Harnessing chaos: Synchronization and secure signal transmission

M. Lakshmanan and K. Murali

Chaotic motion, by definition, corresponds to unpredictable dynamical behaviour in deterministic nonlinear systems, which is sensitively dependent on initial conditions. Surprisingly, in spite of the complex system dynamics, such motions can be tamed and controlled to regular dynamics through minimal predetermined external perturbations. More interestingly, chaos can also be used in a purposeful way, thereby leading to new technological possibilities of spread spectrum secure communications.

CHAOTIC motion is ubiquitous in nature. It is intrinsically unpredictable and sensitively dependent on initial conditions – nearby trajectories of the underlying nonlinear systems exponentially diverge. Consequently the phase trajectories (in the phase space) can take complicated geometrical structures[1–3]. A dissipative nonlinear dynamical system, for e.g., exhibiting chaos can admit a strange attractor of fractional (noninteger) dimension, the so-called fractal[1–3]. Naturally one would expect such a complex motion cannot be controlled or altered unless drastic changes are made to the structure of the system. Surprisingly, recent investigations in this direction have clearly demonstrated[4–8] that not only can chaotic systems be tamed and controlled by minimal preassigned perturbations to avoid any harmful effects to the physical system under consideration but also controlling can be effected in a purposeful way to make the system evolve towards a targeted goal dynamics[4–8]. It is interesting to note that J. von Neumann had anticipated this possibility when he predicted around 1950 that the atmospheric weather can be made to evolve in a specific way by effecting small, carefully predetermined perturbations. Examples of such controlling in daily life include a bicyclist balancing a cycle while riding fast even without using the handlebar by occasional body nudges or the guiding of a communication satellite to its geocentric orbit through small electromagnetic signal nudges from the earth.

Another but related consequence of sensitive dependence on initial conditions is that two identical but independently evolving chaotic systems can never synchronize to be in phase, as any infinitesimal deviations in the starting conditions (or the system specification) can lead to exponentially diverging trajectories making synchronization impossible. Contrast this with the case of linear systems (and also regular motions of nonlinear systems), where the evolution of two identical systems can be very naturally synchronized. In this connection, the recent suggestion of Pecora and Carroll[9,10] that it is possible to synchronize even chaotic systems by introducing appropriate coupling between them has changed our outlook on chaotic systems, synchronization and controlling of chaos, paving ways for new and exciting technological applications: spread spectrum secure communications of analog and digital signals. In this article we essentially wish to bring out these exciting developments.

Chaotic dynamics: example Duffing–van der Pol oscillator

In order to appreciate the concept of chaos synchronization let us consider a typical chaotic dynamical system, namely the Duffing–van der Pol (DVP) oscillator, especially from a circuit theoretic point of view keeping in mind the ensuing signal transmission applications (though other dynamical systems can be equally well considered). A typical circuit realization[11–13] of the DVP oscillator is given in Figure 1. It consists of the linear circuit elements L (inductance), C_1, C_2 (capacitors) and R (resistor) and a nonlinear resistor (N),

