Chandrasekhar and white dwarfs

Peter P. Eggleton

Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge, CB3 0HA, England

White dwarf stars are common but inconspicuous bodies in our Galaxy. Six of the nearest fifty stellar systems contain a white dwarf; the other stars in these systems are all normal, that is to say 'main sequence', stars like the Sun. None of them is the kind of exotic object, such as a red giant or neutron star or black hole, that tends to draw the attention not only of the general public but also of much of the professional astronomical community. Such objects are rarer, by factors of about fifty, ten thousand, and perhaps ten million, respectively.

It is always reckless for astronomers to say 'We understand this kind of object': more detailed observations often throw up surprises, and force us to revise theories that seemed to be quite robust. Nevertheless, it is probably fair to say that white dwarf stars are among the best-understood objects in our Galaxy, even including the commoner main sequence stars. That they can be considered well understood is largely due to the insight of Chandrasekhar some sixty-five years ago.

Nowadays one can calculate the basic structure of a white dwarf on a personal computer in a second or two—though it might take an hour or two to formulate the code that does the calculation. Many students, such as my own, will be set this exercise at an early stage in a course on stellar evolution. When Chandrasekhar did this calculation by hand, in 1933, it must have taken him several months. But the laborious nature of the calculation is much less important than the physical insight that led him to do it. White dwarf structure depends on a combination of quantum mechanics, then a rather new and still somewhat tentative discipline, special relativity (equally new and tentative), statistical mechanics, and gravitation. And yet this synthesis of disciplines turns out to be about the simplest that is capable of accounting for the properties of any class of star, and the results have stood the test of time. To understand even the 'normal' main sequence stars, one needs to add at least three extra major areas of physics: nuclear physics, atomic structure, and turbulent convective heat transport. Interestingly, it was in the first of these three areas that Willy Fowler, who gained his Nobel prize at the same time as Chandrasekhar, did much of his life's work.

Although white dwarfs were the first class of star to have their structure explained, they represent an end-point, not the starting point, to the life of a star. If they are relatively simple, it is because the universal force of gravitation has crushed them to the point where matter is, at least arguably, in its simplest state. Gravity always tends to compress matter, pulling it together. Within planet Earth, this crushing property is opposed by the atomic structure of matter: atoms cannot easily be pushed into each other. But gravity goes up with the square of the mass, and stars are typically a million times more massive than the Earth. Thus in normal stars atoms are crushed to the point where there is rather little structure left—but not so little that one can afford to neglect it. In normal stars gravity temporarily meets its match not because of atomic structure but because of nuclear physics: the deep interior is crushed to the point where nuclear reactions become important, and the immense amount of energy released, flowing outwards steadily through the star, gives a tendency towards expansion that can oppose gravity almost exactly, at least for a time. In a normal star there is a complicated but apparently rather stable balance. Nuclear energy is released at and near the centre; it flows outwards by a combination of, on the one hand, atomic processes in which radiant energy is absorbed and then reradiated by partially but not entirely ionized atoms, and on the other hand by turbulent convective motions not unlike the motion that one can see in heated water in a kettle on a stove.

However, this balance cannot persist indefinitely, because there is only a finite amount of nuclear energy that can be obtained from matter. It is very much more than the amount of *chemical* energy. Wood, or coal, contains a finite amount of chemical energy, that can be released when either burns in air. The same materials also contain nuclear energy about a million times as much, although to release this energy requires a much more powerful trigger than just a lighted match (fortunately!). The crushing effect of gravity in a star provides the trigger, but once the energy starts to be released it can be and will be used up. The sun will take about five thousand million years to use up its store of energy, but it cannot continue indefinitely. Many of the brighter stars that one can see in the night sky, such as Betelgeuse in the constellation of Orion, are using up their energy much more rapidly than the Sun, and may only have a million or less years left to live.

When the nuclear resources are exhausted, gravity wins again, and the star is crushed further. It is possible that it might be crushed quite literally out of existence—into a black hole. But it is at this point that

Chandrasekhar's famous discovery, the 'Chandrasekhar limit', becomes relevant. When the material of the star, or more precisely of its central core, which is where the fuel first becomes exhausted, is crushed to about a million times its original density, the atoms lose all knowledge of their usual structure and are reduced to a 'soup' of nuclei and electrons. The electrons in this soup have no individual identity; and their indistinguishability, combined with the famous quantum-mechanical 'uncertainty principle', means that it is difficult to crush them further *unless* the mass of the core exceeds a limit which is about 40% greater than the mass of the Sun. Below this mass limit, the core can settle down into a stable, long-lived white dwarf configuration; above it, we appear to have to conclude that the core will implode into a black hole.

