ghum and the changing colour of the fruits and seeds in other species allowed an accurate identification of the physiological maturity of seeds. According to results obtained here, on the basis of pod colour changes in *A. lebbek* it can be concluded that the colour of the pods was closely related to the maximum physiological quality of seeds as they started changing colour from green to yellow around 230 DAA, when seeds had attained physiological maturity. Thus, the time when green pods are not found on the plant any longer can be used to characterize the physiological maturity of seeds, and pods can be collected at this stage. Thus, the overall results obtained here indicate that development of hardseededness in seeds of *A. lebbek* succeeds the attainment of physiological maturity, and it is an inherent character of the seed which can, however, be delayed to some extent by harvesting the seeds immediately after maturation. The colour of the pods served as a workable indicator of maturation.

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Fractal description of seismicity of India and inferences regarding earthquake hazard

K. N. Khattri

Wadia Institute of Himalayan Geology, Dehradun 248 001, India

Earthquakes have the quality of fractal structure in their spatial disposition, time sequencing, and magnitude distribution. We analyse the seismicity of India in terms of its spatial fractal structure in various seismic source zones. It is found that the fractal dimensions range between 0.894 and 1.574, indicating that at most the earthquake associated fractures are approaching a two-dimensional space. The low fractal dimension is suggestive of the presence of asperities and barriers of significant sizes in the respective source zones. Using a fractal model the average earthquake slip in primary faults is estimated to be about 3.5 cm/yr in the Himalaya. This gives an average return period of great earthquake of about 285 years. Considering that no great earthquake has occurred in the seismic gaps in the Himalaya for at least 200 years, these gaps are ripe for a future great earthquake to occur within the next hundred years.

To see a world in a grain of sand,
And a Heaven in a wild flower,
Hold Infinity in the palm of your hand,
And Eternity in an hour.

– William Blake

THE earthquakes represent the outcome of complex geomechanical processes having dire consequences for the society. The earth is in an overall steady state of strain, which for most part is caused by plate motions. This is released from time to time in the form of catastrophic motion over faults in the crust, resulting in earthquakes. The structure of the crust is highly inhomogeneous. On a microscale there are crystal defects, cleavages, dislocations, grain boundaries, etc., while on the other end of the scale, i.e. macroscale, by virtue of repeated fracturing and the different shapes of fractures a high degree of heterogeneity of the crust has evolved. The strain field accordingly is also quite heterogeneous. On account of such complexity of the medium, precise prediction of the occurrence of earthquakes in terms of their location, magnitude and time of occurrence is not feasible. Therefore, the earthquakes are describable by statistical means.

Mandelbrot[1] observed that many natural phenomena possess self-similarity at many different scales. He used the term 'fractal' to describe such phenomena. For example, an assemblage of objects with differing sizes and irregular shapes is a fractal set if the number of objects in it with a specified size has a power law dependence on size. The scaling parameter is called the fractal dimension D. The geometry of the fracture surface of rocks is a fractal[1,2]. The natural rock fracture process is a fractal. Earthquakes are associated with fractals by virtue of the accompanying fracturing of rocks. A fractal distribution is scale-invariant. In such a set the complexity of the part is as great as that of the whole. A fractal function such as a curve or a surface is not differentiable and, thus, no local rule can be deduced to generate the function. However, inferences can be drawn about the whole from observations of the phenomenon on any scale; this is the basis of a holistic approach for studying fractal systems[3]. In comparison, in an ordinary system, howsoever complex it may be as a whole, the basic elements are believed to be simple, which is the basis of a reductionist approach.

Earthquakes possess fractal structure with respect to time, space and magnitude. The Gutenberg–Richter relation for frequency vs magnitude is a power law. Similarly, the aftershock decay follows another power law involving time[4]. The two-point spatial correlation function for earthquake epicentres also displays a power law structure[5–7].

Aki[8] proposed that fractal dimension is correlated with the b value of the Gutenberg–Richter relation. The fractal dimension could be interpreted in terms of the nature of crustal deformations accompanying slips on faults, which may be governed by the sizes and spatial distributions of asperities or barriers in the crust[9]. King[10] and Turcotte[11] have used Aki's model to interpret the crustal deformation due to slips on fractal fault systems. However, Hirata[12] has found that for Japan this idea is not satisfactory.