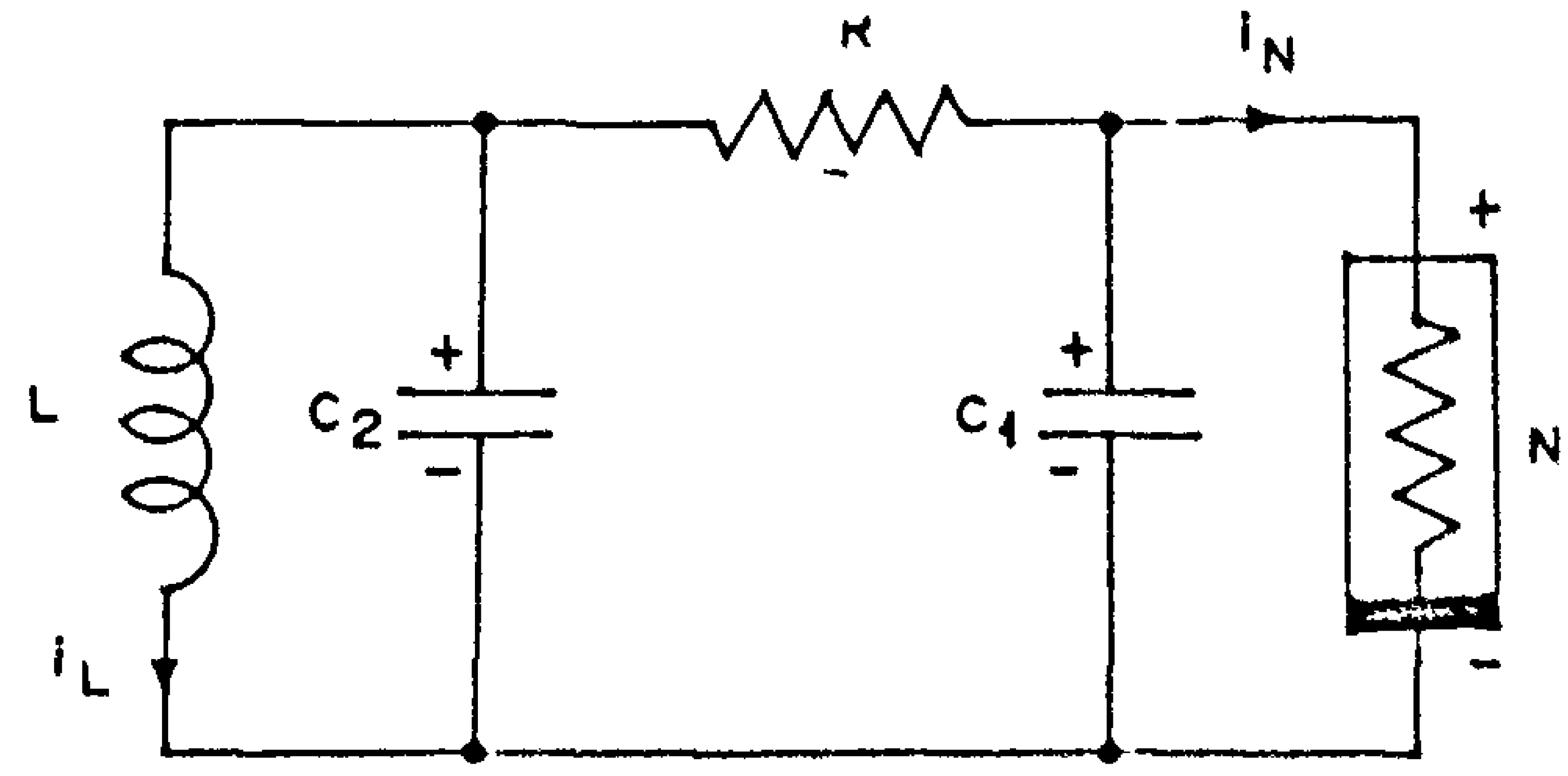

Figure 1. Circuit realization of the DVP oscillator

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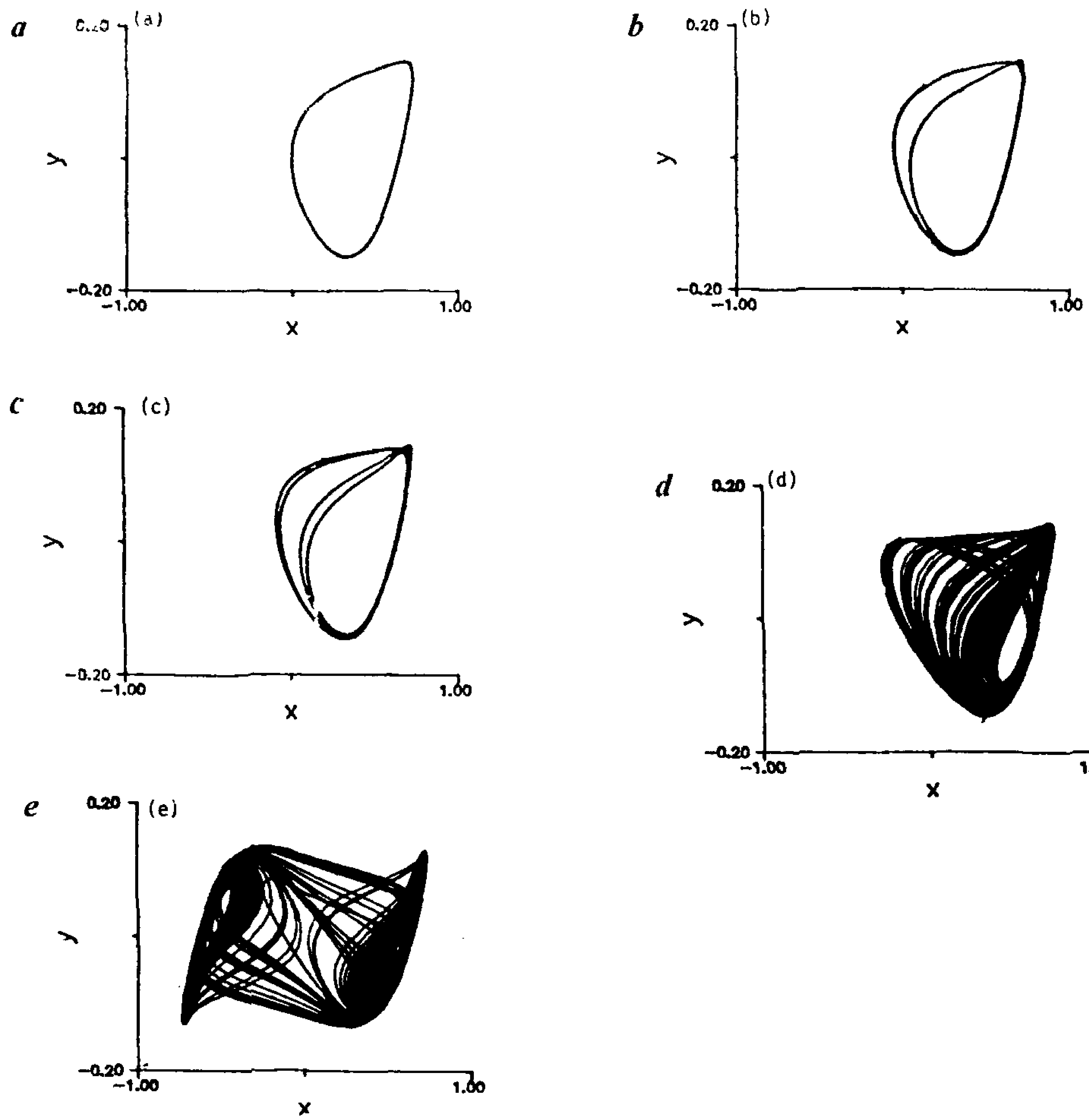


Figure 2. Period doubling bifurcations to chaos of the DVP oscillator Phase portrait for $\alpha = 0.35$, $v = 100.0$ and *a*, $\beta = 800$ (period-1); *b*, $\beta = 750$ (period-2), *c*, $\beta = 710$ (period-4), *d*, $\beta = 600$ (one band chaos), *e*, $\beta = 300$ (double band chaos) of equation (2).