This was a rather surprising conclusion and even, at least to Eddington, an unwelcome one. I have the impression that Eddington, perhaps the leading astrophysicist of his day and the one who had largely laid the foundations of stellar structure in the preceding decade, felt that stars 'had to' be able to end as white dwarfs, and there were many stars around that seemed fairly clearly to be considerably more massive than the Chandrasekhar limit. Eddington's disagreement with Chandrasekhar is well known, and it is also well known that he was wrong, and Chandrasekhar right. We now know that massive stars *do* implode, or at least their cores do. But we also know that stars considerably more massive than the Chandrasekhar limit do succeed in ending up as white dwarfs. How do they do it?

The answer is that stars lose mass, mostly at a late stage in their evolution, and they may lose so much mass that even if they *initially* exceed the Chandrasekhar limit by a factor of five, they finally scrape under it and so survive as white dwarfs. Curiously, the fact of the existence of the Chandrasekhar limit is more or less the cause of this mass loss, though somewhat indirectly. As more and more of the reservoir of fuel in the centre is used up, the central core contracts under gravity to higher and higher density; but at the same time material just outside this core, where there is still fuel, becomes compressed and so is caused to burn more vigorously. The star's energy output increases, by a factor of hundreds or even thousands: in fact it rises until close to a limit called, ironically enough, the 'Eddington limit', which is the largest flux of energy that the envelope can carry and still be in equilibrium. A greater flux than this would simply blow the envelope off.

In practice stars in this phase, of which Betelgeuse may be the most conspicuous example in the sky, probably do not exceed or even reach the Eddington limit, but they approach it sufficiently closely that the outer layers can become unstable. In this they are probably helped by the fact that much of the colossal energy flux, particularly in the outer layers, is carried by turbulent convective motion that must be extremely vigorous. Even the Sun, which is quite a low-luminosity star, has a visibly convective surface, and this convection probably helps to drive the tenuous 'solar wind', which blows outwards through the solar system and causes the aurora phenomenon as it interacts with the Earth as its magnetic poles. But Betelgeuse, and other highly evolved red giants, have winds that are a million or even a thousand million times more copious. In this way the near approach of the star's core to the Chandrasekhar limit may actually rescue the star from its imminent total collapse, in a self-limiting way such that the nearer it is to collapse the more vigorously it loses mass and so avoids collapse. Thus, it is not surprising that the actual limit on the initial mass of a star, allowing it to settle down ultimately as a white dwarf, is several times the Chandrasekhar limit.

In fact most stars of modest mass, say one to three times the mass of the Sun, do not have to approach particularly close to either the Eddington limit or the Chandrasekhar limit in order to lose their entire outer envelope and so settle down as a white dwarf. It is not easy to measure the mass of a white dwarf, but typical masses are thought to be ~ 60–70% of the mass of the Sun, and so less than half the Chandrasekhar mass. Thus, the envelope must in practice be blown off well before either limit is reached. It is not entirely clear what mechanism is responsible. But even at this lower mass a white dwarf is a very dense, small object, and so a star which possesses such a core must be highly luminous and somewhat unstable. There is a type of strongly pulsating star, named after the prototype Mira in the constellation of Cetus, which may represent this almost terminal stage in a star's nuclear-burning lifetime. The variability of Mira was probably known to the ancients: in a cycle of about one year it varies between being almost invisible and quite conspicuous to the naked eye. Such stars are often found, by modern measurements in the infrared, to be surrounded by cool gaseous and dusty envelopes that may have been driven away from the star by the pulsations. In such stars it is likely that the exhausted core has reached perhaps 50% of the Chandrasekhar limiting mass, which has driven the luminosity up to perhaps 10% of the Eddington limiting luminosity. But stars of greater initial mass may have to approach considerably closer to both limits, and perhaps expel their envelopes more vigorously at a very late stage.

