Earthquake ground motion characterization for large or multiply supported structures

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Spatial variation of ground motion has significant effect on the earthquake response of structures that either are elongated with multiple supports or have large dimensions. This paper reviews state-of-the-art approaches to ground motion characterization for these types of structures. Although considerable work is available on characterization of spatial variation of ground accelerations, no consistent design basis has yet evolved. A few studies have investigated spatial variation of ground displacements and differential displacements. Since research in this area is of relatively recent vintage, and not all aspects of the problem are well understood, it is currently most appropriate to use simple models for characterizing spatial variation of ground motion. This paper also highlights the possibilities for further investigations, particularly in the Indian context.

Earthquake ground motion is typically defined at a single point for structures supported on rigid foundations with relatively small dimensions. This characterization is in the form of a response spectrum or an acceleration record for deterministic response analysis, and in the form of power spectral density functions for stochastic response analysis. Several early studies, however, illustrated the effect of ground motion variations over short distances (less than a kilometre) on the response of structures with large or multiple supports. Recent studies on structures with large rigid foundations, bridges, lifelines, and multi-supported beams and arches have further shown that response estimates assuming uniform excitation may be unconservative for these structures.

Therefore, appropriate characterization of the spatial variation of ground motion over short distances is essential for response analysis of certain types of structures. Fortunately, availability of databases from dense strong motion seismograph arrays in various parts of the world has facilitated this process. This paper briefly reviews current practice in this area, and identifies various aspects that need further study, especially in India.

Basic concepts

Spatial variation of ground motion is caused by two separate aspects of the earthquake process: wave passage and incoherence effects. The passage effect, which is due to the different times of wave arrivals at different points on the ground surface, causes a phase lag between ground motions at two separate stations. The incoherence effect, which is due to wave scattering and extended fault rupture propagation, causes a loss of 'likeness' between ground motions at two separate stations. Both these factors have important bearing on the response of structures, and their effects on spatial variation depends on the earthquake mechanism itself.

Two distinct approaches are available in the literature for characterizing ground motion spatial variation: deterministic and stochastic approaches (see Figure 1). The deterministic approach, which is comparatively older, implicitly assumes ground motion to be coherent, and therefore, considers only the wave passage effect directly. The stochastic approach, which has only been in use recently, can consider both aspects that cause spatial variation.

Wave passage characterization

The standard approach for characterizing the wave passage effect considers ground motion to be caused by a
single coherent plane wave (for all wave frequencies) propagating along the ground surface with a constant velocity. Then, the effect on ground motions at two stations, separated by a distance $d$, measured along the direction of wave propagation, can be defined either by a time lag $\Delta t$ or an equivalent phase lag $\eta_c(f)$. These lag terms are given as

$$\Delta t = d/c; \quad \eta_c(f) = 2\pi f d/c,$$

(1)

where $c$ denotes the apparent speed of wave propagation, and $f$ denotes the earthquake wave frequency. Therefore, the passage effect can be completely characterized by the apparent wave propagation speed parameter $c$.

A more recent development considers both plane and random waves to model the wave passage effect, which is represented by a complex-valued phase spectrum $\phi(r, f)$ that can be expressed as

$$\phi(r, f) = [h(r, f)] \exp(i \eta_c(f)) +$$

$$[1 - h(r, f)] \exp(i \eta(f)), \quad (2)$$

where $r$ denotes the separation distance vector between two stations, and $h(r, f)$ gives the relative power at wave frequency $f$ that can be represented by a single plane wave. The complex exponential terms represent the phase lag effect, where $\eta_c(f)$ is the single wave phase lag term in equation (1) and $\eta(f)$ denotes a random phase lag term. This expression defines the wave passage effect in its most general form, because the standard approach mentioned above is a special case when $h(r, f)$ is unity at all frequencies. The function $h(r, f)$, however, has to be defined in the more general characterization approach.