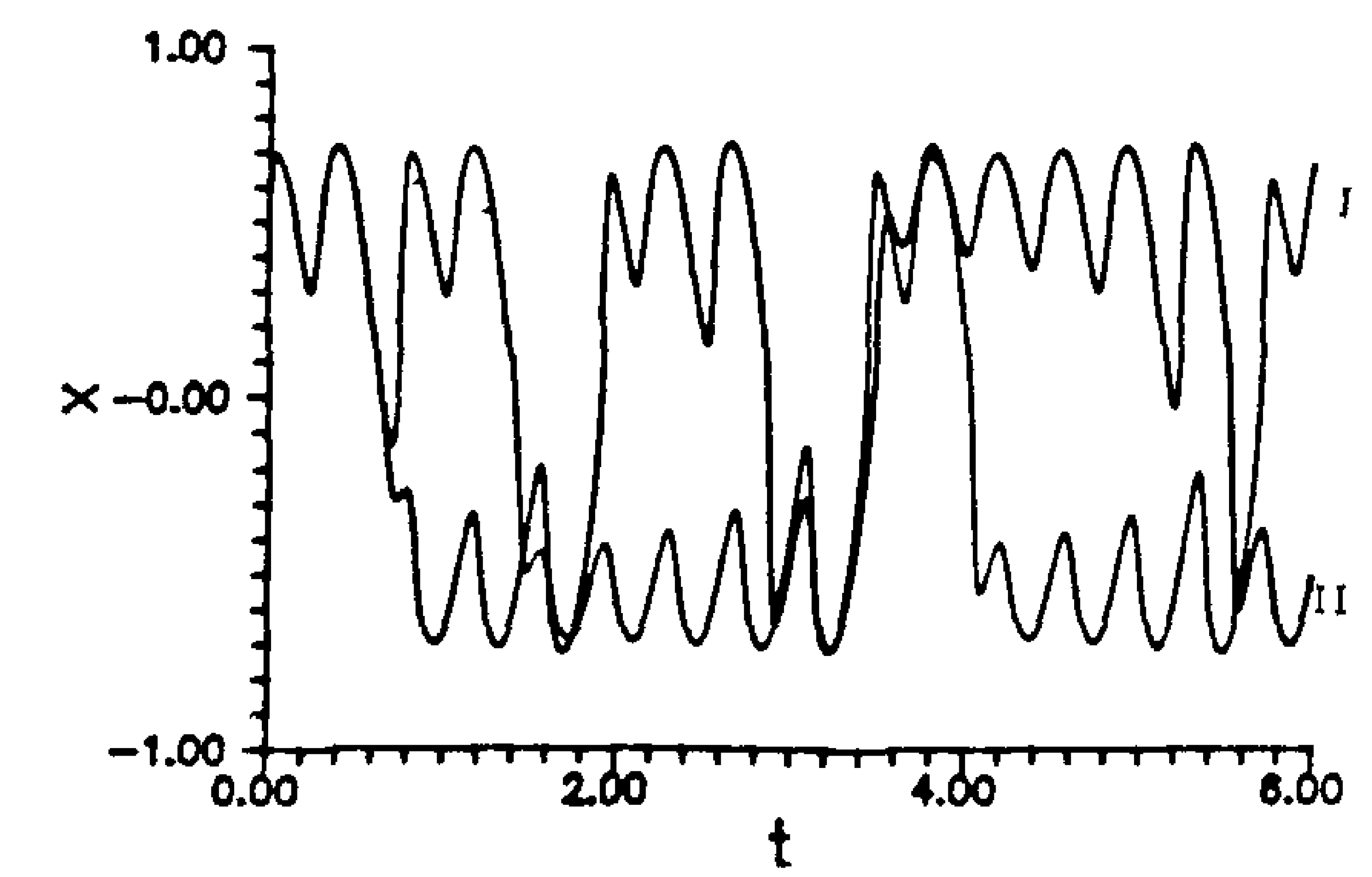


Figure 3. Sensitive dependence on initial conditions· Plot of solution curve x of equation (2) for two different initial conditions *I*, ($x(0) = 0.5$, $y(0) = 0.1$, $z(0) = 0.1$), *II*, ($x(0) = 0.52$, $y(0) = 0.1$, $z(0) = 0.1$)

whose current–voltage characteristics can be represented as $i_N = aV_N + bV_N^3$ $(a < 0,\ b > 0)$. Using Kirchoff's laws to the various branches of the circuit of Figure 1, one can obtain the following dynamical equations:

$$C_1 \frac{dV_{C1}}{dt} = (1/R)(V_{C2} - V_{C1}) - g(V_{C1}), \quad (1a)$$

$$C_2 \frac{dV_{C2}}{dt} = (1/R)(V_{C1} - V_{C2}) - i_L, \quad (1b)$$

$$L \frac{di_L}{dt} = V_{C2}, \quad g(V_{C1}) = aV_{C1} + bV_{C1}^3\ (a < 0, b > 0). \quad (1c)$$

Typically, the nonlinear resistor in Figure 1 can be constructed using a set of diodes and operational amplifier[11], or it can be approximated by the well-known

Chua's diode corresponding to piecewise linearity[14]. To see chaos, we can numerically analyse equations (1) after a rescaling to rewrite equation (1) in the form

$$\dot{x} = -v[x^3 - \alpha x - y], \quad (2a)$$

$$\dot{y} = x - y - z, \quad (2b)$$

$$\dot{z} = \beta y, \qquad (\cdot = \mathrm{d}/\mathrm{d}\tau) \quad (2c)$$

where the scaled variables are related to the unscaled ones by

$$x = \sqrt{bR}\, V_{C1},\ y = \sqrt{bR}\, V_{C2},\ z = \sqrt{bR^3}\, i_L,$$
$$\tau = t/RC_2,\ \alpha = -(1 + aR),\ \beta = C_2R^2/L,\ v = C_2/C_1.$$

A numerical simulation of equation (2) with fixed values of $v = 100$ and $\alpha = 0.35$ exhibits period-doubling bifurcations leading to chaos as the parameter β is decreased from a large value. One observes period-1, period-2, period-4 limit cycles, one-band and double-band chaos respectively at $\beta = 800$, 750, 710, 600 and 300 (see Figure 2). In these simulations, the initial conditions were chosen as $x(0) = x(t = 0) = 0.5$, $y(0) = 0.1$ and $z(0) = 0.1$. Naturally, in the chaotic regime, the motions from two nearby initial conditions diverge exponentially until they become completely uncorrelated. Figure 3 gives the solution curves for $x(t)$ with two different initial conditions (0.5, 0.1, 0.1) and (0.52, 0.1, 0.1).

Synchronization of chaos in the DVP oscillator

Driving a nonlinear system with a periodic signal is a common feature in nonlinear dynamics. However, the idea of using a chaotic signal to drive a nonlinear system in phase and amplitude and so for synchronization is rather new. For example, two identical chaotic systems started at nearly the same initial conditions have by

