Discovery of top quark at Fermilab

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The CDF collaboration at Fermi National Accelerator Laboratory has recently announced evidence for the existence of a top quark with a mass of 174 GeV/c². This discovery provides us now with a completed picture of the matter content of the standard model of elementary particle physics. This article is aimed at providing a self-contained introduction to this report from a non-specialist's point of view.

Recently, there have appeared some reports\textsuperscript{1-3}, including a newspaper item, announcing the observation of the top quark, with its mass quoted as 174 GeV/c². This has caused some excitement in the particle physics community. These reports are based on the experimental data gathered by the CDF (collider detector at Fermilab) collaboration working at the Fermi National Accelerator Laboratory (FNAL) in Batavia, Illinois, in the United States.

Even though no practising experimental or theoretical particle physicist questioned in her heart the existence of top quark prior to this reported discovery, it is nonetheless very satisfying to have an experimental confirmation of this belief. This is because the detection offers now a completed picture of the matter content of the so-called standard model of particle physics. What delayed the detection of the top quark so far was the very high mass of it, quoted above. No experimental facility available before the Tevatron accelerator at FNAL could boast of the initial energy and luminosity required for observing the signatures of top quark production with a reasonable rate of detection. Even the presently reported detection by CDF is based on comparatively not a very large statistics of events. The confidence of the collaboration in the truthfulness of this detection is based on their experimental skills with the detectors and very detailed statistical analyses performed on the data.

In this article we shall first take a cursory look at the standard model of particle physics and the place of top quark in it. This will include some justification, from the point of view of a theorist, of its existence. We will then turn to a brief survey of the methods of experimental particle physics along with a list of existing facilities. This will be followed by a non-specialist's discussion of the Fermilab experiment and its subsequent analysis which has led to the announced discovery.

The standard model

Though called a model, it actually represents a systematic organization of, by now, fairly universally accepted theoretical ideas and experimental facts about the elementary particles and their dynamics. It enjoys the status of a theory with the exception that there are several parameters in it which cannot be explained in a fundamental way. From a mathematical viewpoint, the standard model is a relativistic quantum field theory based on the gauge symmetry of SU(3)_c × SU(2)_L × U(1)_Y group. The principle of special relativity is incorporated in it by demanding Lorentz invariance of this field theory. The gauge symmetry (also called gauge invariance principle) serves a threefold purpose. It provides, firstly, a unique set of labels to each of the particles for identification purpose, protects some of them from acquiring a mass until the symmetry is broken by the Higgs sector and, thirdly, determines the dynamics of the particles in the standard model by dictating the structure of allowed interactions amongst them. It is worthwhile to note here that the physical reality in terms of the masses, charges of the particles, etc., and their behaviour emerges only upon breaking of gauge symmetry to a lower residual one.

The particle content of the standard model, from a book-keeping angle, can be divided into three main parts: matter, gauge and Higgs sectors. The matter sector consists of quarks and leptons, which are the elementary constituents of the matter we are made up of. They are elementary in the sense that they do not have any further substructure. Some candidate theories have been proposed in which they are treated as composites, but they represent deviations from the standard model and we shall not indulge in these. The particle content of the matter sector is displayed below:

**Leptons**

\[
\begin{pmatrix}
\nu_e^L \\
\nu_\mu^L \\
\nu_\tau^L \\
e^-_R, \mu^-_R, \tau^-_R
\end{pmatrix}
\]

**Quarks**

\[
\begin{pmatrix}
u^c_L \\
d^c_L \\
\bar{u}^-_R, \bar{d}^-_R, \bar{c}^-_R, \bar{s}^-_R, \bar{t}^-_R, \bar{b}^-_R
\end{pmatrix}
\]

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The labels L and R refer to the left- or right-handedness of the various particles, and it is apparent from the way the particle content is displayed above that there is an inherent asymmetry in Nature about handedness. This is an observed fact and a lot of effort has gone in to correctly take account of it in the formulation of the model.

