Modelling of coseismic and postseismic crustal deformations associated with strike-slip faulting

Sarva Jit Singh and Sunita Rani

Department of Mathematics, Maharshi Dayanand University, Rohtak 124 001, India

In order to model the coseismic and postseismic crustal deformations associated with faulting at a transform plate boundary, we consider the problem of a long inclined strike-slip fault in a layer overlying a uniform half-space. Closed-form expressions for the static displacements are obtained when the two media are elastic. In addition, the quasi-static field is obtained when the layer is elastic and the half-space is Maxwell viscoelastic. In this model the layer represents the lithosphere and the Maxwell half-space represents the asthenosphere. The coseismic field is modelled by the static response and the postseismic field is modelled by the quasi-static response minus the static response.

THE crustal deformation cycle can be divided into four time phases relative to the earthquake: interseismic, preseismic, coseismic and postseismic. The interseismic phase is the strain accumulation phase and is generally attributed to locking of the uppermost segment of the fault while aseismic slip on the fault continues at secular rate at depth. In the preseismic phase, the strain accumulation rate increases and the medium behaves anelastically. In the coseismic phase, the strain energy accumulated during the interseismic and preseismic phases is converted into kinetic energy and is released in the form of seismic waves. The postseismic phase can be explained as viscoelastic relaxation of the coseismic stresses.

The elasticity theory of dislocations has been developed and applied by Steketee[1], Maruyama[2,3] and others. As a mathematical model of faulting, Steketee assumed a displacement dislocation surface, i.e., a surface across which the displacement vector is discontinuous. Extensive reviews of the applications of the elasticity theory of dislocations to earthquake faulting have been given by Savage[4] and Rybicki[5]. These reviews include thorough discussions of 2-D models of faulting. Although a 2-D model is an oversimplification of the physical system, these models are very useful in gaining insight into the relationship among various fault parameters. Moreover, there are faults, the most obvious being the San Andreas fault in California, which are sufficiently long and shallow that the 2-D approximation may be used.

Rybicki[6] considered the problem of a long vertical strike-slip fault in a two-layer model of the Earth. In order to determine the characteristics of the time-dependent deformation which follows a sudden slip on large earthquake faults, Nur and Mavko[7] considered the lithosphere-asthenosphere composite as an elastic layer overlying a viscoelastic half-space. Cohen[8] discussed the postseismic surface deformation due to lithospheric and asthenospheric viscoelasticity. Cohen[9] performed finite element calculations to evaluate the time-dependent deformation due to a long strike-slip fault in a multi-layered model of the Earth. The model consists of an elastic upper lithosphere, an SLS viscoelastic lower lithosphere, a Maxwell viscoelastic asthenosphere and an elastic mesosphere. Bonafede *et al.*[10] modelled a microplate as an elastic plate with two long strike-slip boundaries lying over a Maxwell viscoelastic asthenosphere. Garg and Singh[11] studied the quasi-static deformation of a layered half-space by a long vertical strike-slip fault.

In this paper, we consider the deformation of a lithosphere-asthenosphere Earth model consisting of a layer of uniform thickness overlying a half-space caused by a long inclined strike-slip fault of finite width in the layer. The static displacement field is obtained when the two media are elastic. The correspondence principle of linear viscoelasticity[12–15] is used to obtain the quasi-static field from the static field when the layer is elastic and the half-space is Maxwell viscoelastic. The static field is used to model the coseismic deformation following a strike-slip earthquake at a transform plate boundary and the quasi-static field minus the static field is used to model the postseismic deformation. The effect of the fault-depth on the coseismic and postseismic deformations is studied by performing detailed numerical computations for four locations of the fault: one surface-breaking fault and three deep crustal faults. It is found that the surface field caused by a surface-breaking fault is characteristically different from the field caused by a fault at depth. Graphs showing the effect of the dip angle on the variation of the coseismic and postseismic surface displacement with distance from the fault are presented. The famous Koyna earthquake of 10 December 1967 is considered to be a strike-slip earthquake. Therefore the results presented here can be used to study the earthquake deformation cycle in the Koyna earthquake region.

Theory

We consider an Earth model consisting of a homogeneous, isotropic elastic layer of thickness H lying over a homogeneous, isotropic, Maxwell viscoelastic half-space (Figure 1). We place the origin of a cartesian co-ordinate system (x_1, x_2, x_3) at the free surface and the x_3-axis is drawn into the medium. Let a long inclined strike-slip 2fault, with strike along the x_1-axis, be situated in the layer.