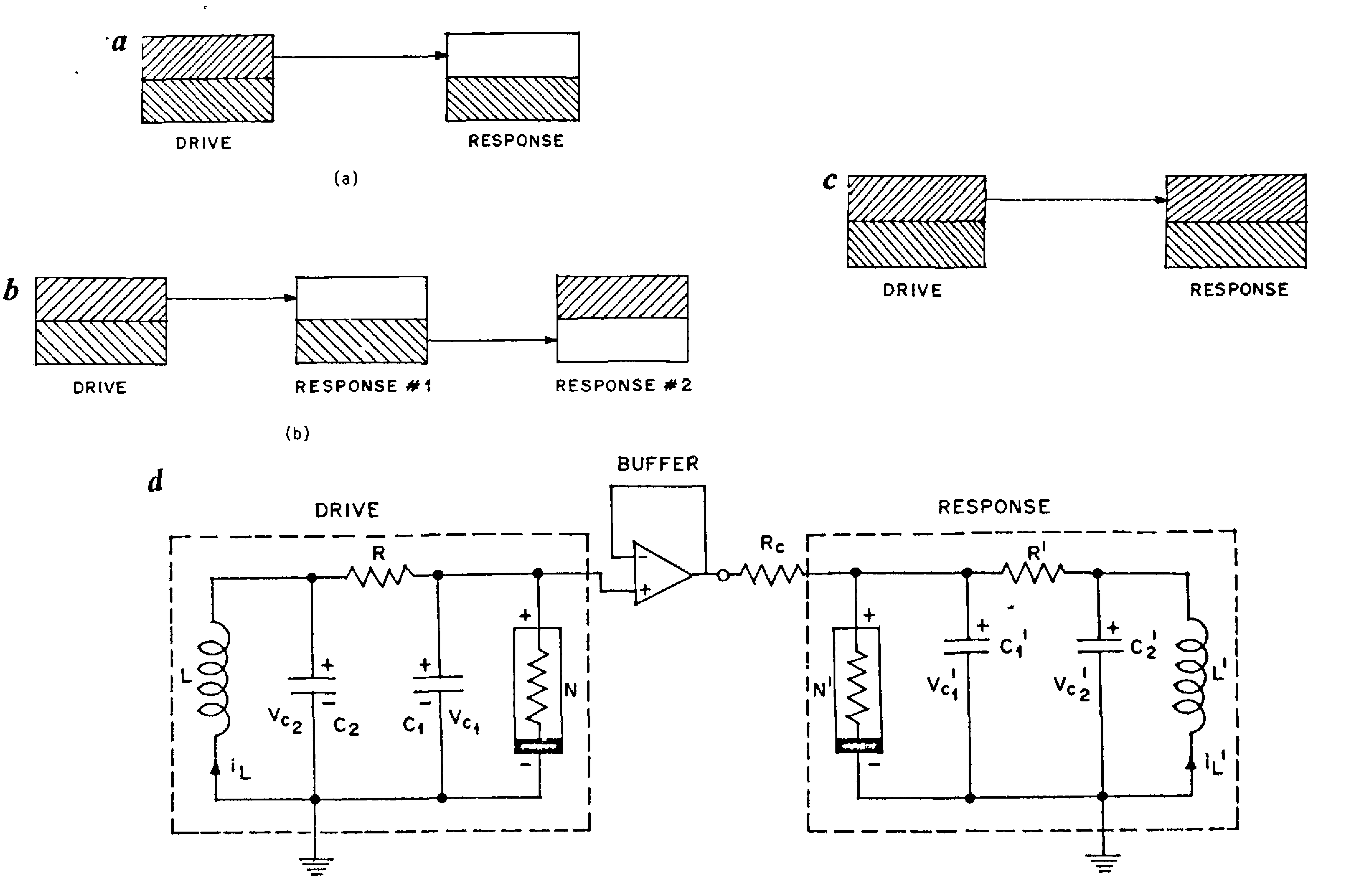


Figure 4. ***a***, Schematic of drive-response scenario of chaos synchronization. ▭ Variables used to drive response system; ▭ Replica part of the drive system used in response; ▭ Drive variables which are fed directly into the response system, ***b***, Schematic of cascading synchronization systems. *Drive and response #1.* ▨ Variables used to drive the response system #1, ▭ Replica part of drive system used in response, ▭ Drive variables which are fed directly into the response system #1; *Response #2* ▭ Variables which are fed directly into the response system #2 from response system #1; ▭ Replica part of the drive system, ***c***, Schematic representation of two identical chaotic oscillators with one-way driving; ***d***, Circuit realization of two uncoupled DVP oscillators

definition trajectories which quickly become completely uncorrelated, even though each system traces the same strange attractor in phase-space for the same set of parametric values. It was the idea of Pecora and Carroll[9,10] that a subsystem of a chaotic system can be synchronized with a separate chaotic system under certain conditions. The idea is to treat one of the chaotic systems as a drive (master) system and the other as a response (slave) system so that the first chaotic system drives the second one to induce synchronization[9,10,12,15–18]. It has also been demonstrated recently that such a synchronization can be achieved through a kind of one-way coupling between the two chaotic systems[13] as in the case of transmission of signals (transmitter → receiver).

According to the original scheme of Pecora and Carroll, one can identify those variables which have negative Lyapunov exponents (corresponding to contraction of phase space in these directions) and retain only those parts in the response system, while the chaotic signal of the remaining drive part (corresponding to positive Lyapunov exponent) is fed directly into the response system as shown schematically in Figure 4*a*. As a consequence for treating the DVP system (2) the variable x is identified as the chaotic signal with positive Lyapunov exponent, while y and z variables correspond to negative Lyapunov exponents. Then the *response system* (denoted by prime variables) becomes

$$x' = x, \tag{3a}$$

$$\dot{y}' = x - y' - z', \tag{3b}$$

$$\dot{z}' = \beta y', \tag{3c}$$

to obtain full synchronization $y \to y'$, $z \to z'$ asymptotically. Thus one can generate from equation (3) the variables y' and z' which are identical to y and z of equation (2) under the influence of the single driving variable x. One can generalize further and develop a cascading procedure to regenerate all the variables identically as shown in the Figure 4*b*, so that equations (3) are replaced by

Response 1:

$$\dot{y}' = x - y' - z', \tag{4a}$$

$$\dot{z}' = \beta y', \tag{4b}$$

Response 2:

$$\dot{x}'' = -\nu[(x'')^3 - \alpha x'' - y'], \tag{4c}$$

so as to obtain the full synchronization $x \to x''$, $y \to y'$, $z \to z'$ asymptotically[12].

Recently we have pointed out that one can obtain complete synchronization even for two identical systems by an appropriate one way coupling[13] (Figure 4*c*). For example for the DVP oscillator the drive system is just equation (2), while the response system (Figure 4*d*) is