The left-handed species occur naturally in doublet representation of the weak-isospin symmetry described by the SU(2) labels, which governs the weak interactions amongst particles. This property is shared by both kinds of fermions, i.e., leptons and quarks. In the leptonic sector, the lower member of an individual doublet is either the familiar electron (e⁻) or its siblings, i.e., a muon (μ⁻) or a tau (τ⁻) lepton. The superscripted minus sign indicates the electric charge of a given lepton. Their doublet partners are the neutrinos (ν) carrying a subscript which identifies them by their charged partners. The neutrinos are chargeless and are considered to be extremely light. The notion of massive neutrinos, e.g., of the Majorana kind, is very interesting from the point of view of particle physics phenomenology, but it represents deviations from the standard model and so we shall refrain from considering it. The right-handed leptonic sector contains only the charged leptons; there are no right-handed neutrinos. This clearly indicates the left–right asymmetry.

The quarks also occur as left-handed doublets of the SU(2) gauge symmetry, and so experience the weak interactions. One difference with the leptons is that both the members of a left-handed quark doublet have their right-handed counterparts. They, however, occur as singlets of the SU(2) group, i.e., do not feel the weak interactions. In addition to the weak interactions, the quarks also experience the force of strong interactions. This is encoded in the theory by assigning them labels under the SU(3)colour symmetry denoted by the superscript α. This index takes values from one to three. The strong force is felt by quarks of both handedness. The interaction is called strong for the reason that it eventually leads to the binding of various quarks together to form the observed nucleons: e.g., a proton is composed of two up (u) and one down (d) quarks, whereas a neutron has two d’s and one u. This composition ratio, along with the fact that the corresponding electric charges are +1 and 0, respectively, is sufficient to deduce that the electric charges of a u-quark is +2/3 and that of a d-quark is −1/3. In order to save the clumping up of notation, these are usually not displayed. Additionally, both the quarks and leptons carry a weak hypercharge (Y), which is also not displayed usually. It is known to contribute partially to the net electric charge of a particle and thus participates to a known extent in the electromagnetic interactions of them. It is due to this ‘mixing’ of the weak and electromagnetic interactions that they are sometimes referred to as electroweak interactions.

The totality of the left-handed electron–neutrino doublet, the up–down quark doublet and their right-handed counterparts, e_L, u_R and d_R, represent one family or a generation of fermions. It contains in all fifteen states. It remains an outstanding puzzle of particle physics as to why Nature has chosen to replicate these generations thrice, with identical charges under the symmetry group of the standard model. These three generations are known to mix to a small extent and this mixing is believed to have a very deep connection with the observed violation of two discrete symmetries of Nature, viz., charge conjugation and parity, taken together.

The gauge sector includes the part of the standard model responsible for mediation of various forces. All the species of particles, according to the rules of underlying quantum field theory, are supposed to interact via an exchange of the so-called gauge bosons. This idea is just an extension of the familiar notion of two electrically charged particles interacting via an exchange of photons. There is, however, one little complication: the weak and strong interactions are nonlinear, i.e., they permit self-interactions amongst the gauge bosons which transmit the corresponding forces. This is a property not shared by photons; they do not interact amongst themselves in the absence of matter whereas the gauge bosons of the other two kinds most certainly do.

If a gauge symmetry is described by a SU(n) group structure then the simple rule is that it correspondingly has n² − 1 kinds of gauge bosons: e.g., the SU(2)_L symmetry of weak interactions manifests itself in the existence of three gauge bosons: W⁺, W⁻ and Z⁰. Here the superscript denotes the electric charge of a species. These are collectively referred to as weak bosons. It is a triumph of technology of experimental particle physics that they have actually been produced and detected in the laboratory. Their masses and decay widths have been measured to a great accuracy and show excellent agreement with theory, almost to the extent of embarrassing the proponents of possible deviations from the standard model.

Similarly, the SU(3)_c symmetry of strong interactions, correspondingly, has eight gauge bosons called gluons. They are in turn responsible for binding the quarks together to form nucleons and the residual interaction leftover manifests itself as the nuclear force. The gluons are themselves charged under the force of strong interactions and carry SU(3)_c labels appropriate to the fact that they mediate the force between a quark (q) and an antiquark (q̅). An interesting feature, not shared by the electroweak bosons, is the formation of 'glueballs' as a result of self-interactions of gluons. These bound states do not require the presence of any matter, just the strong nature of the force energetically favours binding. The SU(3)_c symmetry remains unbroken in Nature, and a profound theorem in quantum field theory implies that the gluons remain exactly massless, as a result. This is
in contrast with the electroweak gauge bosons, which acquire masses as a result of symmetry breaking due to the Higgs sector.