(y_1, y_2, y_3) is any point on the fault $(0 \le y_3 \le H)$. Let μ_1 and μ_2 be the rigidities of the layer and of the half-space, respectively.

We first obtain the elastic response to a long inclined strike-slip fault in the corresponding elastic model. The correspondence principle of linear viscoelasticity is then used to obtain the quasi-static response.

Elastic solution

Consider an antiplane strain problem for which the displacement components are of the form

$$u = u(x_2, x_3), \quad v = w = 0. \tag{1}$$

For zero body forces, the equilibrium equations reduce to

$$(\partial^2 u/\partial x_2^2) + (\partial^2 u/\partial x_3^2) = 0. \tag{2}$$

The displacement field due to an inclined strike-slip line dislocation can be expressed in terms of the displacements due to a horizontal strike-slip line dislocation and a vertical strike-slip line dislocation in the form[16]

$$u = \cos\delta\, u_{\mathrm{I}} - \sin\delta\, u_{\mathrm{II}}, \tag{3}$$

where δ is the dip angle (Figure 1), u_{I} is the displacement for a horizontal strike-slip line dislocation and u_{II} is the displacement for a vertical strike-slip line dislocation. Using the results for the horizontal and vertical strike-slip line dislocations given by Garg and Sharma[17] in equation (3), we obtain the following expression for the displacement at any point of the layer

$$u = \frac{b\,ds}{2\pi}\left\{\cos\delta\left[\frac{x_3-y_3}{R^2} - \frac{x_3+y_3}{S^2} + \sum_{n=1}^{\infty} r^n\left\{(2nH-x_3-y_3)\frac{1}{T^2} + (2nH+x_3-y_3)\frac{1}{V^2} - (2nH-x_3+y_3)\frac{1}{U^2} - (2nH+x_3+y_3)\frac{1}{W^2}\right\}\right] - \sin\delta\,(x_2-y_2)\left[\frac{1}{R^2} + \frac{1}{S^2} + \sum_{n=1}^{\infty} r^n\left(\frac{1}{T^2} + \frac{1}{V^2} + \frac{1}{U^2} + \frac{1}{W^2}\right)\right]\right\}, \tag{4}$$

where

$$\begin{aligned}
r &= (\mu_1-\mu_2)/(\mu_1+\mu_2)\\
R^2 &= (x_2-y_2)^2 + (x_3-y_3)^2\\
S^2 &= (x_2-y_2)^2 + (x_3+y_3)^2\\
T^2 &= (x_2-y_2)^2 + (2nH-x_3-y_3)^2\\
U^2 &= (x_2-y_2)^2 + (2nH-x_3+y_3)^2\\
V^2 &= (x_2-y_2)^2 + (2nH+x_3-y_3)^2\\
W^2 &= (x_2-y_2)^2 + (2nH+x_3+y_3)^2,
\end{aligned} \tag{5}$$

b is the slip and ds is the width of the line dislocation.

Changing to polar coordinates (Figure 1)