$$\dot{x}' = -\nu[(x')^3 - \alpha x' - y'] + \nu\varepsilon(x - x'), \tag{5a}$$

$$\dot{y}' = x' - y' - z', \tag{5b}$$

$$\dot{z}' = \beta y'. \tag{5c}$$

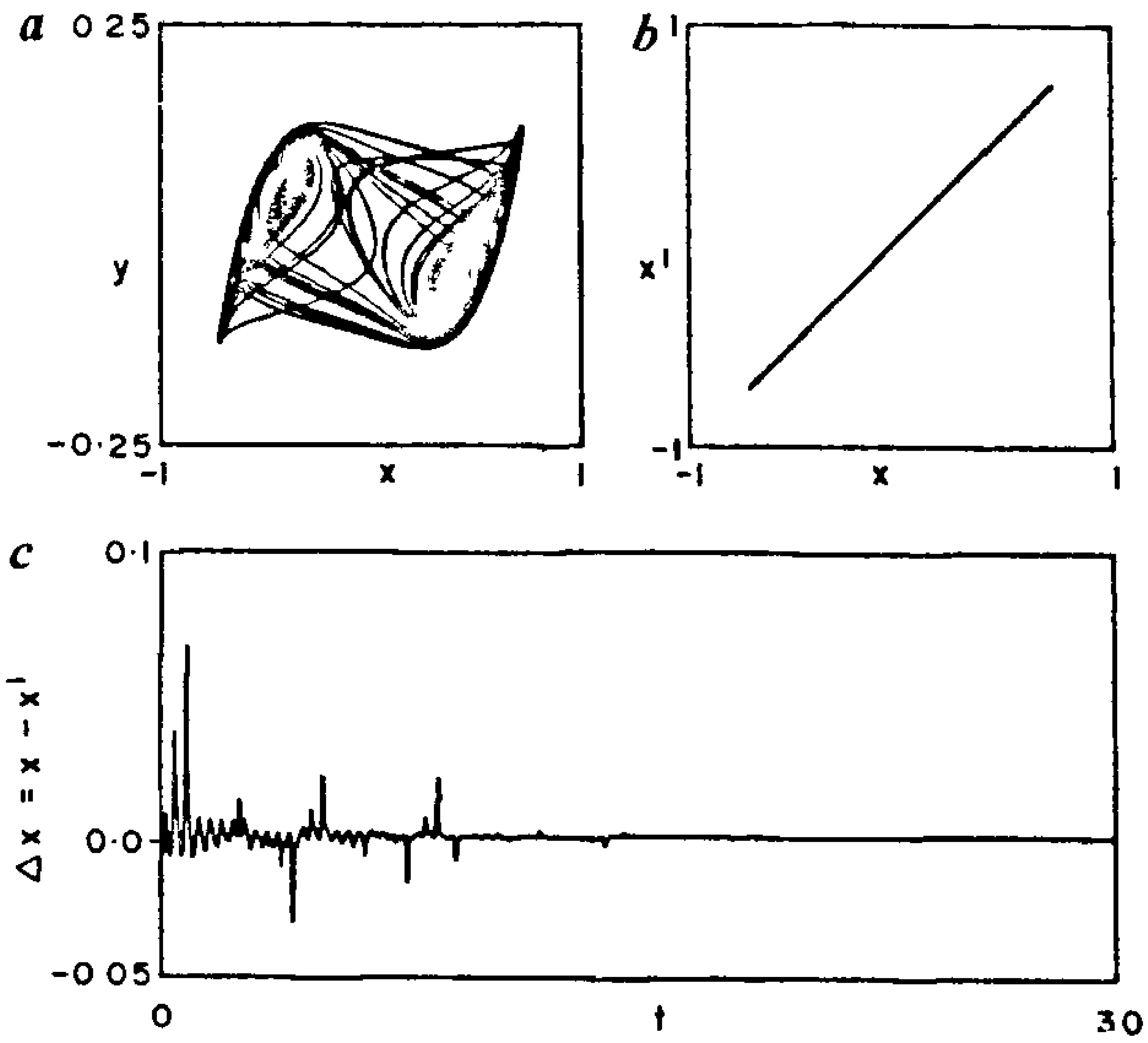


Figure 5. Synchronization of chaos of equation (5) Here $\varepsilon = 1.0$, $\alpha = 0.35$, $\nu = 100$ and $\beta = 300$ ***a***, Phase portrait in (x–y) plane, ***b***, Plot in $x - x'$ plane ***c***, $\Delta x = (x - x')$ vs t plot