In this paper we use Aki's[8] model to determine the fractal dimension of seismicity in various source zones in the Indian region and make comparative inferences regarding fault segmentation, asperity–barrier relationships and consequences for seismic hazard estimation. We further use Turcotte's[11] formalism to estimate the fraction of slip taking place on the main fault, the rest being accommodated on a smaller fault system. Thus, we estimate the strain rate and make prognostications about the average return periods of largest earthquakes.

The Gutenberg–Richter frequency magnitude relation is given by

$$\log N\,(m > M) = a - bM, \tag{1}$$

where N is the cumulative number of earthquakes in a time window in an area having magnitudes m greater than M, a and b are parameters of correlation. This relation allows us to estimate the average return period of a future earthquake of a given magnitude, and is, therefore, the basis of preparing probabilistic seismic hazard maps[13].

The Gutenberg–Richter relation represents a power law distribution of magnitudes. Since magnitudes are related to the fault size involved in the earthquake generation, a power law relation for the faults causing earthquakes may be inferred[3].

Another power law relation has been obtained between the magnitude M (surface wave) and the moment M_0 of an earthquake[14]:

$$\log M_0 = d - cM, \tag{2}$$

where c and d are parameters of correlation.

Let both L and l represent linear dimensions. Assuming that the distribution of the earthquake sizes is self-similar and, therefore, the seismic moment M_0 is proportional to L^3 (volume), Aki[8] has shown that a relation, between the cumulative number of faults $N(l > L)$ and L, of the following type may be obtained:

$$\log N(l > L) = e - (3b/c) \log L, \tag{3}$$

where e is a constant.

The power law relation discussed above characterizes the earthquake process. It is appropriate to examine it with the analytical framework for fractals. The parameter characterizing a fractal set is its fractal dimension D.

Let N_n be the number of fragments with characteristic linear dimension r_n in a fractal set; then a power law of the type given below exists for the set[15].

$$N_n = f\, r_n^{-D}, \tag{4}$$

where f is a constant of proportionality and D is the fractal dimension. D lies between D_T (topological) and D_E (Euclidean) and may be a fractional number.

Comparing (3) and (4) the fractal dimension for the earthquake process is $3b/c$. An average value of c is estimated to be 1.5 (ref. 13). Thus, on an average, $D = 2b$.

The value of b in various seismogenic zones of the earth has been found to range between about 0.5 and 1.5. Thus, the corresponding fractal dimension can range between 1 and 3. A value close to 3 implies that earthquake fractures are filling up a volume of the crust, a value close to 2 suggests that it is a plane that is being filled up and a value close to 1 means that line sources are predominant[8].

Slip over faults distributed over a region form a major mode of crustal deformation. At the major plate boundaries this slip is driven by relative velocity between the plates. Of the total slip a fraction occurs over the primary fault system causing the largest (and also smaller) earthquakes and the rest on the secondary fault system[11]. An estimate of the proportion of the slip occurring over the primary faults can permit a better estimation of the return periods of the largest earthquakes.

Representing symmetric slip at triple junctions by rhombic plates that together form a regular hexagon, King[10] showed that for fractal distribution of such faults the corresponding earthquakes follow the Gutenberg–Richter relation. Using this concept of fractal matrix of faults Turcotte[11] represents the slip over an extended fault by superposition of slips over the primary fault and that on a fractal distribution of self-similar system of smaller faults. For such a scheme, the following relation for the ratio of the slip over the primary fault to the total slip over the fault system (i.e. due to the relative motion of the plates with respect to each other) is given:

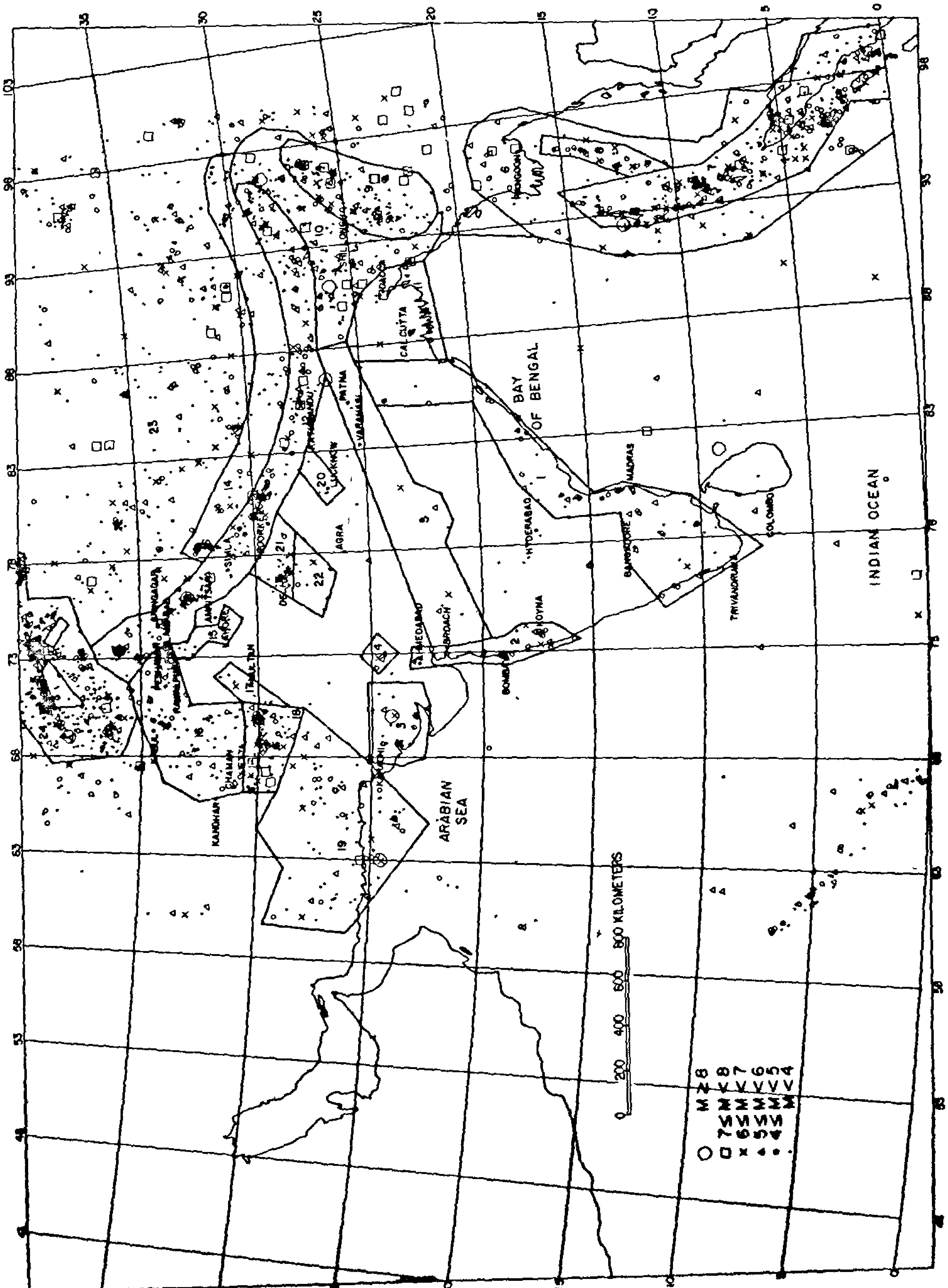

Figure 1. The seismic source zones of Indian region (after Khattri *et al* [13])

$$s_p/s = 1 - 2^{-(3-D)}, \quad (5)$$

where s_p is the slip on the primary fault, s the total slip and D the fractal dimension. For $D = 3$ all the slip occurs on smaller faults, whereas on the other end, for the limiting case of $D = 0$, nearly 88% of the slip occurs on the primary faults. The slip that occurs on a fractal system of subfaults contributes to gradual processes like fold formation and other similar crustal deformations.

The seismicity of the Indian region has been examined in detail by Khattri *et al.*[13]. They identified earthquake source regions on the basis of quasi-homogeneity of seismotectonic processes in the respective source regions (Figure 1). The seismicity catalogue was examined and compensated for incompleteness, and the b values were estimated. In some source zones, those in Peninsular India, the seismicity data were not adequate for an independent and reliable estimate of the b values to be made. For such source zones respective data sets were combined and an estimate of the b values was made. The estimates of the b values and the corresponding D values are shown in Table 1.

We note that the fractal dimensions for various source zones range between 0.894 for the Shillong region and 1.574 for the Peninsular region.

The fractal dimensions estimated above indicate that the fractures in the various source zones are filling up either linear regions (low fractal dimensions of ~ 1) or nearly two-dimensional surfaces (fractal dimension of ~ 1.5). None of the source zones has a volume distribution of sources.