The Higgs sector consists of a left-handed doublet of scalar particles, with the upper member $\Phi^+$ carrying a positive charge whereas the lower member $\Phi^0$ is neutral. According to the Higgs mechanism, the ground state of the standard model prefers to be less symmetric compared to the allowed symmetry corresponding to the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. This fact is signalled by the lower member of the Higgs doublet $\Phi^0$ acquiring a nonzero vacuum expectation value (VEV) of about 246 GeV. The masses of the electroweak gauge bosons and fermions are directly related to this VEV. Although the Higgs mechanism is a technically satisfactory way of generating the observed masses in the standard model, it is the most poorly understood part of the whole theory, in a fundamental way. The Higgs mass is a completely free parameter and the origin of it is another outstanding problem of particle physics. Since we have no a priori clue about it, direct detection of the Higgs particle is very difficult. The on-going searches for it are based on the upper and lower bounds on the Higgs mass obtained from dynamical considerations. In fact, the top quark discovery announced by Fermilab is an outcome of the experimental setup whose ultimate aim is to look for the Higgs particle.

**Dynamics of the standard model**

So far, we had restricted our discussion to the description of the table of elementary particles, i.e. their nomenclature and the labels associated with them for identification purpose. The dynamics of these particles, viz. how these particles interact with themselves and others, what decides whether a particular interaction is allowed in the standard model, what would be the outcome of a typical scattering experiment involving these particles, etc., are the kind of questions that are answered by the underlying mathematical structure. This structure is specified by the lagrangian of the relativistic quantum field theory with the gauge symmetry of the standard model.

The lagrangian incorporates a quantum field (which depends on the spacetime coordinates as a function) for each degree of freedom associated with the elementary particles. In the corresponding mathematical expression, there are basically two kinds of terms. The first term represents the sum of kinetic energy terms for individual species while the latter is the total potential energy of various particle interactions. In general, the allowed terms are computed by explicitly incorporating invariance of the lagrangian under Lorentz and gauge symmetries. All the possible terms satisfying these requirements constitute the final lagrangian and fix the dynamics. From the lagrangian, one can derive the Feynman rules (as originally suggested by Richard Feynman in the context of quantum electrodynamics). These can be used to compute scattering cross-sections, probabilities, etc., corresponding to an experimental situation involving interactions of various particles, as a function of their charges, masses, momenta and energies. One can also compute the rates of production and decay of any particle. This theoretical technology is at the heart of designing of actual laboratory experiments set up to produce various elementary particles and observe their properties.

**Collider physics**

As remarked previously, stable matter consists of the first-generation fermions alone. The members of the second or third families of quarks and leptons do not occur naturally since they have reasonably short times to live before decaying eventually to the first-generation products. However, nothing prohibits their existence for these periods. They can be produced by colliding extremely energetic stable particles and thereby creating a very small region in space with very high energy density. This artificially created fireball then subsequently relaxes by producing a variety of particles, including those belonging to the higher generations. These particles can travel nontrivial distances, determined by their masses, energies and lifetimes, before they decay further, producing a cascade. The final products are typically first-generation particles, and/or stable nuclear matter and photons. The relative chances of production of various particles are specified by the branching ratios and are calculable from the Feynman diagram technology. The leptonic final product states are usually 'clean' in the sense that they consist of the leptons themselves, whereas the situation with final states involving quarks is quite murky. This again is an artefact of the strong nature of the force of quantum chromodynamics and what is called confinement of colour. Loosely speaking, the latter means that the only observable quark composites are the combinations which are neutral under the SU(3) colour symmetry. This causes hadronization of the constituent quarks into mesons and baryons, i.e. nucleons in general, rapidly after they are produced in a collision. The process of hadronization is a very complicated one since it involves many particles forming a quantum-mechanical bound state. It also includes gobbling up of gluons. The final picture which emerges is that the quarks produced at the interaction point (also called a vertex) in a collision experiment travel a short distance and hadronize thereafter. For a detector placed sufficiently away from the collision site, they therefore appear as streams of nucleons, and are usually referred to as jets.