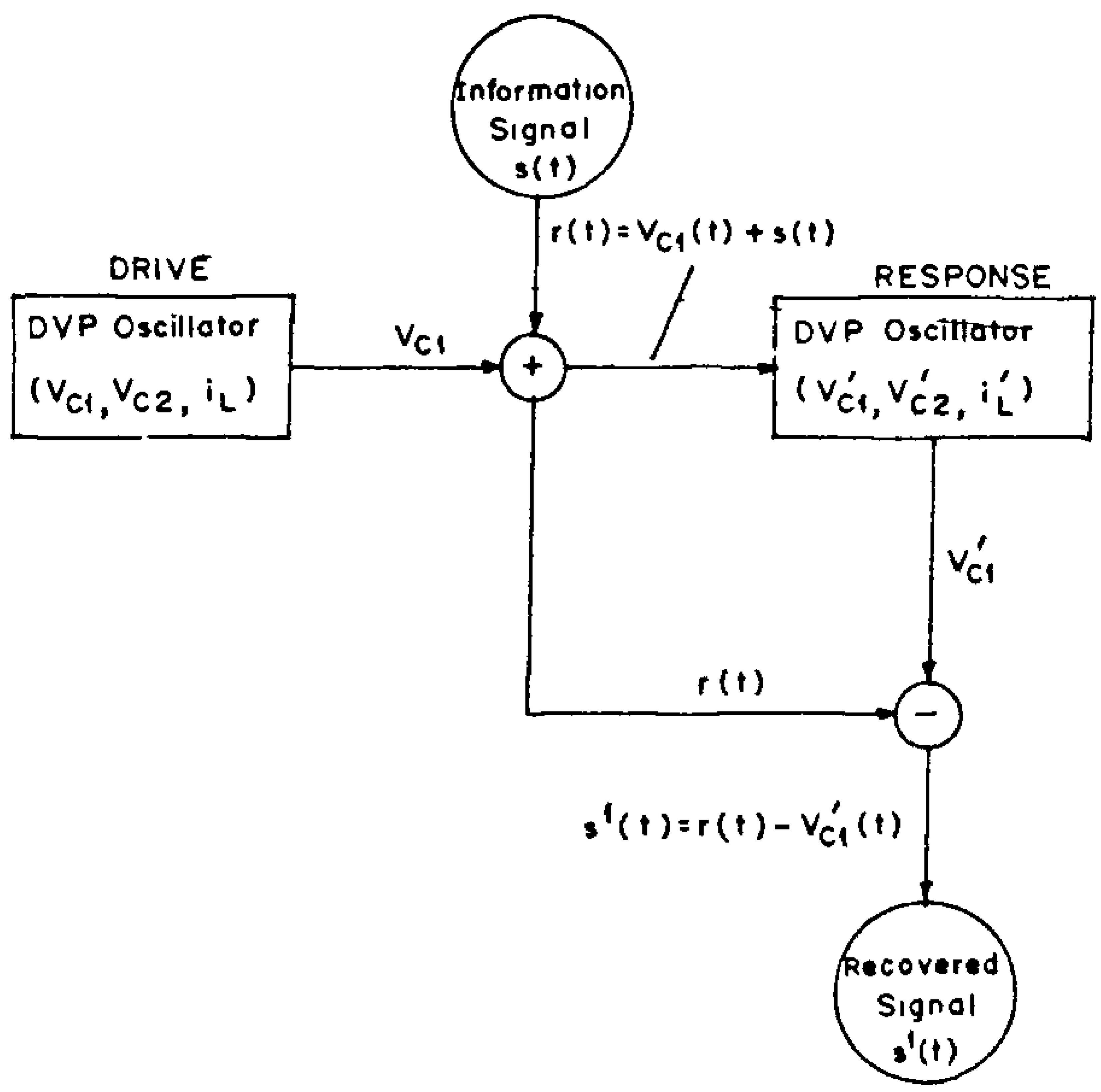


Figure 6. Schematic of 'signal masking' technique for analog signal transmission

Here $\varepsilon = (R'/R_c)$ is the coupling parameter. From the numerical simulation of equation (5), the synchronized chaotic behaviour between x and x' variables for $\varepsilon = 1.0$, $\alpha = 0.35$, $\nu = 100$ and $\beta = 300$ is shown in Figure 5. The initial conditions have been fixed as $x(0) = 0.1$, $y(0) = 0.1$, $z(0) = 0.2$, $x'(0) = 0.15$, $y'(0) = 0.2$, $z'(0) = 0.3$. We have tested the workability of this method for a wide variety of second-order nonautonomous systems and third order autonomous systems.

Chaotic signal masking and transmission of analog signals[12,13,19–22]

We now focus on the use of synchronized chaotic signals as vehicles for effective transmission of analog signals in the context of secure communications. We illustrate the idea again with the use of the chaotic DVP oscillator. The point is that when a chaotic signal is transmitted it cannot be deciphered by a second person unless one has the full information about the transmitting (chaotic nonlinear) system to couple with it appropriately for synchronization. Thus the signal which is to be transmitted is masked by the noise-like chaotic signal by adding it at the transmitter to the information bearing signal $s(t)$ and at the receiver the masking is removed. The basic idea is to use the received signal to regenerate the information-bearing signal by subtracting the masking chaotic signal (received separately through synchronization) to obtain $s(t)$. This task is feasible with the synchronizing receiver system since the ability to synchronize is robust, that is, it is not highly sensitive to perturbations in the drive signal. It is assumed that for masking, the power level of $s(t)$ is significantly lower than that of $x(t)$. Then one can exploit the robustness of synchronization using $r(t)$ as the synchronizing drive signal at the receiver. If the receiver or response has synchronized with $r(t)$ as the drive signal, then $x_r(t) \approx x(t)$ and consequently $s(t)$ is recovered as $s^1(t) = r(t) - x_r(t)$. Figure 6 illustrates this approach schematically.

The above procedure can be demonstrated with the aid of the DVP oscillator system (2) easily through numerical simulation by using $x(t)$ signal of the drive system as the masking signal for the information signal $s(t)$. The response system is now modified as

$$\dot{x}' = -\nu[(x')^3 - \alpha x' - y'] + \nu\varepsilon(r(t) - x'), \qquad (6a)$$