One of the most massive white dwarfs known is also one of the nearest, a binary companion to Sirius, the brightest star in the sky. This white dwarf has much the same mass as the Sun, and therefore ~ 70% of the Chandrasekhar mass. The visible component is more than twice as massive still, but presumably its companion, the progenitor of the white dwarf, was originally even more massive, perhaps four or five times the mass of the Sun. It must have been an extraordinarily bright object at its brightest—but this would have been several million years ago. Only a few white dwarfs are known, or suspected, to have masses that are actually quite close to the Chandrasekhar limit, and they may be descendants of stars whose original masses were six to eight times the mass of the Sun.

Stars that are more massive still cannot avoid a drastic gravitational collapse of the core, and a consequential supernova explosion. The core however does not inevitably have to collapse to a black hole: there is one more possible equilibrium configuration that the core can collapse to, even if it is above the Chandrasekhar limit and so too massive to form a white dwarf. This configuration is a neutron star, with a density perhaps a thousand million times greater: an entire star of mass like the Sun would shrink to perhaps 10–15 kilometers in radius. At these enormous densities the electrons and atomic nuclei are forced into each other, and can co-exist only as neutrons, which are almost touching each other. The physics of these neutrons is in some respects not unlike the physics of the electrons that preceded them, and as a consequence there must also be an upper limit to the mass of a neutron star just as the Chandrasekhar mass is the upper limit to the mass of a star whose main pressure support is from degenerate electrons. But there is sufficient extra complication in neutron-star physics that we do not yet know this upper limit: it might be 50%, or perhaps even as much as 100%, more than the Chandrasekhar limit.

However there are now, thanks to extraordinarily detailed measurements of radio pulsars in binary systems, a number of very well-determined neutron star masses. In fact the remarkable properties of pulsars enable their masses, in some fortunate cases, to be better determined than those of white dwarfs. All those that are accurately known are ~ 0–10% *less* than the Chandrasekhar limit. This is testimony to the fact that it is still Chandrasekhar's limit which actually governs the ultimate fate of stars. It is not too difficult to see why. Since it is lower (though by an unknown factor) than the neutron star mass limit, a star's core will first get into trouble as it reaches the Chandrasekhar limit, and only as it collapses to much greater density does it 'discover' that it can stabilize at neutronic (i.e. nuclear) densities. During the collapse, an enormous amount of *gravitational* energy is released (mainly in the form of neutrinos), which actually exceeds by a factor of about 10 the *nuclear* energy that was available in the preceding stage when flux of nuclear energy was just sufficient to balance the inward gravitational pull. This enormous amount of energy, possibly supplemented by nuclear energy that remains in the material outside the core, can apparently be enough to eject all of the envelope that surrounded the core shortly before its collapse. In a supernova explosion we see the material being ejected at something like one tenth of the speed of light. It is not yet clearly understood how the energy from core collapse is transferred into envelope expansion, but that much of it is transferred can hardly be denied.

But the collapse of stellar core is not invariably halted by neutronization of the electron–nucleon core. A very few stars are known to have compact companions that are not only more massive than the Chandrasekhar mass, but also than any reasonable upper limit to neutron stars. These must contain black holes. One of them, a very inconspicuous object in the constellation of Cygnus (V 404 Cyg), has a companion which is at least four times as massive as the Chandrasekhar limit, and yet which is almost invisible except for sporadic outbursts which may happen because the normal component occasionally has a flaring eruption (as does the Sun) which throws out material, and some of this material is accreted by the black-hole component, emitting an enormous amount of (ultimately) gravitational energy. Such black holes might in principle exist in quite large numbers, but be completely invisible except in the fortunate circumstance that they have a normal companion. A major question that remains to be answered is why some stellar cores have their collapse halted as they reach neutron-star densities, while others continue and become black holes.

It is not entirely fanciful to say that if the Chandrasekhar limit did not exist, then planet Earth, and human civilization, could not exist. Terrestrial planets, as well as about 2% of the material making up normal stars, are made from elements that have been synthesized in stars by nuclear reactions involving the primordial elements hydrogen and helium. Much of this material was ejected by a previous generation of stars, either in supernova explosions or in the mass-loss phase of red giant stars as their cores approach the Chandrasekhar limit. The distribution of certain elements and isotopes on Earth shows clear signs of their origin in such processes. If stars were able to die quietly, in the absence of a limiting core mass, this material would be trapped in them and not available for a later generation of stars and terrestrial planets. Thus the pioneering work of Chandrasekhar, as a student in his early twenties, has had repercussions not only for remote astronomical objects but for all life on Earth.