The Academy Fellowship problem – no quick fixes

The analysis of the academy fellowship problem by J. V. Narlikar (*Curr. Sci.*, 1995, **69**, 969–970) is most interesting, and I have spent an instructive and entertaining couple of hours going through his beautiful solution. I have (i) a small addition to make to the latter, and (ii) a comment on why the simple example he uses to illustrate the behaviour of the solution is too optimistic as far as the rate of improvement of the median quality (the rate of increase of M_n with n) is concerned, even in the framework of the 'toy' model proposed.

i) Equation (9), namely,

$$M_n = M_{n-1} + \left[2\sum_{r=1}^{n}(1-M_{r-1})^{-1} + 2N_0N^{-1}\right]^{-1},$$

where $M_0 = 1/2$ does have a simple closed form solution. It is $M_n = (1/2)[N_0 + (2n+1)N]/[N_0 + (n+1)N]$, which is not too different in structure from the continuum solution exhibited in eq. (13).

ii) The most interesting aspect of the implicit relation (7) for M_n is the strong dependence on the *initial* cumulative distribution function $f_0(x)$. (It is evident that the solution for subsequent evolution remains valid even if the time origin $t = 0$ is set at any epoch in the evolution of the fellowship population, i.e., it is independent of the manner in which $f_0(x)$ itself was arrived at.) The approach of M_n to the value 1 (the state of perfection!) as n increases is strongly governed by the initial distribution $f_0(x)$. Now, the assumption of a uniform density in the quality factor x for the *general* pool of scientists ($g'(x) = 1$, $g(x) = x$) is quite reasonable, and it is hard to see how this can be improved upon significantly, e.g., $g(x)$ made to go like x^p, $p > 1$. However, the assumption of a uniform distribution for the existing or initial fellowship, $f_0'(x)/N_0 = 1$ or $f_0(x) = N_0x$, is more open to question. (Among other factors, it must be recalled that the quality variable x appearing in $f_0(x)$ is the *same* variable that appears in $g(x)$.) This aspect is all the more important because, given the fact that N_0 is necessarily $\gg N > 1$, it is the form of $f_0(x)$ that controls the manner in which the median M_n approaches unity as n increases. It is far more likely that $f_0(x)$ will be more depleted for small x than N_0x, while enjoying a greater probability mass for larger x – after all, earlier selections *must* have, by and large, focused on the 'superior' part of the general pool! One thus has strong reason to believe that the density function $f_0'(x)$ is actually *peaked* at some value x_p that is <1, but which is not less than 1/2, however uncharitably one views the existing fellowship. The cumulative distribution thus has a typical sigmoidal shape. In keeping with the premises of the toy model, let us assume a symmetric $f_0'(x)$ with a peak at $x = 1/2$ (so that $M_0 = 1/2$). A possible functional form is $f_0(x) = x^2(3-2x)N_0$. However, the ensuing cubic recursion relation for M_n is not too tractable. There is, however, a very simple way of making a slight modification of the original assumption $f_0(x) = N_0x$ that captures the relevant features of a peaked density – namely, the piecewise linear function (roughly sigmoidal in shape)

$$f_0(x) = \begin{cases} 0, \text{ for } 0 \le x < \frac{1}{2}-\delta \\ \frac{N_0}{2\delta}(x-\frac{1}{2})+\frac{1}{2}N_0, \text{ for } \frac{1}{2}-\delta \le x \le \frac{1}{2}+\delta \\ N_0, \text{ for } \frac{1}{2}+\delta < x \le 1. \end{cases}$$

For $\delta = 1/2$, this reduces to $f_0(x) = N_0x$. The width of the peak in the density $f_0'(x)$ is measured by δ. The point is that δ is *not* likely to be of the order of unity (e.g., $\delta = 1/2$ or something of that order of magnitude); rather, δ is most likely to be *small* in comparison with unity, i.e., $\delta \sim N_0^{-\alpha}$ where α is a positive index (simple arguments suggest $\alpha = 1/2$ or even $\alpha = 1$, for instance). Whatever δ is, we can use the foregoing form for $f_0(x)$ in eq. (7) of Narlikar to arrive at the solution

$$M_n = (1/2)\,[N_0 + (2n+1)N\delta]/[N_0 + (n+1)N\delta].$$

This is just the original solution with N replaced by $N\delta$, but, since δ is expected to be small compared to unity, the time-scale on which one has significant increase in x now changes from $n \sim N_0/N$ to $n \sim N_0/(N\delta)$. And, since δ is small, this makes a big difference. A rough estimate shows that even $\alpha = 1/2$ leads to an improvement time scale that is at least an order of magnitude larger than the earlier estimate: with the values $N_0 = 500$, $N = 20$ used by Narlikar to illustrate the solution, and with $\delta \sim N^{-1/2}$, we see that it would take about 250 years, rather than 25 years, to effect a sizeable improvement in the median value from its initial value of 1/2. The assumption of immortality is, of course, absurd now, but a little more analysis shows that taking a steady and uniformly distributed depletion into account does not change the qualitative conclusions we have arrived at.

Although the model is a 'toy' model as already emphasized by Narlikar, and numerous refinements can be thought of, it seems to be quite reliable in its basic message regarding the strategy required for improvement, and the time scale on which the latter can be expected to occur. The strategy of selecting fellows in year n such that all of them are 'superior' to the preceding year's median (or some variant thereof) is not a bad one; it will lead to a *steady*, but not *quick*, improvement in quality. There seem to be no quick fixes in this matter. And if one thinks about it, 200–300 years is precisely the time-scale on which the great science academies of Europe have reached a stage at which the *median* quality is excellent. In the case of the USA, the 'quick fix' was undeniably fuelled by the *en masse* transportation of talent from elsewhere during several periods in this century, not to mention the out-of-the-ordinary circumstances pertaining to the cataclysm of the Second World War and its aftermath.

In conclusion, I feel that this is a matter in which we have no option but to take the long view. It is understandable that, as individuals, we should like to see things improve significantly in 25 years or so; but the weight of the initial conditions would appear only to permit this to occur (and that too if the effort is *sustained* relentlessly) in 250 years or so. But the effort is surely worthwhile.

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