$$\dot{y}' = x' - y' - z', \qquad (6b)$$

$$\dot{z}' = \beta y', \qquad r(t) = s(t) + x(t). \qquad (6c)$$

The information signal is recovered as

$$s(t) = r(t) - x'(t) = x(t) + s(t) - x'(t) \approx s^1(t). \qquad (7)$$

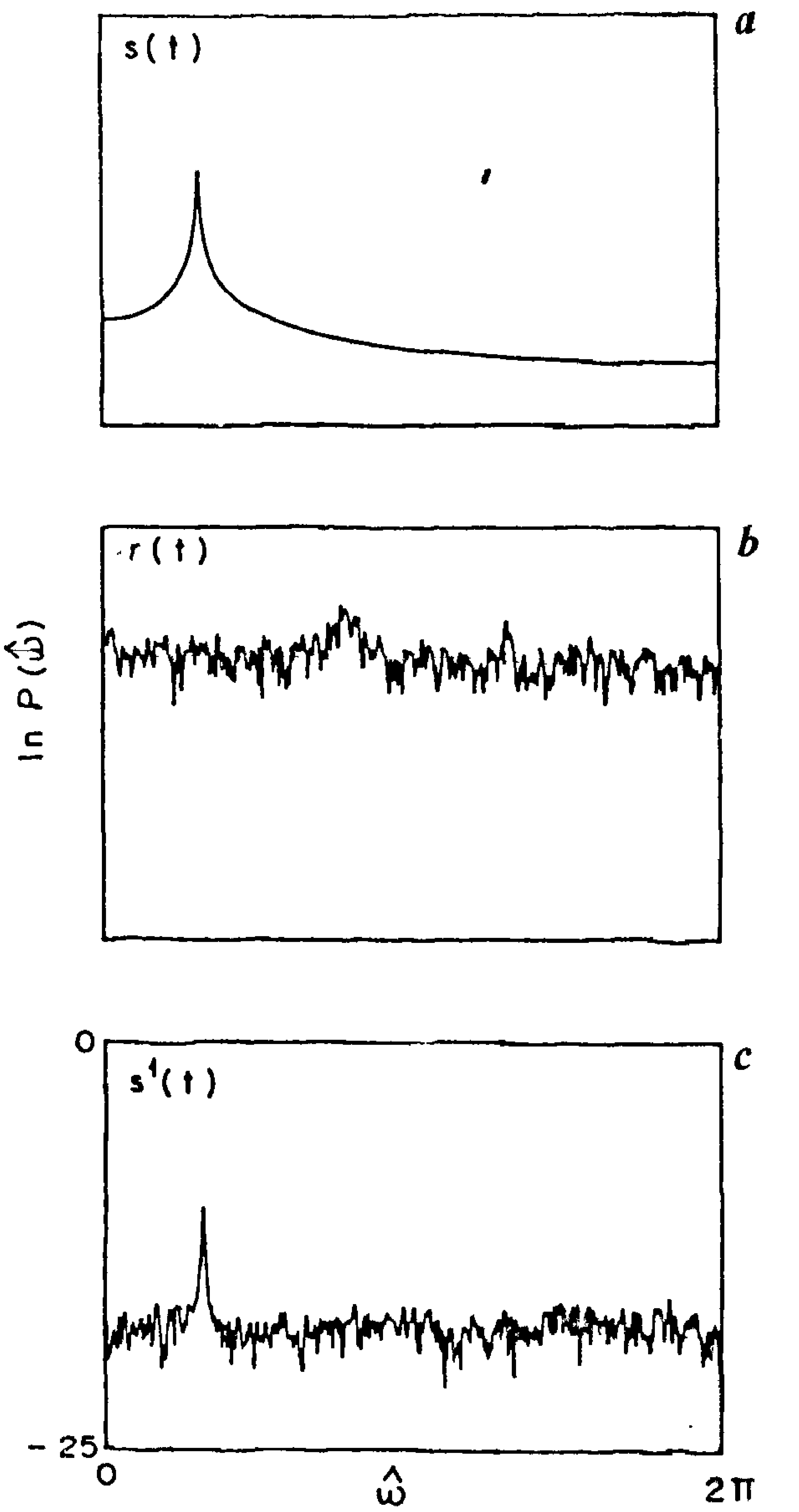


Figure 7. Power spectra of the signals. ***a***, $s(t) = 0.02 \sin(t)$; ***b***, $r(t) = x(t) + s(t)$; ***c***, $s^1(t)$.

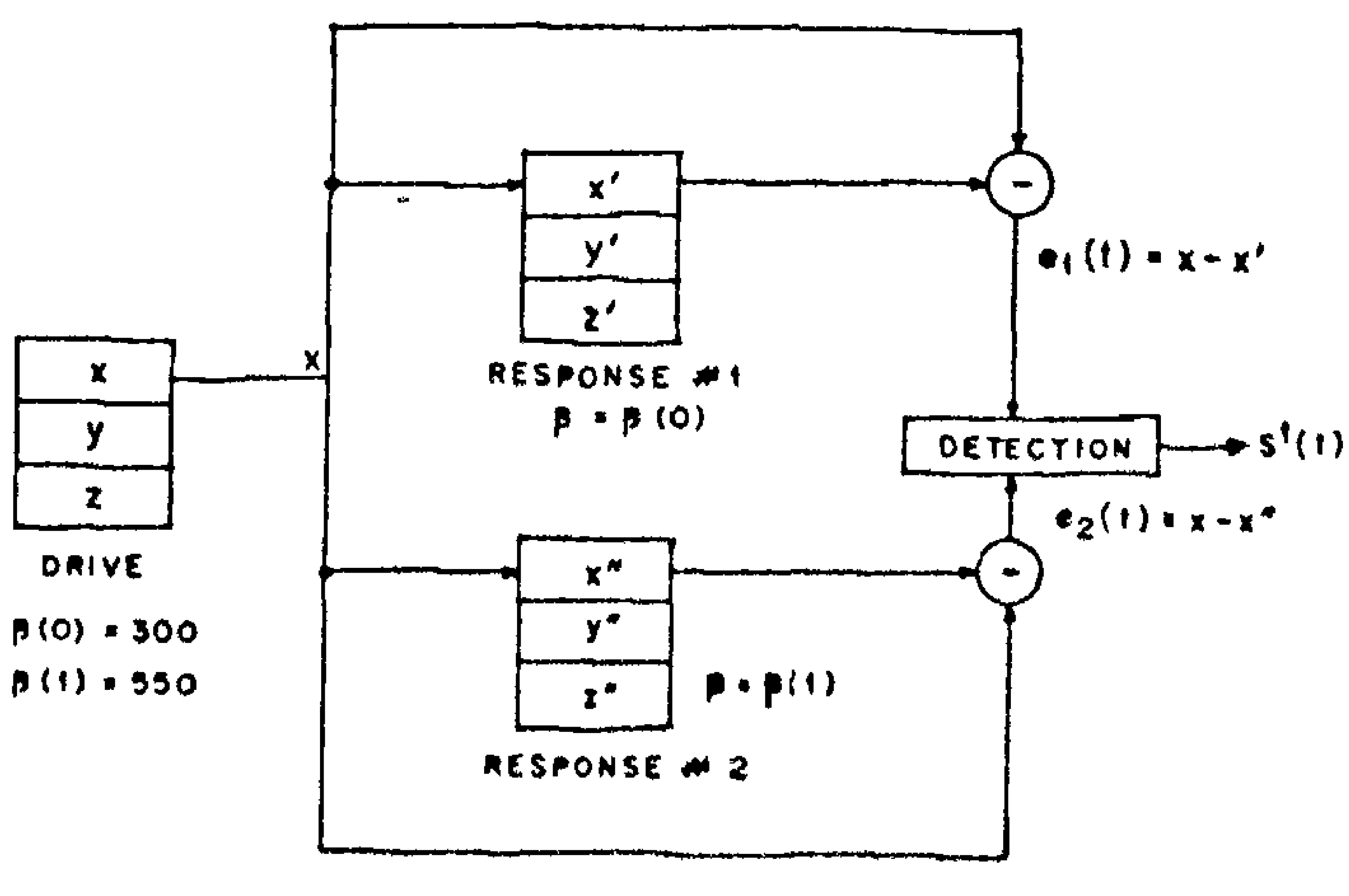
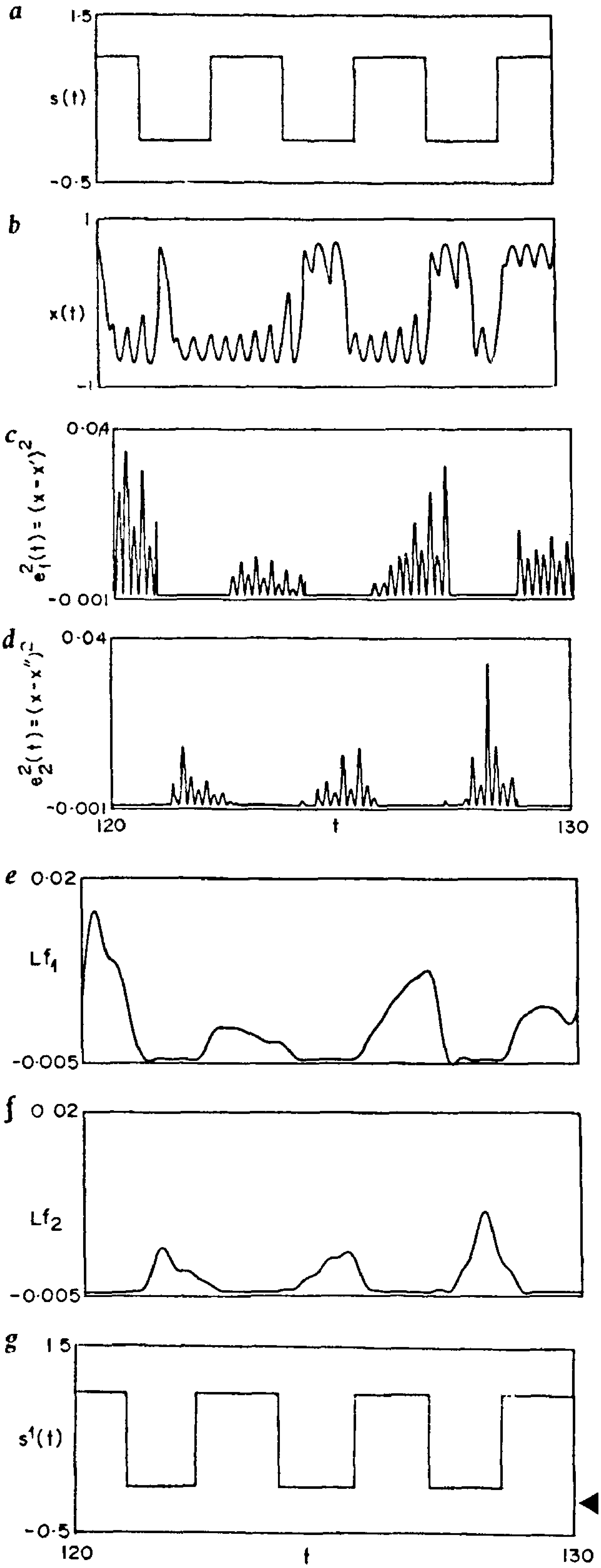