Einstein's mass—energy relation tells us that there is a threshold which must be exceeded by the total energy—momentum of the initial colliding particles, in order to
produce particles of other kinds in a collision, viz. the rest mass of the produced unstable particle. If the initial total energy (conveniently parametrized by a variable \( \sqrt{s} \)) is continuously increased, we would meet successively the mass thresholds corresponding to more and more massive particles. This way the entire hierarchy of elementary particles can be probed, since the known branching ratios and the information from observed final states can be used to invert the experimental situation and infer the production of a massive particle as an intermediate state. This is precisely the manner in which existence of particles from our table is established. It is now clear that the heavier the particle, the higher is the initial energy needed to produce it. The top quark being a member of the third generation and the heaviest of the lot, we now understand why it was not detected in earlier collider experiments, i.e. until the collider energy was cranked up sufficiently at Fermilab.

In practice, a collider experimental set-up consists of a particle accelerator, a collision chamber to which various detectors are attached and computers for gathering the data output from the electronics of detectors as well as for performing the subsequent analysis. The accelerators are usually linear or circular in shape. The collisions are produced in either a fixed target manner or in a head-on way, commonly referred to as 'in the centre of mass'. Typical parameters of an accelerator are: the type of particles accelerated, the beam energy, and the luminosity, i.e. the number of particles in a beam per unit area of cross-section per unit time. Colliders have been conceived and operated with all the three possible combinations: electron–positron (e+e−), electron–proton (ep) and proton–(anti)proton (pp). The major facilities around the world are DESY in Germany, KEK in Japan, CESR and SLAC in USA for the electron–positron type, DESY for the electron–proton kind and CERN in Europe and Fermilab in USA for the hadronic colliders. The electron–positron machines are the cleanest in terms of analysis of the data produced but they have inherent limitations on the highest energy they can reach. Presently, efforts are being made to push this upper limit using innovative technologies. The hadronic colliders do not have this problem but they are more expensive to build and operate since they are less efficient in the sense that much higher beam energies are required compared to the leptonic colliders for producing particles of a given rest mass. The reason underlying this is again the strong nature of the force; a lot of energy is wasted in hadronizing the nucleons to produce reasonably energetic quarks which actually participate in the collisions. All the quarks and leptons as well as the electroweak gauge bosons have been produced and studied in some variant or the other of a collider experiment by one of the facilities cited above.

In the remainder of this article, we shall see from a non-specialist's point of view how the recently announced discovery of the top quark was made at the Tevatron in Fermilab. Before proceeding with that, it is worthwhile to pause and wonder as to why physicists believed in the existence of a top, prior to its discovery. The motivation arose from theoretical considerations. According to the standard model, left-handed quarks occur as doublets of the SU(2)\(_L\) gauge symmetry. This fact is encoded into the weak-isospin charge assignments to the upper and lower components in a doublet. They are plus or minus half respectively, for a left handed doublet. These charges are not mere labels, but they actually govern the dynamics of the standard model by means of the conservation laws associated with them. They are measurable in an experiment: e.g. prior to the recent discovery of the top, it was known experimentally that the bottom (b) quark carried the SU(2)\(_L\) charge of minus half. Internal consistency of the theory then required that it be partners with some quark forming a doublet, which would carry plus half charge. The up (u) and charm (c) are possible candidates, but they are known to form doublets with the down (d) and strange (s) quarks and, therefore, are unavailable as a partner for the bottom quark. Also, the would-be partner had to be heavier than any of the known quarks in order to have had eluded experimental detection so far. This would-be partner is christened as the top (t) quark. Its mass is a parameter in the standard model and so only bounds on it could be established using theoretical consistency arguments. A lower bound of 130 GeV/c\(^2\) was already available from previous experiments at Fermilab, whereas a reasonable upper bound of about 1 TeV/c\(^2\) is inferred from the viability of the standard model.