$$y_2 = s\cos\delta, \quad y_3 = s\sin\delta \tag{6}$$

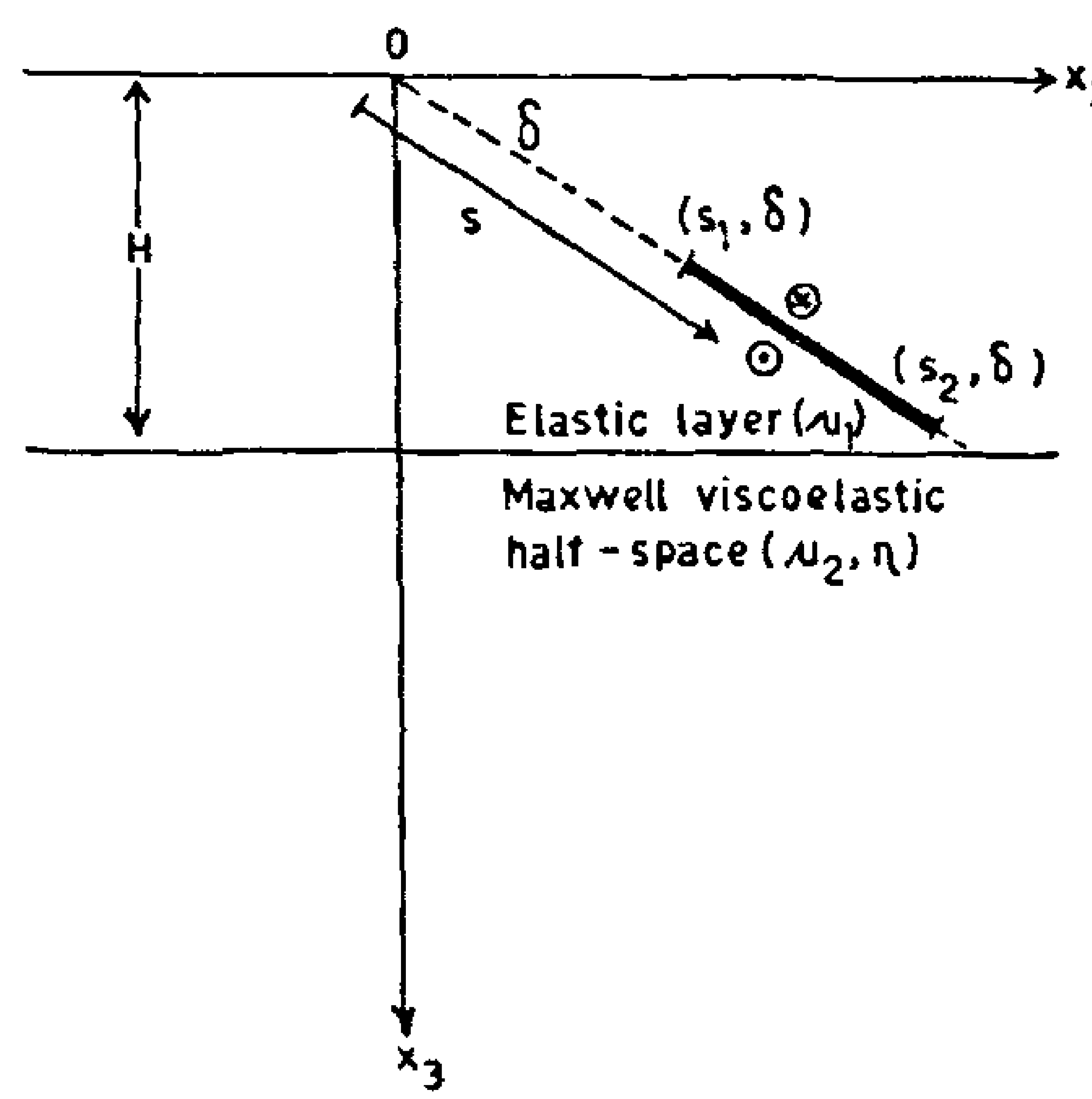

Figure 1. Geometry of a long strike-slip fault situated in a layer of uniform thickness H lying over a half-space. The displacement discontinuity on the fault is parallel to the x_1-axis. The sign ⊙ indicates displacement in the direction of the x_1-axis, the sign ⊗ in the opposite direction. The cartesian coordinates of a point on the fault are (y_2, y_3) and its polar coordinates are (s, δ), where δ is the dip angle.

in equation (4) and integrating over s between the limits (s_1, s_2), we obtain the following expression for the displacement due to a long strike-slip fault of finite width $s_2 - s_1$:

$$u = \frac{1}{2\pi}\left\{ M_0\left[\tan^{-1}\left(\frac{s - x_2\cos\delta - x_3\sin\delta}{x_3\cos\delta - x_2\sin\delta}\right) - \tan^{-1}\left(\frac{s - x_2\cos\delta + x_3\sin\delta}{x_3\cos\delta + x_2\sin\delta}\right)\right] + \sum_{n=1}^{\infty} M_n\left[\tan^{-1}\left(\frac{s - x_2\cos\delta + x_3\sin\delta - 2nH\sin\delta}{2nH\cos\delta - x_3\cos\delta - x_2\sin\delta}\right) + \tan^{-1}\left(\frac{s - x_2\cos\delta - x_3\sin\delta - 2nH\sin\delta}{2nH\cos\delta + x_3\cos\delta - x_2\sin\delta}\right) - \tan^{-1}\left(\frac{s - x_2\cos\delta - x_3\sin\delta + 2nH\sin\delta}{2nH\cos\delta - x_3\cos\delta + x_2\sin\delta}\right) - \tan^{-1}\left(\frac{s - x_2\cos\delta + x_3\sin\delta + 2nH\sin\delta}{2nH\cos\delta + x_3\cos\delta + x_2\sin\delta}\right)\right]\right\}\Bigg|_{s_1}^{s_2}, \quad (7)$$

where

$$f(s)\Big|_{s_1}^{s_2} = f(s_2) - f(s_1),$$

$$M_n = b\left(\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\right)^n. \quad (8)$$