The largest b value of 0.787 is estimated for the source zones of Peninsular India, for which, as mentioned above, the earthquake catalogues were combined. The compositing of the catalogues of earthquakes for various source zones, possibly having different geodynamic processes, may have a biasing effect towards higher estimates for the b value, as we find in the present analysis. This could be a reason why we get a b value estimate of nearly 1 for the global catalogue of earthquakes. In this light the estimates of D for source zones in the Peninsular region are not reliable.

The low b value and the corresponding low fractal dimensionality obtained for the Shillong region (zone 10) is noteworthy and indicates that there is a tendency for few faults with large attendant earthquakes on them. This may be interpreted to be indicative of a relative lack of fracturing and attendant weaker zones in the region. In other words, the region is characterized by the presence of large asperities or barriers which are relatively unfractured and strong. Similarly, the low fractal dimensionalities of source zone 6 in the Andaman region and 18 in the Sulaiman–Kirthar region imply the presence of significant asperities or barriers.

The Himalaya (source region 12) and the source zone 14 to the north have fractal dimensions of 1.514 and 1.226, respectively. This suggests that the fracture distributions are tending to fill a two-dimensional (2-D) space.

Table 1. The b and D values for the seismic source zones

Source zone	b value	D value	Region
1–5	0.787	1.574	Peninsula
6	0.533	1.066	Andaman
7	0.678	1.356	Andaman
8, 9	0.580	1.160	Andaman, Arakan
10	0.447	0.894	Shillong
11	0.787	1.574	Bengal Basin
12	0.757	1.514	Himalaya
14	0.613	1.226	Trans Himalaya
15	0.787	1.574	Himalaya foredeep
16	0.581	1.162	Sulaiman
17	0.787	1.574	Sulaiman foredeep
18	0.485	0.970	Sulaiman–Kirthar
19	0.581	1.161	Kirthar–Makran
20, 21	0.787	1.574	Himalaya foredeep
22	0.787	1.574	Peninsula
23	0.543	1.086	Tibet
24	0.561	1.122	Hindukush

The other source zones are characterized by fractal fracture distributions ranging in between the two end types discussed above.

Following Hirata[12], we calculated the 2-D fractal dimension D_2 using the correlation integral $C(r)$ defined with respect to the spatial distribution of epicentres. It is defined by

$$C(r) = [2/\{N(N-1)\}]N(R < r), \quad (6)$$

where $N(R < r)$ is the number of pairs of epicentres with a mutual distance R smaller than r. N is taken to be 865 events. The value obtained is 0.275. Now D (in Aki's definition) is the volume dimension (3-D). Thus,

$$D = D_2 + 1,$$

from which we obtain a value of 1.275 for D. The value obtained from the b value is 1.514. The two estimates are reasonably close. As noted above, it was not so in the case of the Japanese earthquakes[12]. Thus, we consider that the present analysis is valid.

We next consider the estimation of the slip on the primary fault in the Himalayan collision zone. The convergence rate between India and Asia is estimated to be 5.5 cm/yr. Of this a fraction is released on the primary fault and the rest on the fractal distribution of subfaults. The primary fault is considered to be the basement thrust[16], on which the largest earthquakes occur. This is a north-dipping surface constituting a megathrust extending along the Himalaya for about 2400 km and having an average width of approximately 80 km. It is desirable to estimate the fraction of the total convergence rate released over this fault, as that would enable one to estimate the average return period of great damaging earthquakes. In the present analysis the source zones 12 and 14 are considered to be the main players in