The top quark search at Fermilab

The experimental set-up at FNAL consists of a pair of circular rings producing highly energetic proton and antiproton beams which are eventually made to collide head-on in a small region called the interaction chamber. Various detectors, analysers and calorimeters are fitted on the outside of it, transverse to the beam direction. Amongst these are the CDF and D0 detectors, which played a major role in the top quark search. The beam energy for this experiment corresponded to the parameter value \( \sqrt{s} \) of 1.8 TeV. The experiment ran for several months and the data collected over this period are called the 1992–93 run of the Tevatron. The luminosity summed over this entire running period, and therefore appropriately called integrated luminosity, was \( 19.3 \times 10^{33} \text{cm}^{-2} \).

Theoretically, the most probable mechanism of top quark production at the above collider energies is via the production of top–antitop (tt) pairs. The T-quark is supposed to have the same mass as a top but it carries exactly the opposite charges of the top quark otherwise. Existence of such antiparticles is forced upon us by the
relativistic nature of the underlying quantum field theory describing the standard model. This is in conformity with the conservation laws for various charges since a particle–antiparticle pair does not carry any net charge and, therefore, would not violate any symmetries if produced in a $q\bar{q}$ or $p\bar{p}$ collision. The Feynman rules corresponding to processes involving antiparticles can simply be obtained from those for ordinary particles by reversing all the charges and directions of flow of all the momenta. The standard model processes that involve $t\bar{t}$ production from the initial $p\bar{p}$ collisions are summarized in Figure 1. For the purpose of our discussion, time can be thought of as flowing from left to right, i.e., subsequent decays will appear to the right of the event denoting the production of a species.

In each of these diagrams, a black dot indicates the initial interaction vertex, viz. the point of collision of $q\bar{q}$ in the collider. It produces a $t\bar{t}$ pair. Once produced, a top quark (from here on we shall follow only the $t$-branch, analogous statements can be replicated for the $\bar{t}$-branch) loves to decay into a $W^+$ gauge boson and a $b$-quark. This requires that the energy carried by the top is greater than the sum of the rest masses of the $b$ and the $W^+$. This process and the analogous one for the $\bar{t}$ is common to all the three diagrams. Further distinction arises from what is the decay mode of the $W^+$ arising from the $t$-decay and the $W^-$ from the $\bar{t}$-decay. According to the standard model, a $W^+$ (or a $W^-$) can decay either to a lepton–(anti)neutrino pair (e.g., $e^+ + \nu_e$ or $\mu^+ + \bar{\nu}_\mu$), generically denoted as $l\bar{\nu}$ pair or a $q\bar{q}$ pair. These are, respectively, referred to as leptonic or hadronic decay modes of a $W$-gauge boson.

If both the $W$'s decay leptonically, then the final state detected by the detectors comprises of two leptons and two jets arising from the hadronization of the $b$ and $\bar{b}$ quarks produced earlier in the decay of the top. The (anti)neutrinos escape direct detection owing to their charge neutrality as well as very weak coupling to matter. Their production is usually inferred kinematically from the missing energy–momentum needed to satisfy the associated conservation law. This chain of events is depicted in Figure 1a and is referred to by the CDF collaboration as the dilepton mode. Figure 1b corresponds to the possibility that one of the $W$'s decays leptonically while the other hadronically. This produces the final state comprising of a lepton $l$, (anti)neutrino $\bar{\nu}_l$ and four jets arising from the hadronization of the $b\bar{b}$ and $q\bar{q}$ pairs. This is called leptons+jets mode. The final possibility (Figure 1c) is that both the $W$'s decay hadronically, which corresponds to six jets in the final state and no leptons. This is the pure-jets mode.

The experimental situation, however, is nowhere as close to the clearly separated three possibilities described above. The actual data gathered are extremely messy and it is quite a task to isolate the 'signal' corresponding to the $t\bar{t}$ production and decay. Since the detectors detect only the final states, the signal is usually corroborated by various alternate allowed physical processes that can mimic the final state but which do not involve the top. Their contribution to the final detection
is collectively referred to as background and represents the noise in the data. The background contribution varies from mode to mode and also significantly depends on the on-shell mass of the top.