Characterization of incoherence

The incoherence effect can be characterized only by quantifying the loss of similarity in ground motions at different stations. This is not possible in the deterministic approach, because of its implicit assumption of coherent ground motion. The stochastic approach uses either the spatial coherency function $\gamma(r, f)$ or the spatial correlation function $\rho(r)$ for this quantification. These functions can be expressed in terms of the cross-power spectral density function $S_{jk}(r, f)$ and the auto-power spectral density functions $S_{jj}(f)$ and $S_{kk}(f)$ of the ground motion at two stations $j$ and $k$, separated by a distance vector denoted by $r$, as

$$\gamma(r, f) = S_{jk}(r, f)/\sqrt{S_{jj}(f) S_{kk}(f)}, \quad (3)$$

$$\rho(r) = \left[ \int \text{Re} \left[ S_{jk}(r, f) \right] df \right]/$$

$$\left[ \int \left( S_{jj}(f) S_{kk}(f) \right) df \right], \quad (4)$$

where $\text{Re}[.]$ denotes the real part of a complex-valued function.

The coherency function is complex-valued with a magnitude range of zero to unity, where the two asymptotic values represent, respectively, completely incoherent and coherent ground motions. Ground motions at two stations are completely coherent if they are identical barring the phase effect, and are completely incoherent if they are statistically independent (i.e. they do not influence each other). The spatial correlation function, on the other hand, is real-valued with a range from $-1$ to $1$. This function has a unit value if two ground motions are identical and in-phase, and a value of $-1$ if they are identical, but completely out-of-phase. The correlation is zero for two statistically independent ground motions.

Spatial coherency function

The coherency function $\gamma(r, f)$ is typically expressed as a product of its magnitude, defined as the spatial coherency spectrum $|\gamma(r, f)|$, and the unit-magnitude complex-valued phase spectrum $\phi(r, f)$ given by equation (2). Therefore,

$$\gamma(r, f) = |\gamma(r, f)| \phi(r, f). \quad (5)$$

Since this function specifies the wave passage and incoherence effects separately, it provides the best definition of the spatial variation of ground motion currently available.

Ground acceleration spatial variation models

Initial research perhaps owing to lack of a substantial database, concentrated on analysing and characterizing the effects of spatial variation, rather than characterizing spatial variation itself. The derivation of standard pseudo-acceleration response spectra at a station, defined as the in-phase spectrum, incorporated the reduction in spectral values due to spatial variation. In addition, the out-of-phase nature of ground motion at two separate stations was incorporated either by defining the ground motion covariance or an out-of-phase response spectrum.

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Subsequent studies in this area, however, have concentrated on developing acceleration spatial coherency function models. The phase and coherency spectra [see equation (5)] are characterized separately in every model, although some models do not explicitly define the separation and are restricted to coherency spectrum characterization only.

**Phase spectrum models**

When the wave passage effect is considered to be caused by a single plane wave at all frequencies, the phase spectrum is completely defined [see equation (1)] by determining the apparent speed $c$ of wave propagation. For a particular earthquake event, this parameter is generally estimated from the frequency-wave number spectrum that is obtained through analysis of dense array records for that event.

Estimates of apparent propagation speed given in the literature range from 1 km/sec to 5.2 km/sec. However, most researchers report values between 2.17 km/sec and 3.5 km/sec for earthquake events that have been recorded by Lotung arrays in Taiwan. Analysis of events recorded in California, USA, confirm the above range of values. It has been observed that the apparent propagation speed is higher for earthquakes with larger focal depths. A possible reason for this is that a larger component of ground motion for such earthquakes is due to vertically propagating body waves which reduce the phase lag between ground motions at two separate stations on the ground surface.

The study by Abrahamson *et al.* is the only investigation known to the author that considers both plane and random waves to model the wave passage effect. They, therefore, define the phase spectrum in its most general form, as given in equation (2). According to them, the wave passage effect is properly characterized if $c$ is considered to be 2.7 km/sec, and the function $h(r, f)$ in equation (2), which denotes the relative power at frequency $f$ that can be represented by a single plane wave, is given by the following expression:

$$h(r, f) = 1 + \left(\frac{f}{19}\right)^4$$

(6)

This expression leads to the conclusion that at least 95% of the power can be represented by a single plane wave for wave frequencies less than 10 Hz. The randomness of the wave is important only for higher wave frequencies of ground motion. However, most of the power of strong earthquake ground motions is concentrated at wave frequencies less than 10 Hz. Therefore, the approximations involved in considering ground motions to be caused by a single plane wave at all frequencies are typically within the bounds of engineering accuracy, and result in a significantly simpler model for characterizing the wave passage effect.

