

- 5 S Ramanujan, *The Lost Notebook and Other Unpublished Papers* Berlin: Springer-Verlag (1988) A facsimile edition, which contains Littlewood's judgement in a letter.
- 6 E H Neville, 'The Late Srinivasa Ramanujan' *Nature* 106, 661-662 (1921)
- 7 R Kanigel, *The Man Who Knew Infinity A Life of the Genius Ramanujan* London: Scribners (1991)
- 8 S R Ranganathan, *Ramanujan. The Man and the Mathematician* London: Asia Publishing House (1967)
- 9 J. R. Newman, *The World of Mathematics* vol. I p 366 New York: Simon & Schuster (1956).
- 10 R A Rankin, 'Ramanujan as a Patient'. *Proc. Indian Acad. Sci. Math Sci.*, 93, 79-100 (1984).
- 11 'I had ridden in a taxicab No 1729, and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavourable omen. "No", he replied, "it is a very interesting number, it is the smallest number expressible as a sum of two cubes in two different ways" (i.e. $12^3 + 1^3 = 1729 = 10^3 + 9^3$). As Littlewood remarked, "every positive integer was one of his personal friends".' G. H. Hardy (note 4).
- 12 P. V. Seshu Aiyar & R. Ramachandra Rao. Introduction. pp xi-xix, note 4.
13. Quoted from Rankin (note 10) This letter is also given in full elsewhere (*S Ramanujan Commemoration Volume* Eds S S Aiyar & R. Bharathi p 75 Madras: Jupiter Press, 1974), where the key sentence contains an additional phrase (italicized here); i.e., 'from some obscure and only partially diagnosed source of blood poisoning which has now dried up'.
- 14 See Neville (note 6) and Aiyar and Rao (note 12) Also evident in 2 photographs (a) group photo taken in March 1916 when Ramanujan received his BA degree (frontispiece for the Lost Notebook, note 5); (b) full length photo of him seated, in western dress, taken just after his arrival in England in 1914 (Kanigel, note 7), shown in Aiyer & Bharathi p 36 (note 13).
15. T. K. Munro, *Manual of Medicine*. 4th ed London. Bailliere, Tindall & Cox (1917)
- 16 T. D. Savill, *System of Clinical Medicine* 8th ed London: Edward Arnold (1930).
- 17 E. B. Adams & I. N. MacLeod., 'Invasive Amoebiasis II Amoebic Liver Abscess and its Complications', *Medicine* 56, 325-334 (1977)
18. O. Felsenfeld & V. M. Young, in *Clinical Tropical Medicine*, eds R. B. H. Gradwohl, L. B. Soto & O. Felsenfeld. Chap 4, pp 35-70 London: Henry Kimpton (1951).
- 19 A. J. Wilmot, *Clinical Amoebiasis*. Oxford: Blackwell (1962)
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22. T. D. Savill, *System of Clinical Medicine*. 13th ed. Ed. E. C. Warner London: Arnold (1950).
23. Chopra, R. N., B. Mukerji & I. C. Chopra, *A Treatise on Tropical Therapeutics* 2nd ed vol 1, pp. 261-310 Calcutta: U. N. Durr (1950)
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- 25 R. A. Rankin, 'Srinivasa Ramanujan (1887-1920)', *Bull. Inst. Math. Appl.* 23, 145-152 (1987)
- 26 C. F. Craig, *Amebiasis and Amebic Dysentery* London: Bailliere, Tindall & Cox (1934)
- 27 E. B. Adams & I. N. MacLeod, 'Invasive Amebiasis I Amebic Dysentery & its Complications'. *Medicine* 56, 315-323 (1977)
- 28 G. Klatskin, 'Amebiasis of the Liver Classification, Diagnosis and Treatment' *Ann. Int. Med.* 25, 601-631 (1946)
- 29 F. Murgatroyd, in *Modern Practice in Infectious Fevers* Ed H. S. Banks vol. II pt 6, pp. 881-889 London: Butterworth (1951)
30. S. L. Reed & A. I. Braude, in *Amebiasis Human Infection by Entamoeba histolytica* Ed J. I. Ravdin chap. 33, pp 511-532 New York: John Wiley (1988).
31. S. Chandrasekhar, *An Incident in the Life of S. Ramanujan FRS Conversations with G. H. Hardy FRS & J. E. Littlewood FRS and their Sequel* Papers deposited with the Royal Society (18/8/1977)
- 32 S. Chandrasekhar, 'On Ramanujan' in *Ramanujan Revisited Proc. Centenary Conf. Univ. Illinois, 1987*. Eds G. E. Andrews, R. A. Askey, B. C. Berndt, K. G. Ramathan & R. A. Rankin p 2 New York: Academic Press (1988)
33. T. D. Savill, *System of Clinical Medicine*. 5th ed London: Edward Arnold (1918)

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Mr. S. Ramanujam's work in England

The age we are living in has been one of many great national upheavals. We are today claiming for the wider recognition of our powers, active and dormant. Politically we are issuing into a united nationhood; and materially we hope soon to be abreast of the more civilized countries of the world. Intellectually too, our literary and scientific achievement has not been behind-hand but has been receiving world-wide recognition. The poet went out, sang and was honoured with a prize and a knighthood. The scientist struck famous

academies of Europe and America in tremulous wonder; and Mr S. Ramanujam is in a fair way to do a similar thing for mathematics.