The CDF detector at Tevatron is equipped to isolate the signal coming from the dilepton and lepton+jets processes. The background for the fully hadronic final state is so heavy that it makes impossible for the experimentalists to say with any degree of confidence that even a single event of the pure-jets mode involves an intermediate $t\bar{t}$ production. This is primarily due to the fact that the jets contain all kinds of hadrons as well as that hadronic jets can be produced with equal ease in the primary interaction region, as a result of the $q\bar{q}$ collisions. The CDF analysis of the data simply rejects all the events in the fully hadronic channel and concentrates on the data corresponding to the dilepton and lepton+jets mode. These are the modes depicted in Figure 1a and b. Theoretical estimates of the probabilities for channels corresponding to Figure 1a and b are 5% and 30%, respectively, while the remaining 65% corresponds to the rejected fully hadronic channel. Needless to say, majority of the data are rejected and retrieving the $t\bar{t}$ production signal is like looking for a needle in a haystack. In order to reduce the background faking the desired signal in the remaining modes, CDF collaboration has devised several ingenious methods. Amongst other things, they introduce several kinematic cuts on the data to further reject the events from the data which show only two or three jets. By running Monte Carlo simulations corresponding to various background possibilities emulating the signal, they estimate the number of background events, and excess over which can then qualify to be the real signal indicating intermediate $t\bar{t}$ production.

The CDF collaboration's quest for the true $t\bar{t}$ events does not stop at this stage. They employ further two clever techniques called SVX and SLT tagging. SVX tagging involves an experimental set-up called the secondary vertex detector (SVX). It has the ability to distinguish between different points in the interaction region from where different b-jets can originate. It can, e.g. differentiate between the main interaction vertex of the $p\bar{p}$ collision and a secondary vertex from which a b-jet can originate. The events which are SVX-tagged in this manner have a better chance of having been produced through the $t\bar{t}$ channel as opposed to the non-SVX tagged events. The other technique is called soft lepton tagging (SLT). The b-quarks produced as a result of the $t\rightarrow Wb$ decay are capable of decaying further, instead of simply hadronizing into jets. A typical decay of this type would be $b\rightarrow l\nu X$, where $l$ is a lepton, $\nu$ is a neutrino and $X$ is everything else. The lepton produced from such a b-decay typically has much lower energy, hence the adjective 'soft' as compared to a lepton produced from, say, a W-decay. The latter contributes to the background. The reason for this is that a W-boson is much heavier than a b-quark. The upshot of all this event rejection analysis, various kinematic cuts introduced on the data and folding in of the statistical and systematic errors is that CDF claims to see an excess of dilepton, SVX- or SLT-tagged events above the estimated background when the number of jets in the final state exceeds three. This excess is interpreted as evidence for top quark production. For one or two jets there is no signal above the background. Just to appreciate the labour involved in data analysis, besides the experimental intricacies, the final number of candidate events is a mere fifteen whereas the total number of events gathered in the 1992–93 run is about $10^{13}$. Using the candidate events which survive all the acid tests mentioned above, and the kinematics of a typical event, CDF collaboration made a likelihood fit to the mass of the top quark. They quote its most probable value as $174 \pm 10^{+3}_{-5}$ GeV/c$^2$. The first uncertainty in this quote is due to statistical errors while the latter is due to systematics.

The elaborate analysis performed by CDF and their conclusion, though convincing, are not altogether free of criticism. This is primarily due to the very small number of candidate events. Another objection is due to the claim by the other collaboration D0 at Fermilab (which incidentally does not have the SVX facility), that their data are consistent with the hypothesis of no top. Some theoreticians object to the fairly large value of $M_{top}$ quoted by CDF, since if one preferred living out on the edge of errors with the CDF data, the chances of correctness of the standard model as the theory of elementary particles are significantly reduced. These objections have, of course, been carefully noted by CDF collaboration and only additional data from the 1993–94 run of the Tevatron can improve the statistics further. In the meanwhile, it is, however, not very discomfiting to accept the present CDF analysis and even to rejoice the discovery of the top quark!

Note added in proof. According to the recently announced reports based on the 1993–94 run of the Tevatron the D0 collaboration also has confirmed the observation of the top quark made earlier by the CDF collaboration referred to in this article.

1. Fermilab preprint PUB-94/097-E, CDF/PUB/TOP/PUBLIC/2561, CDF collaboration
2. Evidence for Top Quark Production in pp collisions at $\sqrt{s} = 1.8$ TeV, CDF collaboration, Phys. Rev. Lett., 1994, 73, 225
3. Phys Today, May and June 1994 issues

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