Temporal clustering of earthquakes – A fractal approach

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Temporal clustering of earthquakes occurring in two distinct seismogenic provinces, viz. the BEAS network located in NW Himalaya and the MOBILE network located in Delhi and its surrounding regions (India), is modelled using fractal approach. The earthquake sequences in these regions strongly deviate from a uniform distribution and suggest a combination of scale-invariant clustering and random events. The fractal dimensions (D) obtained are 0.254 and 0.193 for the BEAS and MOBILE network regions respectively. The application of fractal approach to seismic clustering is demonstrated to discriminate the temporal distribution of earthquakes under different seismogenic provinces.

EARTHQUAKE occurrence in time has been, hitherto, modelled by various workers using Poisson process. However, some recent studies on regional seismicity[1, 2] have described the seismic distributions as non-Poissonian. Fractal statistics, which model scale-invariant processes have, of late, been applied to quantify the deviations from a Poisson distribution. The fractal theory was first developed by Mandelbrot[3] and applied to rocky coastline. He introduced the concept of fractal dimension to describe the roughness of the coast of Britain. According to the definition of the fractal curve, the length or perimeter of a curve (P) is related to the length of the measuring rod (l) by

$$P \sim l^{(1-D)}, \qquad (1)$$

where D is the fractal dimension. Typical values of D ($1 < D < 2$) for coastlines are found to be about 1.2. Alternatively, the number of objects N with a characteristic linear dimension greater than r is given by,

$$N \sim r^{-D}. \qquad (2)$$

Smalley *et al.*[4] applied the concept of fractals to the temporal clustering of earthquakes in the New Hebrides region. The method provides for a means of testing whether clustering in time is a scale-invariant process, and the degree of clustering is explained by fractal dimension (D). The present study is a similar attempt to investigate the temporal clustering of earthquakes in two distinct seismogenic provinces, viz. the BEAS and MOBILE network regions of India.

The India Meteorological Department has been operating BEAS and MOBILE seismic networks since early sixties. Under the BEAS network, 13 seismological observatories are in operation (between latitude 30°–34°N and longitude 74°–78°E) to monitor the reservoir-induced seismic activity around the Beas–Sutlej reservoir, where a large dam has been constructed. Another network (MOBILE) of four seismological observatories (excluding the NDI observatory) is in operation (between latitude 28°–29°N and longitude 76°–78°E) to monitor the seismicity of Delhi and its neighbourhood. Both the networks have yielded continuous reliable data for over 25 years. The two networks, thus, offer a unique opportunity to study and compare the temporal clustering of events associated with natural and reservoir-induced seismicity.

The application of fractal theory to seismic clusters is best explained by a Cantor Set model[4]. The problem essentially involves in analysing the temporal distribution of N earthquakes occurring in a time interval of r_0. Depending upon the time resolution of the events and the period for which data are available, the total duration of data is rescaled so that the width of an individual element of the truncated nth order Cantor set is unity giving a total duration of 3^n units with 2^n elements. In the present study, the width of an individual element is taken as 1 day and the total duration is rescaled to give a