Figure 8. Schematic of digital signal transmission technique. Here #1 and #2 are two separate DVP oscillators



For the numerical values $\alpha = 0.35$, $v = 100$, $\beta = 300$, $\varepsilon = 1.35$, and for a single-tone signal, $s(t) = 0.02 \sin t$, the power spectra of the signal, the masked signal and the successfully recovered signal are given in Figure 7. Details for other signals can be had from refs. 12, 13. In view of the typical broad-band spectrum, the chaotic signal $x(t)$ becomes an ideal candidate for spread-spectrum communication applications. Recently using the approach of chaos synchronization, Cuomo and Oppenheim[20, 21] demonstrated the successful transmission of voice signal by using Lorenz model.

Chaotic digital signal transmission[20–22]

It is not only the analog signals which can be securely transmitted through chaos synchronization. Binary valued bit signals can be equally well transmitted by this method. Here the idea is to essentially modulate a component parameter associated with the transmitter using the information bearing digital waveform and accordingly transmit the chaotic signal. At the receiver, the coefficient modulation will produce a synchronization error between the received drive signal and receiver's regenerated drive signal, with an error signal amplitude that depends on the modulation. Using the synchronization error the modulation can be detected. This method has also been recently demonstrated experimentally for typical nonlinear systems[20–22].

Now we explain the method for the DVP oscillator, which is illustrated schematically in Figure 8. Here the coefficient β in equation (2) is modulated by the information wave form $s(t)$, which is a binary coded signal. The information is carried over the channel by the chaotic signal $x(t)$, which serves as the driving input to the receivers #1 and #2. At the receivers the modulation is detected by forming the difference between $x(t)$ and the reproduced signals $x'(t)$ of #1 and $x''(t)$ of #2. Then the synchronization error $e_1(t) = x(t) - x'(t)$ will be relatively large in amplitude during the time period when '1' is transmitted and small in amplitude during the '0' transmission. Also, the synchronization error $e_2(t) = x(t) - x''(t)$ will have an opposite nature to the previous one. Thus the synchronization receivers can be recognized as a form of matched filters for the chaotic transmitted signal $x(t)$. A sample illustration is provided in Figure 9.

Conclusion and outlook

We have come a long way from the feeling that chaotic motions are complex, harmful and to be avoided to a

Figure 9. Secure digital signal transmission ***a***, $s(t)$-digital information signal; ***b***, actual transmitted signal, ***c***, error signal $e_1^2(t)$, ***d***, error signal $e_2^2(t)$; ***e*** low pass filtered signal of *c*; ***f***, low pass filtered signal of *d*; ***g***, recovered information signal $s^1(t)$

stage where we realize that the introduction of chaos can be quite useful or desirable. Generally, so far, in physical, mechanical and engineering systems the stress has been to avoid nonlinearity so as to be free from any unpredictable motions of dynamical systems. Only now have researchers begun to use the advantages of designing systems to exploit, rather than avoid, nonlinearity and chaos. One dividend for such an outlook is the realization of the possibility of synchronization of chaotic orbits, leading to new vistas in secure signal transmission and thereby harnessing chaos.

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REVIEW ARTICLE

The biology of human granulocyte colony stimulating factor

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Human granulocyte colony stimulating factor (hGCSF) is one of the important haemopoietic growth factors which mediates granulopoiesis and has several potential therapeutic applications. It is a glycoprotein with the number of amino acids 177 and 174 for its two forms which are produced as a result of alternative splicing of precursor mRNA. In the body it is produced by monocytes, fibroblasts and vascular endothelial cells in very minute quantities. In clinical practice hGCSF is used in the patient with agranulocytosis and granulocytopenia, congenital in nature or as a result of bone marrow transplant or myelosuppressive chemotherapy during the treatment of several types of cancers and AIDS.

THE production of mature blood cells from pluripotent stem cells, derived from bone marrow and incapable of unstimulated division, is a complex and intricate process that requires the well-coordinated and concerted action of several protein molecules which are collectively known as hemopoietic growth factors. Of these, colony stimulating factors (CSFs) are a group of proteins which regulate the proliferation and differentiation of the cells of granulocyte and macrophage lineages. In murine and human beings, the presence of four CSFs have been clearly established. Of these, two factors namely, interleukin-3 (Multi-CSF) and granulocyte-macrophage colony stimulating factor (GM-CSF) are broad range in their activities supporting the growth of granulocyte, macrophage, eosinophil and other hemopoietic pro-