**Coherency spectrum models**

Coherency spectrum models can be classified either as semi-empirical or empirical. Semi-empirical models are based on analytical idealizations of the earthquake wave propagation process. Empirical models have only a heuristic basis of providing a best-fit expression for the analysed data set. The parameters in both types of models are obtained through regression analyses of coherency spectra obtained from array records for a set of past earthquake events. Only a few of the more popular models are presented and critically examined in this paper.

**Luco model.** Various semi-empirical coherency spectrum models are available in the literature—e.g. Loli and Harada and Shinozuka. However, the model proposed by Luco *et al.* is used often, because of its simplicity. This model considers coherency decay to depend on the ground motion wavelength only. The coherency spectrum is given by the following expression:

$$\gamma(r, f) = \exp\left(-2\pi\varepsilon f d \right)^2,$$

(7)

where $d$ denotes the separation distance, and $\varepsilon$ denotes the model parameter. Luco suggests that $\varepsilon$ lies in the range of $2 \times 10^{-4}$ m$^{-1}$ s to $3 \times 10^{-4}$ m$^{-1}$ s for actual earthquake events.

**Harichandran model.** The first investigation, to the best of this author’s knowledge, to attempt a rigorous development of an empirical model suggested that the coherency spectrum could be defined using a weighted summation of two exponential functions:

$$\gamma(r, f) = A \exp\left[-\frac{2d}{\alpha \theta(f)} \right] \left(1 - A + \alpha A \right)$$

(8)

where

$$\theta(f) = k\sqrt{1 + (f/f_n)^2}$$

(9)

and $A$, $\alpha$, $k$, $f_n$ and $b$ denote the model parameters. A typical set of values for these parameters that defines the coherency spectrum for the horizontal ground motion component along the epicentral direction, which has been determined through an analysis of SMART-1 array.
records for a particular earthquake event are (from ref. 34):

\[
\begin{array}{cccc}
A & \alpha & k & f_0 & b \\
0.736 & 0.147 & 5210 & 1.09 & 2.78 \\
\end{array}
\]

**Hao model.** One of the main advantages of a dense strong-motion array is the availability of records for components of ground motion that have different alignments to the epicentral direction for a particular earthquake. Hao et al. and his co-workers have used this feature to propose a two-dimensional empirical model for the coherency spectrum of horizontal ground motion. This coherency magnitude is given by the following expression:

\[
|\gamma(r, f)| = \exp \left[ -\beta_1 d_1 - \beta_2 d_2 \right] \times \\
\exp \left[ -\left( \alpha_1(f) \sqrt{d_1^2 + \alpha_2(f) \sqrt{d_2^2}} \right) f^2 \right],
\]

where \(d_1\) and \(d_2\) are, respectively, the projected distances along and transverse to the wave propagation direction, and

\[
\alpha_1(f) = a/f + bf + c \quad \alpha_2(f) = d/f + ef + g,
\]

where \(\beta_1, \beta_2, a, b, c, d, e,\) and \(g\) denote the model parameters. A typical set of values for these parameters that defines the coherency spectrum, determined by Hao et al. for a particular earthquake event recorded by the SMART-1 array, are:

\[
\begin{array}{cccc}
\beta_1 & \beta_2 & a & b \\
2.25 \times 10^{-4} & 5.1 \times 10^{-4} & 1.07 \times 10^{-2} & 2.65 \times 10^{-5} \\
\end{array}
\]

\[
\begin{array}{cccc}
c & d & e & g \\
-1.0 \times 10^{-4} & 6.66 \times 10^{-3} & 5.88 \times 10^{-5} & 1.1 \times 10^{-3} \\
\end{array}
\]

It should be noted that this is the only model currently available that can be used to define the coherency spectrum for a horizontal ground motion component aligned at a specified angle to the earthquake epicentral direction. However, if the epicentral direction is not known, it is currently best to assume that this direction is aligned with the two stations, i.e., the separation distance components, \(d_1\) and \(d_2\), are equal to \(d\) and zero, respectively.