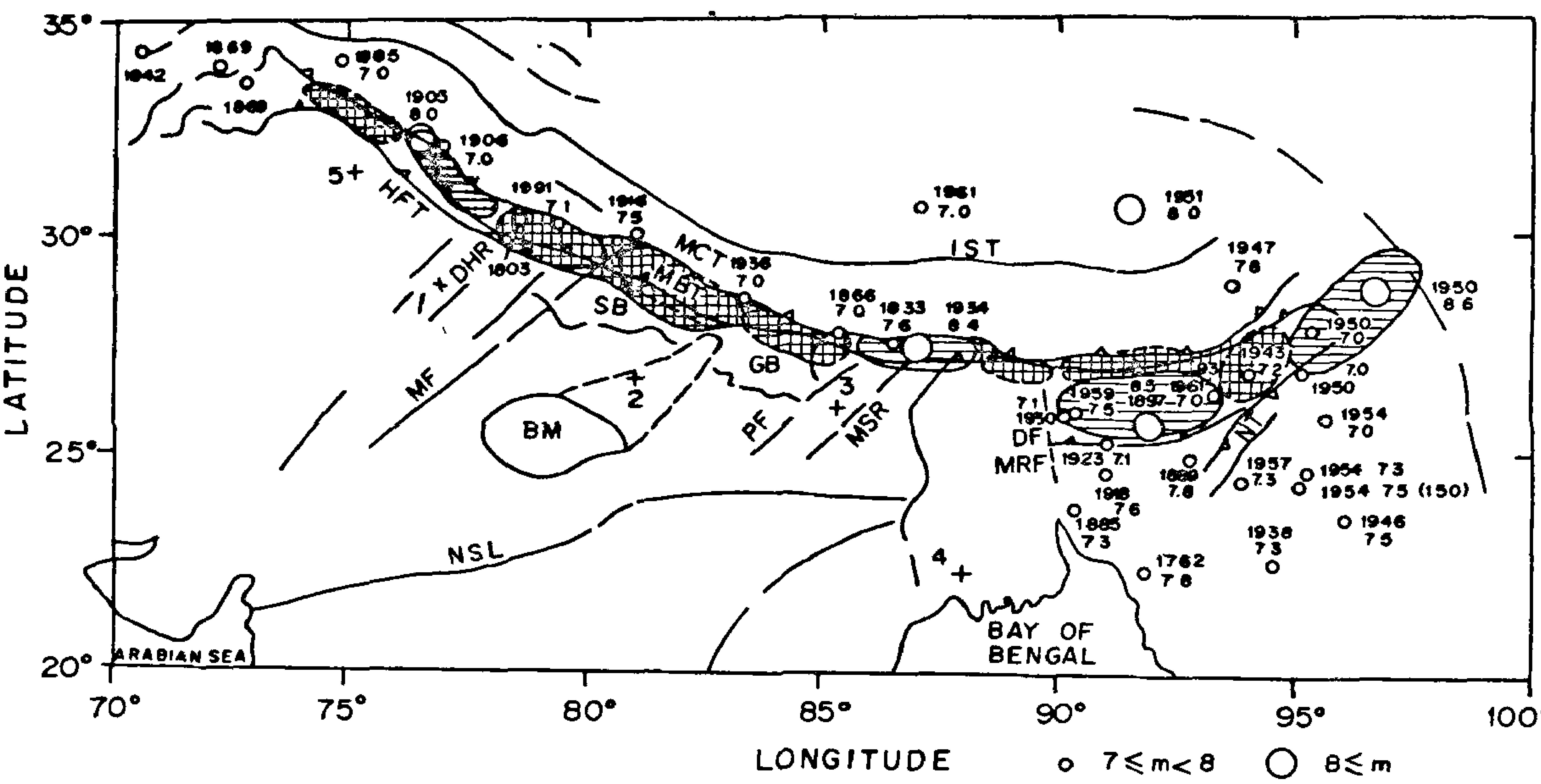

Figure 2. The larger earthquakes of Himalaya The rupture zones of the great earthquakes are shown by hatured areas The outlines of the sections of the seismic gaps that may rupture in a single earthquake are marked by dashed lines It is possible that the central seismic gap may rupture in a single great earthquake[20]. 1: Delhi; 2· Lucknow; 3. Patna, 4· Calcutta; 5. Lahore; BB Bundelkhand massif, DF Dauki fault, DHR. Delhi–Hardwar ridge; GB Gandak basin; HFT: Himalaya frontal thrust, MF· Moradabad fault, MRF: Madhupur fault; MBT· main boundary thrust, MCT main central thrust; MSF Monghyr–Saharsa ridge, NSL. Narmada Son lineament, NT. Naga thrust; PF: Patna fault; SB Sarda basin (after Khattri[22])

