critical. In the early seventies, Russell Hulse and Joe Taylor discovered the first binary pulsar and the effect of radiation reaction on the orbits of such bodies now had direct observational consequences. If one had a crystal ball to foresee these developments before choosing research problems, one could not have made better choices than the ones Chandra's inner voice led him to.

The next major topic Chandra took up was black-holes. His work in this area brings out a number of his unique traits. First, Chandra always liked to develop his own perspective of the subject, his own ways of tackling problems. Second, he disliked loose ends and incompleteness. Therefore, he decided to re-examine perturbations of the Schwarzschild black-hole and brought harmony and coherence to the subject by explicitly showing that the various equations that had emerged in different approaches were in fact equivalent. While most of us would be satisfied with this accomplishment, Chandra did not stop here. He deeply believed in looking at the problem from different angles, understanding all its ramifications and bringing out the 'wholeness' of the subject. (Indeed, he was often critical of astrophysicists who lacked this broader perspective and were interested in solving only the immediate problem in a 'narrow' way.) As a result, he had the leisure to discover an amazing fact. The equations governing the perturbations of this black-hole can be written as (time independent) Schrödinger equations in *two different* ways, i.e. with two entirely different potentials. Yet, because they deal with the same physical problem, all the transmission and reflection coefficients are the same. For this to happen, it turns out, the two potentials have to satisfy an infinite number of identities. Therefore, in general, it is very hard to construct such pairs of potentials explicitly. The mathematical theory of black holes provides such a pair naturally! Chandra was very pleased by this result. His work on black holes also brings out Chandra's deep intuition. From his experience with Newtonian gravity, he came to the conclusion that the quasi-normal modes were a key to the understanding of black hole perturbations (now for the general rotating case, i.e. not necessarily Schwarzschild). At the time, the problem was being looked at entirely differently, using Green's functions. Subsequent years have shown that not only did Chandra's insight simplify the problem mathematically but that the radiation in these modes in fact dominates. Again, this insight stands as a landmark in the subject.

However, from a personal point of view, Chandra derived the greatest satisfaction from another, more mathematical result. While studying perturbations of the Kerr black-hole, his desire for completeness and coherence drove him to study the Dirac equation on the Kerr space-time. He managed to separate the equation, reducing the four-dimensional problem to one dimensional ones. This was the sort of the problem that best suited his temperament. No 'standard' method was available. Intuition was called for. One had to speak to the equation in its own intrinsic language and gently coax it to yield. And the final solution was elegant and the derivation simple. An ideal problem for a classical mathematical physicist! It is no wonder that this effort of Chandra's was appreciated more by applied mathematicians than hard core astrophysicists. In the subsequent years, it has led to some 50 papers and several new theorems in the subject of partial differential equations.

Chandra was nearly seventy-five when he completed the work on black holes. Most scientists would have then retired from active research. But Chandra went on to his next topic, which was to be the least one within general relativity: colliding gravitational waves. In a certain sense, this work is a natural extension of his research on black holes. However, it does have quite a different flavour. In this case, Chandra did not begin with specific physical problems or situations of direct astrophysical interest. Rather, he wanted to explore the wondrous mathematical landscape of general relativity, especially the mysterious tunnels that connect different aspects of the theory. This was also the first time he collaborated extensively with younger colleagues – Basilis Xanthopoulos and Valeria Ferrari – who were *not* his students. With them, Chandra first discovered that there is an underlying similarity and unity between the mathematical theory of black-holes and of colliding waves, the extent of which no one had anticipated. (Chandra has commented on this point in detail in ref. 6.) And then they used this fact very effectively to discover a variety of new radiating solutions of Einstein's equations. Given Chandra's record, one day we may realize that even these aesthetically inspired mathematical results have a deep physical significance as well.

There is no doubt that Chandra was enchanted by general relativity. He worked in this area for over thirty years, nearly half of his career. He found in it the 'strangeness in proportion' which to him was an essential feature of beauty. He had heard from Henry Moore that 'great sculptures should be viewed from all distances since new aspects of beauty will be revealed at every scale'. And he found that general relativity, and especially the mathematical theory of black-holes, did just that. While he worked on a large variety of problems, there was an underlying pattern in his work. He chose problems that were well-formulated and, with the exception of his work on colliding waves, were motivated by central physical issues. His main aim was to extract the true content of general relativity. Invariably, he wanted to find exact solutions to his well-posed problems. And he succeeded in this extremely ambitious task because he had a profound intuition for equations of mathematical physics, unmatched among his contemporaries. He sought coherence, completeness and simplicity. He did not always succeed. The Kerr–Newmann perturbations resisted him and hundreds of pages of calculations on properties of black-hole could not be simplified to his satisfaction. However, these exceptions were rare. He almost always met his goals and by doing so made the subject more comprehensible to the rest of us.

His quest during these 'general relativity years' is perhaps best summarized in the passage from Virginia Woolf's *The Waves* that he himself liked to quote:

> *There is a square; there is an oblong. The players take the square and place it upon the oblong. They place it very accurately; they make a perfect dwelling-place. Very little is left outside. The structure is now visible; what is inchoate is here stated; we are not so various or so mean; we have made oblongs and stood them upon squares. This is our triumph; this is our consolation.*

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Chandra remembered

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The article contains a brief biographical sketch of S. Chandrasekhar, his many accomplishments and his multi-faceted personality.

With the sudden death of S. Chandrasekhar on 21 August 1995, the world lost one of the foremost scientists of the twentieth century. Known simply as Chandra throughout the scientific world, he dominated, by his prolific contributions, the fields of theoretical astrophysics, physics and applied mathematics. Writing about his death, scientists from around the world highlighted Chandra's scientific achievements and his unique style of research – how he would occupy himself with a particular area of research for a period of five to ten years, publish a long series of papers, and when he felt that he had accumulated a sufficient body of knowledge in that area and developed a view of his own, he would present it as a coherent account with 'order, form and structure' in a monograph.

Chandra was born on 19 October 1910 (19–10–1910, as Chandra was fond of saying), in Lahore, Pakistan (then a part of colonial British India). His father, C. S. Ayyar was in the Government Service, serving as the Deputy Auditor General of the Northwestern Railways. Chandra was the first son and the third child in a family of four sons and six daughters. His mother was a remarkable woman of great talent and intellectual attainments. Although she had received only a few years of formal elementary education, she had learned English on her own well enough to translate Ibsen's *A Doll House* into Tamil. Intensely ambitious for her children, she played a pivotal role in Chandra's pursuit of a career in pure science. Chandra's early education, till he was twelve, was at home, taught by his parents and private tutors. When he was twelve, his father was transferred to Madras, and Chandra began his regular school in the Hindu High School, Triplicane, during the years 1922–25, followed by his university education at the Presidency College, Madras, graduating with a B Sc. (Hons.) degree in 1930. Lalitha Doraiswamy, his future wife, was a fellow undergraduate studying physics.

An exceptionally brilliant student throughout his student career, Chandra had become nationally known by presenting an original research paper in physics as an undergraduate during the Indian Science Congress Meetings in 1929. Before even he graduated, he learned that he was awarded a Government of India Scholarship to continue his studies in England. By then he had established contact with R. H. Fowler in Cambridge and had decided to work under his guidance for his doctoral degree. On his long voyage across the Indian Ocean in 1930, young Chandra was to pioneer the discovery of the critical mass condition known as the 'Chandrasekhar Limit', which is now hailed as one of the fundamental discoveries of this century.

The Chandrasekhar limiting mass condition says that a star whose mass is greater than a certain critical mass cannot become a white dwarf. This was an inevitable