**Abrahamson model.** The models discussed above have all been developed using the SMART-1 array database, which has a minimum separation distance of 100 m between two stations. Coherency decay must, however, be defined over much shorter distances, as supports of most structures have dimensions or separation distances of less than 50 m. This was mostly done by arbitrarily considering the above models to be valid for all distances. Abrahamson et al. however, have used the database from the more recently installed smaller LSST (EPRI) array in Taiwan to develop an empirical coherency spectrum model that is specifically valid for separation distances in the range of 6 to 85 m. The coherency magnitude for this model is given by the following expression:

\[
\tanh^{-1} |\gamma(r, f)| = (a_1 + a_2 d) \times \\
\left[ \exp \left\{ (b_1 + b_2 d) f \right\} + V_3 f^5 \right] + 0.35,
\]

where \(d\) denotes the separation distance, and \(a_1, a_2, b_1, b_2,\) and \(c\) denote the model parameters. A typical set of these parameters that have been obtained by analysing 15 separate earthquake events recorded by the LSST array are given below:

\[
\begin{array}{cccccc}
a_1 & a_2 & b_1 & b_2 & c \\
2.535 & -0.0118 & -0.115 & -8.37 \times 10^{-4} & -0.878
\end{array}
\]

This study concluded that station alignment with respect to the epicentral direction does not significantly affect the coherency spectrum. Therefore, the coherency spectra for all horizontal ground motion components are identical.

**Simple design model.** The empirical models presented above are all multi-parameter models with complex functions defining coherency spectra. In addition, a set of model parametric values is strictly valid for the earthquake database chosen to determine that set. This set of values exhibits a large scatter when coherency spectra of different earthquake events are fitted to the model function for any particular model. These models, therefore, have limited application in characterization of spatial variation, as a design set of parametric values cannot be obtained for any model at present.

A coherency spectrum model, if it is to be useful for characterizing spatial variation of a design-basis earthquake, should be simple and have no more than two parameters in the spectrum expression (also emphasized by Novak). The Luco model (refer equation (7)), therefore, should be an excellent model for separation distances greater than 100 m, because for relatively larger distances coherency decay has been observed to depend, as assumed by this model, on wavelength only. However, as is evident from Figures 2 and 3, in which the coherency spectrum based on the Luco model has been plotted as a function of both separation distance and wave frequency for a value of \(c\) equal to \(2 \times 10^{-4} \text{ m}^{-1} \text{s}\), this model considers the rate of coherency decay.
Figure 2. Variation of ground acceleration coherency with separation distance obtained using different empirical models derived from SMART-1 array data.

Figure 3. Variation of ground acceleration coherency with excitation frequency obtained using different empirical models derived from SMART-1 array data.

decay with both distance and frequency to be significantly larger than observed in actual earthquakes, as represented by the best-fit empirical models proposed by Harichandran and Hao. In this author's opinion, a lower order variation of the function exponent for coherency magnitude in equation (7) than the proposed quadratic form, e.g., a linear exponent variation, may give greater correlation with observed coherencies.

The problem associated with using coherency spectrum models obtained from the SMART-1 array for separation distances below 100 m is illustrated in Figure 4, in which the Abrahamson model is considered to be the proper model. For these short separation distances, which are more often encountered in real structures, the coherency magnitude decays faster with increasing wave frequency than with increasing separation distance. Therefore, coherency decay predicted by every SMART-1 array-derived model is different from the actual observed decay.

However, since the Abrahamson model is complex with multiple parameters [refer equation (12)], it is not a suitable design model. A simple two-parameter model that has recently been proposed is probably more appropriate for characterizing spatial variation of the horizontal component of earthquake ground motion. The expression for the coherency spectrum is given as follows:

\[ |\gamma(r, f)| = \exp\left[-a(f^2 d)^b\right], \quad (13) \]

where \(a\) and \(b\) are the model parameters. If \(a = 0.0007\) and \(b = 0.75\) are considered to be the design values then the coherency spectrum predicted by this model is close to that predicted by the Abrahamson model for separation distances less than about 100 m and wave frequencies less than 15 Hz (refer Figure 5). Since these distance and frequency ranges also define the ranges for most practical situations, it can be reasonably concluded that the coherency spectrum model given by the expression in equation (13) is a good design-basis model.
Effects of earthquake and local site parameters on coherency spectrum

Since coherency models cannot be specifically developed for all types of sites, it is essential that there is a rational basis for characterization of spatial variation, so that a particular design coherency spectrum can be justifiably used for a particular site. Some recent investigations\textsuperscript{22,31}, have, therefore, concentrated on studying the effect that various earthquake source, travel path, and site factors have on coherency spectra. However, it must be stated that the conclusions from these investigations are based on observations from a relatively small number of earthquake events, and are at best preliminary indications that have to be verified through further studies.