releasing the strain. Further, the source zone 14 comprises of only the fractal subfaults, the primary fault being confined to the source zone 12. Since the fractal dimensions in the two source zones are quite close, we take the value for the same from source zone 12. Using this value in equation (5) we obtain a value for slip rate s_p on the basement thrust to be approximately 3.5 cm/yr. The remaining slip is released on the smaller fractally distributed subfaults. Thus, on an average, for a magnitude 8+ earthquake involving 10 m of slip a time interval of 285 years will be required. The average slip rate for underthrusting is estimated to be of the order of 1.0–2.5 cm/yr (ref. 17) from moments of large earthquakes and geological evidence, 1.5–2.2 cm/yr (ref. 18) from GPS measurements in Nepal section of the Himalaya, and 7 ± 4 cm/yr (ref. 19) from geodetic measurements using levelling. All these estimates are of the same order of magnitude, with the estimate for the slip rate obtained here being somewhat higher. If part, say, one-third, of the slip on primary faults is taken up in source zone 14, then the estimate obtained here will be almost totally congruent with the other estimates. We note that while the geological estimate is an average over several million years, the estimate from GPS data is instantaneous. One of the two estimates derived from earthquake data is based on a period of most recent, about a couple of hundred years, data and magnitudes ranging from smaller to largest earthquakes, while the other one is derived on the basis of great earthquakes only. The estimate from levelling data refer to a period of a few decades. The consistency of estimates derived by such a diversity of databases as well as concepts serves to instill a measure of confidence in the estimated value.

The Himalayan plate boundary has experienced four great earthquakes ($M > 8$) in 1897, 1905, 1934 and 1950, which ruptured and relieved accumulated strains over about 200–300 km long sections (Figure 2). These earthquakes have left sections of the plate boundary where the accumulated strains have not been relieved for at least 200 years; these sections form the seismic gaps[16–26]. The large earthquakes in the Himalaya are recorded from 1803 onwards and a total of 15 with magnitudes (estimated from damage reports for earthquakes in the previous century) between 7.0 and 7.6 have occurred. Of these, two are aftershocks of the great 1950 Assam earthquake and one of the great 1905 Kangra earthquake. One of magnitude 7.6 occurred in 1866 in the rupture zone of the subsequent great 1934 Bihar–Nepal earthquake. Four events, in 1923, 1930, 1959 and 1960 have occurred in the rupture zone of the great Assam earthquake of 1897 and may be considered to be its aftershocks One event has occurred in 1943 in the Assam gap.

An 800 km long central seismic gap exists between the rupture zones of the Kangra and the Bihar earthquakes. It is possibly divided by transverse geological structures into three sectors. The first lies between the Delhi–Hardwar ridge (eastern margin of the 1905 Kangra rupture zone) and the Moradabad fault, the next is between the Moradabad fault and the Bundelkhand massif promontory, and the third one is between the promontory and the Patna fault. The first sector has had earthquakes in 1803 and 1991. One event occurred in line with the Moradabad fault, another event occurred in 1866 in a region aligned with the Patna fault. The average slip in a $M \sim 7$ earthquake is of the order of 1–2 m. This has been released over faults having relatively small areas, in various parts of the central sector, through such earthquakes since 1803. The cumulative fault area (overall $M \sim 7$ earthquakes) is estimated to be merely about 4500 km^2. On the other hand, the seismic gap has accumulated a slip of at least 5–7 m over an area of 48,000 km^2 (length of about 800 km). This slip accumulation would be much more in case the last great earthquake in the gap occurred 300 or more years ago. Thus, this and similarly the Assam gap have very high potential for giving rise to great earthquakes in the not too distant a future (within about 100 years); depending on how far back in time the last great earthquake ruptured these seismic gaps, the next one may be advanced in time accordingly. In this connection it may be noted that the earliest report of a damaging earthquake in northwest Himalaya appeared in 1669 and in Nepal Himalaya in 1833 ($M = 7.6$). Thus, we may reasonably conclude that an earlier great earthquake in the Himalaya has occurred prior to the 18th century. This already gives at least 300 years of strain accumulation.

The seismic zone of the Himalaya has a fractal dimension lying between 1 and 2, and may be taken to behave approximately as a planer zone. On the other hand, the Shillong massif region has a fractal dimension of less than 1. Thus, this region is relatively dominated by the presence of asperities and/or barriers.

The results of the present study supply confirmatory evidence supporting the conclusions arrived at by earlier studies[16–26] that the seismic gaps in the Himalaya have considerably increased potential for the great earthquakes to occur within the next 100 years or so.

Therefore, there is an urgent need to recognize the above findings regarding the lurking earthquake disasters. The public and government opinions need to be mobilized for a well-planned effort for intensive earthquake studies in the Himalaya, which at present are at a very rudimentary plane. Also there is an urgent need for mounting on a war footing a systematic earthquake hazard mitigation plan to ameliorate the disastrous consequences from future earthquakes.

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