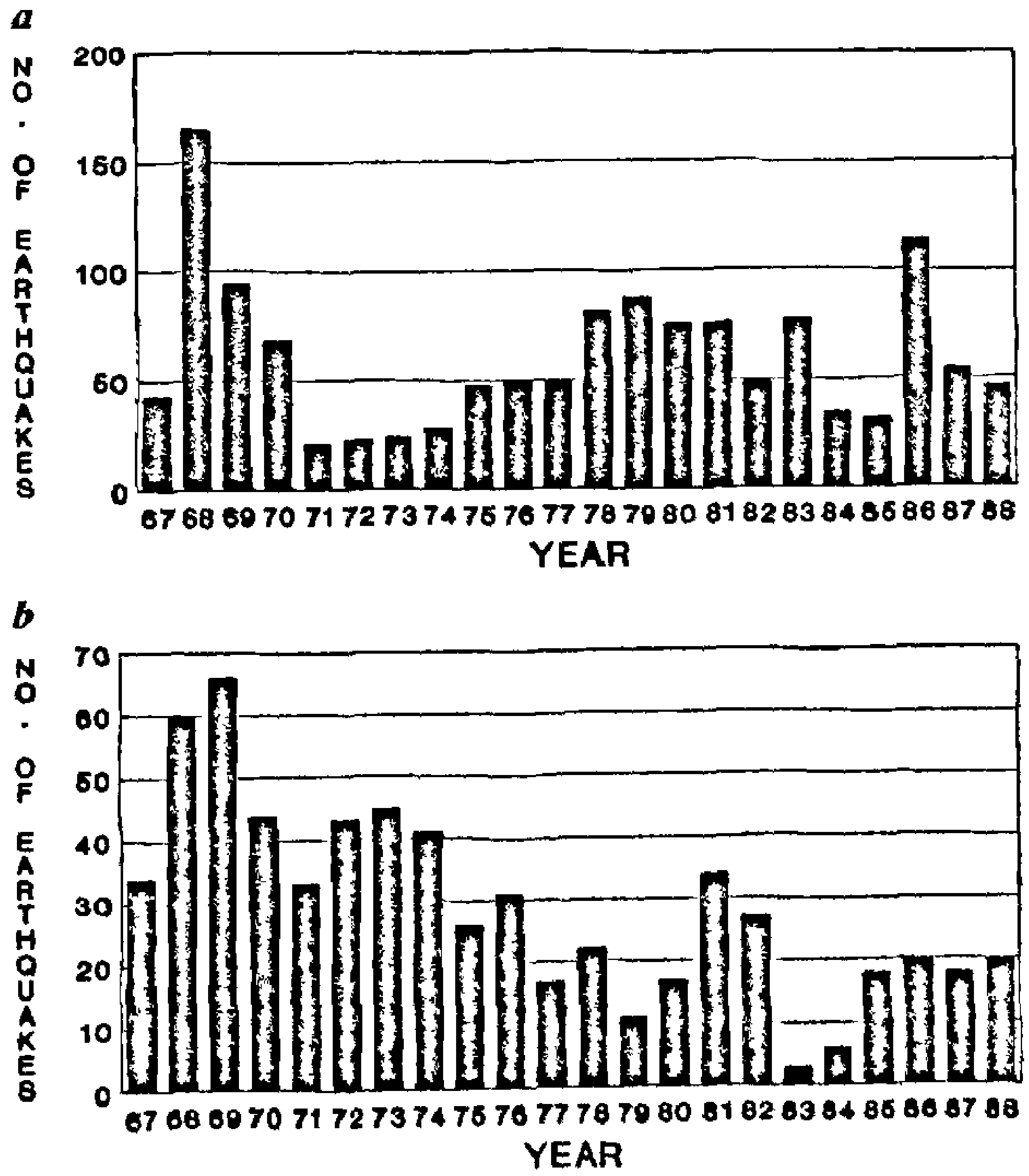

Figure 1. Histogram showing yearly distribution of earthquakes for (*a*) BEAS network region and (*b*) MOBILE network region

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total length of 6561 (3^8) days with 256 (2^8) elements for an eighth order Cantor set. Accordingly, events occurring between 29 July 1966 and 14 July 1984 (6561 days) are considered to study the temporal cluster in the two network regions. The total number of events during the period are 1103 and 587 for BEAS and MOBILE networks respectively. A histogram showing the yearly distribution of events for the two networks is shown in Figure 1 *a* and *b*.

Using the box method suggested by Smalley *et al.*[4], the total duration is divided into intrinsic intervals of length $r = 3^n$ ($n = 0, 1, 2, \ldots 8$) and the fraction (X) of intervals that include at least one element is determined. The fraction of intervals that include at least one event, is related to the interval length by,

$$X_i \sim r_i^{(1-D)}. \qquad (3)$$

From the slope of the log–log plot between X_i and r_i, the fractal dimension (D) is obtained.