Analyses of Taiwan earthquakes show that small magnitude earthquake coherencies may be lower than those for large earthquakes, especially for low to moderate wave frequencies, although opposite trends are noticed for high wave frequencies. Similarly, near-field coherencies may be lower than those in the far-field for low to moderate wave frequencies; these observations, however, definitely need further verification, as the trends may be more due to the frequency attenuation effects of nonlinear soil behaviour in the soft alluvium in Taiwan, than any inherent dependence. Therefore, it can be concluded that current knowledge suggests no clear dependence of acceleration spatial coherency on either magnitude or source distance.

Observations also suggest that coherency decay does not depend on the type of foundation-soil medium, if source effects due to extended fault rupture are inconsequential, i.e. for small magnitude earthquakes or at far-field sites for large magnitude earthquakes. Therefore, spatial coherencies measured from small magnitude earthquakes can be used to characterize spatial variation of ground motion when source effects are small. However, if a particular site has significant lateral heterogeneity in the foundation medium, e.g. it is located...
on folded sedimentary rock, coherency decay does not exhibit the strong dependence on separation distance and wave frequency that it does for an essentially laterally homogeneous medium. In fact, higher spatial decay has been observed at small separation distances and low frequencies at sites with significant lateral heterogeneity.\(^{38}\)

**Ground displacement spatial variation models**

Definition of the spatial variation of ground displacement during an earthquake event is important, because the differential ground movement caused by this spatial variation significantly adds to the structural distress caused by the inertial effects of ground acceleration. There have, however, been relatively fewer studies on spatial variation of ground displacement than of ground acceleration. Some researchers have suggested that both ground acceleration and displacement show similar spatial variation. But this logic is tenuous because the two processes are primarily affected by different wave frequencies—acceleration by the high frequency (short period) waves, and displacement by the low frequency (long period) waves—and should, therefore, exhibit differences in their spatial variation.

A significant part of the effort on characterizing spatial variation of ground displacements has been spent on studying the variation of differential ground displacements with separation distances.\(^{39-41}\) This is because differential ground displacement is the major cause of distress in multiply supported structures that are essentially excited pseudo-statically by earthquake ground motions. However, no uniform basis has evolved through these studies for characterizing differential displacements as a function of separation distance, to the best of this author’s knowledge. Further research, therefore, is required before any definite empirical characterization models can be developed.

**Correlation function models**

A few empirical models that define the variation in ground displacement correlation function \(\rho (r)\) with separation distance are currently available. The earliest model\(^{42}\) is given by the following expression:

\[
\rho (r) = \exp \left[ -\left( \frac{d_0}{d} \right)^2 \right] \left( 1 - 2 \left( \frac{d_0}{d} \right)^2 \right),
\]

where \(d_0\) denotes a model parameter, whose value (as obtained through analysis of a few SMART-1 array records) is specified as \(8.8389 \times 10^{-4}\) m. Two of the subsequent models are given by the following expressions:

\[
\rho (r) = \exp \left[ -(\bar{d} d_1) \cos (2\pi k_0 d) \right] J_0 (k_0 d),
\]

where \(J_0\) denotes a Bessel function of the zeroth order, and \(k_0\) denotes a model parameter, whose empirical value is obtained from SMART-1 array records as \(2.734 \times 10^3/\)m; and

\[
\rho (r) = \exp \left[ -(d/d_0)^{3} \right] \left[ 1 - (d/d_0)^{3} \right],
\]

where \(d_0\) denotes a model parameter, whose empirical value for different earthquakes measured by the Chiba array varies between 380 m and 550 m, with a typical value of 470 m.

A comparison of the above three models is presented in Figure 6. It is noteworthy that whereas in Taiwan ground motions become out-of-phase (negative correlation coefficient) at separation distances of greater than 800 m, in Japan this happens at distances less than 500 m. This seemingly illustrates that ground displacement correlation functions are different for Taiwan (as represented by the Harada and Loh et al. models, which give reasonably similar correlation variation) and Japan (as represented by the Tamura et al. model). However, further studies are required to investigate carefully the causes for these differences.