India is certainly a country for the mathematician; and her early mathematicians were without exception astronomers - votaries of a science at once the grandest and oldest. Brahmagupta, Bhaskara and Mahavira Charya to mention only a few wrought in middle ages what Europe is still acknowledging heavy indebtedness to. A full however set in later, and India is today emerging again from an isolation of over a thousand years. Consequent on the lack of facilities here, Indian

mathematicians have been frequently contributing to *The Educational Times*, to that famous Belgian journal *Mathesis* and quite recently to *Fortschritte, Math Annalen* and *L'intermediare des Sciences*. Such lack of facilities is now being partly removed by the rapid progress of the Indian Mathematical Society.

Considerable interest has, and rightly has, now arisen in this country in the work of the young Madras mathematician Mr S. Ramanujam. His Excellency Lord Pentland in opening the First Conference of the Indian Mathematical Society the other day referred in no uncertain terms to the

unstinted praise that Mr Ramanujam has received from distinguished mathematicians of the present day. Amidst enthusiastic cheers he said 'you know the story of the discovery of his unusual talent and all here will be glad to hear how entirely he is justifying the efforts which were made to give it full scope'; and it is the purpose of this article to show in as concise and non-technical a language as is consistent with this non-scientific journal what Mr S. Ramanujam has done after going to Cambridge and how he has justified such praise.

I shall not say how his old and affectionate parents were induced to temporarily part with their son and prevent him from being one of those 'many a gem of purest ray serene, born to blush unseen.' Suffice it to say that the day on which he sailed to England will now become a red letter day in the history of our national triumph and glory. I shall not also say of the mass of unpublished material containing hundreds of most curious and interesting formulae, some definitely proved and others only conjectured that was in his possession when in this country. But I would only say that even in that unsystematized condition its astonishing individuality and extraordinary ingenuity attracted the attention of those people who have done now to him what they were bound to do in the cause of pure science. I shall only draw attention here to what is remarkable about the papers published in England; but the critical mathematician, if any such linger in this country without having already read his papers from their original sources, must turn to the memoirs themselves, or in a less lengthy form to that report on his work, which has been kindly prepared by Mr Hardy of Trinity College, Cambridge, at the request of the authorities of the Madras University, and which has since been reprinted in *The Journal of the Indian Mathematical Society*, Vol. IX, No. 1.

At the outset it must be stated that as it stands at present Mr Ramanujam is known only in England in any large measure of publicity. The war has naturally had disastrous results on the progress of mathematical research. Mr Hardy deplores 'the war has distracted three quarters of the interest that would otherwise have been taken in his work and has made it almost impossible to bring his results to the notice of the continental mathematicians most certain

to appreciate it'. All his researches have consequently to be published in only the English journals; but it is however a very good augury that just a small note under the combined authorship of Hardy and Ramanujam has very recently (15-1-17) been published in *Comptes Rendus*, a Paris journal in its CLXIV volume.

His first published paper on 'Some definite integrals' appeared in *The Messenger of Mathematics* which has published in the space of two short years as many as half a dozen of his papers. It contains besides some transformation formulae, the values of some curious and interesting definite integrals of Gamma Functions of the variable. Some of these had no doubt previously been obtained by Hardy and Lindelöf by that favourite method of contour integration which though difficult to learn and belongs to more advanced modern analysis is easy to retain and fruitful in application; but Ramanujam's individuality consists in his novelty—a reasoning of an elementary but highly ingenious character. His next two papers to this periodical deal with 'Certain other definite integrals connected with Gauss's sums' and 'Certain infinite series' involving mainly trigonometrical functions. Only very special cases of this result were known before, found partly by Hardy and partly by Kronecker, the latter a German mathematician of no mean repute. What is striking about them is Mr Ramanujam's ingenious application of a theorem of Lerch which Hardy calls 'well known' but which unfortunately is often loosely used. It is in the latter of these two papers again that Mr Ramanujam has first given printed evidence of his knowledge of Elliptic Functions, a knowledge which has since shown itself to be very profound. His two later papers 'Summation of a certain series' and 'Some formulae in the analytic theory of numbers' (1915) push him more and more into the leading investigations of the day. Riemann's Zeta function, Dirichlet's series, Asymptotic formulae and 'representations' claim his attention and bring him abreast with the work of classic mathematicians of the order of Mittag-Leffler, Lindelöf, Marcel Riesz, Vito Volterra, to mention only one from each of the four prominent countries of the continent.

In this connection *The Journal of the Indian Mathematical Society* was

fortunate in getting some arithmetical discussion of the Riemann's Zeta functions, e.g. formulae concerning sums of spare roots of integers. The paper 'On some series for Euler's Constant' published after Hardy's report was written, attacks a problem originally set, I believe, by Dr Vacca (Italy) and contains extensive generalizations of certain formulae of Dr Glashier and in particular of a formula which the latter 'only conjectured'.