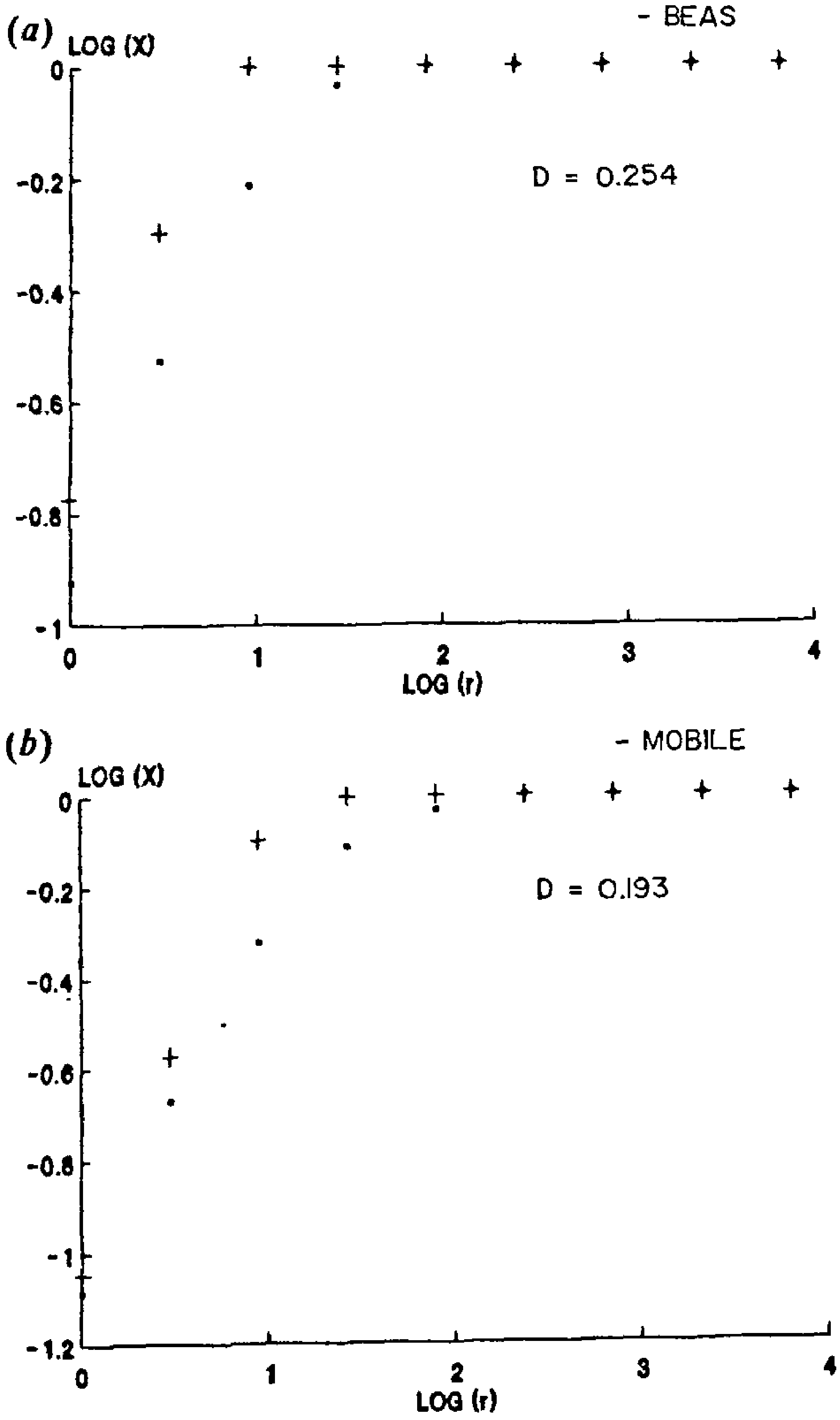

Figure 2 *a* and *b*. Fractal distribution curves for (*a*) BEAS and (*b*) MOBILE networks. The dots are the fraction of intervals that contain at least one event. Pluses are for a uniform distribution of same number of events in respective regions.

If N earthquakes are distributed uniformly (equally spaced in time) over a period of r_0, then we have,

$$X = \begin{cases} \dfrac{rN}{r_0}, & \text{if } r < \dfrac{r_0}{N} \\[2ex] 1, & \text{if } r > \dfrac{r_0}{N}. \end{cases} \qquad (4)$$

The actual earthquake sequences in the two network regions are compared with their respective uniform distributions of similar number of events using eq. (4) to estimate the deviations.

The results of fractal analysis are shown in Figure 2 *a* and *b*. The fraction of intervals (X) that include at least one event is plotted against the interval length (r) for both the regions. For comparison, the results of a uniform distribution of the same number of events in the time interval studied, are also plotted. It may be seen that the actual earthquake sequences are significantly different from their respective uniform distributions. Every interval contains at least one event for $r > 81$ days in case of BEAS network region and $r > 243$ days for MOBILE network region. Although there is considerable curvature of the data, a fractal clustering is found to occur over three orders of time scales for r between 1 and 9 days for both the regions. From the slope of the fractal curve, the fractal dimensions are estimated as 0.254 and 0.193 for BEAS and MOBILE network regions respectively. The results thus suggest that the earthquake sequences in these regions show a combination of scale-invariant clustering and random events.