**Coherency function models**

No detailed investigations on spatial coherency variations of ground displacements with wave frequency and separation distances, as are available for ground accelerations, have yet been reported in the literature. Recently, Tamura et al.\(^{41}\) have used earthquake records from the Chiba

![Figure 6. Comparison of displacement correlation coefficient estimates, as functions of separation distance, obtained using different empirical models derived from dense array records.](image-url)
dense array to suggest a preliminary coherency model for ground displacement. Coherency has been estimated as a function of separation distance at the predominant frequency of ground motion. The coherency function is defined as follows:

\[ \gamma(r) = \exp[-(d/a_0)^2] \]  \hspace{1cm} (17)

where \( a_0 \) denotes a model parameter, whose empirical value for different earthquakes varies between 370 m and 960 m, with a typical value of 760 m. The study, however, does not mention whether this coherency model is equally valid for sites with different predominant frequencies of ground motion. Further studies are, therefore, required for development of more sophisticated and robust coherency models for ground displacement.

Development of a reliable displacement coherency model calls for a satisfactory solution to the problem of high noise-to-signal ratio in the displacement coherency spectrum for a particular earthquake. This problem is likely to be encountered generally because ground displacement is typically affected by a smaller band of wave frequencies than is ground acceleration, and a small frequency bandwidth is, therefore, required in the numerical analysis of ground displacement records. Reducing frequency bandwidth, however, increases noise-to-signal ratio significantly.

Final comments

Spatial variation of ground motion has a significant effect on the earthquake response of structures that either have large dimensions, such as nuclear reactor buildings and dams, or are elongated and multiply-supported, such as bridges and pipelines. This spatial variation has two separate causes: one, arising from the wave passage effect due to the finite apparent velocity of surface propagation of earthquake waves; and the other, defined as the incoherence effect, produced by wave scattering and extended fault rupture propagation.

The coherency function approach of characterization provides a complete definition of spatial variation of ground motion as it quantifies each of the above-mentioned effects separately. Significant amount of work has been done on defining coherency functions for ground acceleration, although there is no design coherency spectrum available as yet. A simple model that has been proposed recently is probably the best approach for characterizing spatial variation of ground acceleration. Relatively fewer studies have concentrated on defining spatial variation of ground displacement. Further research is required in the following areas:

- Development of design ground acceleration coherency spectra for different soil conditions.
- Definition of design normalized differential ground displacements as a function of separation distance, and a study of the effect of earthquake source, travel path, and soil conditions.
- Development of coherency models for ground displacements and subsequent definition of design displacement coherency spectra.

In India, to the best of this author's knowledge, spatial variation of ground motion has never been considered in the earthquake response analysis of any structure. This is probably because no investigations have yet been carried out to characterize the spatial variation of earthquake ground motion in this country. These studies have of course been hampered by the absence of a dense array of digital strong motion accelerographs similar to those installed in Taiwan, Japan and USA. However, since no strong correlation between spatial variation and earthquake magnitude has yet been established, it would be fruitful to carry out the following preliminary investigations to initiate research in this field:

- Instrumentation of a few bridges, preferably with a variety of foundation-soil conditions, in the northeastern region (a high seismicity zone, where moderate intensity earthquakes are fairly common), to measure differential motions between different parts of the structures during an earthquake event. These measurements can be repeated for a few events over a period of a few months.
- Analysis of data obtained from the above instruments, and study of the correlation between measured differential motions on these bridge structures and those obtained analytically using spatial variation models from other parts of the world.
- Development of a mobile 9 or 10 station dense array with broad-band digital triaxial accelerographs linked to a common time base, which, over a period of one year, can be deployed in different parts of the north-east region to generate a reasonable size database of ground motion records. This database can then be used to study spatial variation of ground motion for Indian earthquakes, and compare with studies using data from other parts of the world.

These preliminary investigations are to establish whether spatial variation of ground motion in India has features not found elsewhere in the world, and whether further detailed studies should be undertaken on a country-wide basis.

SPECIAL SECTION

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