The Quarterly Journal of Pure and Applied Mathematics has published up till now only two papers of his. The first on 'Modular equations and approximations to π ' embodies the results of a great deal of his work in India. The difficulties of getting access in this country to all published literature have been very great and as a consequence Mr Ramanujam was not aware that many of his results have been 'anticipated by European mathematicians, in particular by Hermite, Kronecker, Weber and Greenhill'. Nonetheless it still contains a large number of new results; and I believe that the one thing in this paper that strikes the popular eye is the amazing if not bewildering number of approximations to π —the most remarkable being correct to 31 places of decimals. The paper also contains a number of interesting series for $1/\pi$ drawn from the theory of Elliptic Functions. His next paper, viz. that on 'Riemann's Theta functions' is undoubtedly for the specialist. Fourier's double integral theorem is freely used and the paper attacks some outstanding questions of the present decade. The unsolved problems concerning the Zeroes of Riemann's Zeta and Theta functions are among the most obscure and difficult in the whole range of pure mathematics and Mr Hardy thinks it certainly possible to apply Mr Ramanujam's formulae in this direction. Hardy disillusioned himself so as not to feel surprised 'if still more important applications were to be made of Mr Ramanujam's formulae in the future'.

The Proceedings of the London Mathematical Society—the official organ of the London Mathematical Society—has as yet published only one paper, a paper adjudged by its bulk, individuality and originally to be 'one of the most remarkable published in England for many years'. This elaborate memoir on 'Highly Composite

Numbers' – defined as the numbers which have more divisors than any smaller number – is his longest connected piece of work done in England. These are numbers as unlike a prime as a number can be – the first few numbers being 2, 4, 6, 12, 24, 36, etc. Mr Ramanujam's largest highly composite number is 6746328388800. In this paper Mr Ramanujam shows also how, by reasoning of an elementary but highly ingenious character we can obtain surprisingly accurate information as to the structure of highly composite numbers. The paper on 'A problem in the analytic theory of numbers', a subject of continental interest, has been promised but has not yet appeared. Fragments of what is announced to be part of this – viz. 'Asymptotic value on the number of partitions of n ' has however appeared in a very recent issue of *Comptes Rendus* (*loc cit.*). Still another paper on the expression of numbers in the form $ax^2 + by^2 + cz^2 + dw^2$ has been promised but not published yet.

But by far the longest paper that has arrested the critic and merited his respect is 'On Certain Arithmetical Functions' contributed to the *Transactions of the Cambridge Philosophical Society*. Neither the popular reader nor the mathematician of the old school shall claim to know one single line of this remarkable memoir of 35 pages of royal foolscap. For it deals with Riemann's Zeta and Theta functions, determination of maximum 'order', Dirichlet's Series of Real and Complex

Variables, ' $r_2^k(n)$ representations', and Arithmetic functions (his one function including as special cases functions introduced by Dr Glashier in the course of half a dozen lengthy memoirs) – all subjects on which the last word has not been said and on which there is not even a regular systematic printed account in any written language – Landau's 'Prim Zahlen' perhaps comes nearest to such an account in some of these subjects.

I was one of those who had the privilege of looking into Mr Ramanujam's note books when he was here and I may say that in addition to what has been published as shown above, he had in his notes many more curious and interesting results and theorems and that when he comes out with such theorems, specially a definite integral theorem at which I hear he is now working, and his theory of divergent series, I have no doubt he would astonish the mathematical world.

Before concluding it may not be out of place to consider what may next be done for Mr Ramanujam. Hardy has asserted that 'in him India now possesses a Pure Mathematician of the first order, whose achievements suggest the brightest hopes for its scientific future'. And His Excellency drawing pointed attention to this vouchsafed: 'Not a few Indians have earned high distinctions in the Cambridge schools and some of them are I believe members of this Society (Indian Mathematical Society) and they will be able to realize what is required to earn an encomium of

this nature'. All this is not flattery but deserved tribute to an intellectual genius. Cambridge, which has now sent most of its University staff to active war service, may confer on Mr Ramanujam a Fellowship. Certainly it will be an unprecedented honour both to the giver and the recipient. But of what use is that to India from a national point of view? The eager Universities of Northern India or nearer home, the nascent University of Mysore may covet his acceptance of one of their professorial chairs. All credit to them, but such must excuse some amount of provincial patriotism. Mr Ramanujam is a Madrasi, by birth, by adaptation and by habit. It is but fitting that sufficient inducements must be held out here in Madras when he returns so as to enable him to open a well-equipped Mathematical Research Institute, the nucleus of an institution which will ultimately develop into a University College of Science, such as one we have elsewhere in India. In the words of a good mathematician 'The capacity for research in Mathematics is nowhere very plentiful; but it does undoubtedly exist in South India and should be encouraged', and the best way of doing this is to get the inspiration of master minds such as Mr Ramanujam's to encourage research in other promising young men of Madras so that in course of time we may have here a school of mathematicians doing work of a very high order that will command their world-wide recognition.

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