Smalley *et al.*[4], also obtained fractal dimensions ranging between 0.126 and 0.255 for four seismically active regions in the New Hebrides Island arc region. Based on the time resolution, they selected the smallest time interval as 1 min and obtained a strong scale-invariant clustering over several orders of time scales for one of the four regions. However, for the other three regions there is considerable curvature of the data which is interpreted as due to combination of scale-invariant clustering and random events. They also found that the results of a random simulation of the same number of events in the time interval studied are close to the uniform distribution and different from the earthquake data.

The important aspect of the present study is the difference in the fractal dimensions obtained. The fractal dimension obtained for the BEAS network region ($D = 0.254$) is relatively higher than that of MOBILE network region ($D = 0.193$), suggesting a relatively strong clustering for BEAS events. The difference may be attributed to the distinct nature of seismicity of the two regions. While the seismic activity around Delhi region is mainly tectonic in nature, the same in the BEAS network region may have the additional influence of the loading of reservoirs in the region. Chaudhury and Srivastava[5] studied the reservoir-associated seismic activity around Bhakra, Pandoh and Pong dams in the study region. The increase in the seismic activity did not, however, bear any direct relationship with the reservoir level in the post-monsoon months. The region has also witnessed few earthquakes with foreshock–aftershock sequences, prominent among which is the Kinnaur earthquake of 19 January 1975 ($M_B = 6.2$) in which case the aftershocks continued for several years[5]. In view of the complexity of the region which is influenced by tectonically active Kangra region with foreshock–aftershock sequences as well as the activity in the vicinity of reservoirs, it is difficult to attribute these clusters to any particular phenomenon. It is, however, evident that both these phenomena strongly contribute to the temporal clusters observed in BEAS region. Better time resolution and detection capability of events would enable us to quantify these clusters in terms of the associated activity. Fractal approach to seismic clusters is thus an important tool to study and discriminate the temporal cluster of earthquakes under different seismogenic provinces. The method may also be extended to study the spatial cluster of events in a given tectonic set-up.

Temporal distribution of earthquakes in two distinct seismogenic provinces, viz. the BEAS and MOBILE network regions, has been studied using the fractal approach. The results suggest that the earthquake sequences in these regions differ significantly from the uniform distribution and are interpreted as a combination of scale-invariant clustering and random events. The BEAS network region exhibits relatively strong clustering ($D = 0.254$) in comparison to MOBILE network region ($D = 0.193$). The relatively strong clustering observed for BEAS events may be explained as due to the combined effect of tectonically active Kangra region characterized by foreshock–aftershock sequences and the seismic activity in the vicinity of reservoirs in the region.

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Localization of chromosome scaffold by transmitted- and incident-light fluorescence microscopy

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In order to demonstrate the chromosome scaffold the spermatocytes of a grasshopper were fixed in acetomethanol, treated with alkaline 2XSSC and stained with $AgNO_3$ solution. The air-dried preparations were fluorochromed with acridine orange (AO) and excited with either transmitted or incident beam of UV for fluorescence microscopic study. Under transmitted-light the scaffold is very clearly visible as non-fluorescent dark silver deposition along the middle of the chromosomes. However, under incident-illumination the scaffold is invisible and only the fluorescence of the epichromatin is observed. This indicates that the scaffold is situated longitudinally in the centre of a solid cylinder of epichromatin made of DNA-histone.

SINCE the early electron microscopic demonstration of a chromosome core surrounded by the epichromatin[1] and the existence of a proteinaceous scaffold surrounded by DNA loops[2], Howell and Hsu[3] were able to demonstrate a chromosome core at the light microscopic level by developing a silver-deposition technique. Since then a number of authors[4–7] confirmed the presence of such a scaffold. The scaffold is thought to be involved in chiasma-formation[8] and is possibly bipartite in nature[9] and is a permanent component of the chromosomes[10]. After complete removal of histones from the metaphase chromosomes, a very limited set of proteins remains in the scaffold and the enzyme topoisomerase II is one of them[11]. The enzyme topo II provides a structural anchorage for the chromatin loops to the axial scaffold of the chromosomes[12, 13]. The anchorage or the scaffold attachment region (SAR) of DNA is characterized by A-T rich consensus sequence[14]. On the other hand the catalytic activity of topo II in facilitating the strand passage is intimately associated with chromosome condensation as has been demonstrated by a cytological analysis of appropriate mutants in fission yeast[15] and from *in vitro* studies of cell extracts[16]. The present transmitted- and incident-light fluorescence microscopic study of the silver-stained scaffold shows that it is centrally located inside the epichromatin during the