Viscoelastic solution

We use the correspondence principle of linear viscoelasticity to obtain the quasi-static deformation for a model consisting of an elastic layer lying over a Maxwell viscoelastic half-space. In order to obtain the Laplace transformed viscoelastic solution, it is only necessary to replace μ_2 and b by μ_2^* and $\bar{b}$, respectively, in the corresponding elastic solution, where an overbar indicates Laplace transform, $\mu_2^* = p\,\mu_2/(p + \tau^{-1})$ is the transform rigidity, $\tau = \eta/2\mu_2$ the relaxation time, η the viscosity and p the Laplace transform variable. Time-dependence of the dislocation source is taken to be a unit step function, i.e. $b(t) = b_0 H(t)$, where b_0 and $H(t)$ are, respectively, the magnitude of the slip (dislocation) and the Heaviside unit step function. Therefore, $\bar{b} = b_0/p$. From (7) and (8) we notice that μ_2 and b occur only through M_n. Therefore, the Laplace transformed solution of the viscoelastic problem is obtained from (7) on replacing M_n by $\bar{M}_n$ where, from (8),

$$\bar{M}_n = \frac{b_0}{p}\left(\frac{pB + A}{p + A}\right)^n, \quad (9)$$

and

$$A = \frac{\mu_1}{\tau(\mu_1 + \mu_2)}, \quad B = \frac{\mu_1 - \mu_2}{\mu_2 + \mu_2}. \quad (10)$$

In order to find the inverse Laplace transform of $\bar{M}_n$, we use integral transform tables of Erdelyi[18] and obtain

$$L^{-1}[\bar{M}_0] = b_0,$$

$$L^{-1}[\bar{M}_n] = b_0\left[1 + \exp(-At)\left[\sum_{m=1}^{n}\frac{F_{2m}(-A)}{(n-m)!\,(m-1)!}\,t^{n-m}\right]\right], \quad (11)$$

where $t > 0$, $n > 0$ and

$$F_{2m}(p) = \frac{d^{m-1}}{dp^{m-1}}\left[\frac{(pB + A)^n}{p}\right]. \quad (12)$$

Numerical results

We consider the particular case when the rigidities μ_1 and μ_2 are equal, i.e. $\mu_1 = \mu_2 = \mu$ (say). Equation (10) shows that, for this particular case,

$$A = ½\tau, \quad B = 0. \quad (13)$$

Equations (12) and (13) yield

$$F_{2m}(-A) = -(m-1)!/(2\tau)^{n-m}. \quad (14)$$

We wish to compute the coseismic and postseismic surface displacements. The coseismic field is modelled by the static response. The postseismic field is obtained on subtracting the static response from the quasi-static response. In order to study the effect of the depth of the fault on the surface displacements, we consider four positions of the fault:

Source I: $s_1 = 0$, $s_2 = H/4$,
Source II: $s_1 = H/4$, $s_2 = H/2$,
Source III: $s_1 = H/2$, $s_2 = 3H/4$,
Source IV: $s_1 = 3H/4$, $s_2 = H$

Figure 2 *a* shows the variation of the coseismic and postseismic parallel displacement (u) at the surface in units of the slip b for a surface-breaking fault (source I) dipping at $\delta = 60°$. u is asymmetric about the origin. The coseismic displacement is discontinuous at $x_2 = 0$ while the postseismic displacement is continuous at $x_2 = 0$ and vanishes at $x_2 = 0.12\,H$. Figures 2 *b–d* are for sources II, III and IV. The coseismic displacement vanishes at $x_2 = 0$ and the postseismic displacement vanishes at $x_2 = 0.36$ H, 0.57 H and 0.75 H for sources II, III and IV, respectively. Figures 3 *a–d* exhibit the variation of the coseismic and postseismic displacements with the distance from the fault for a vertical strike-slip fault. The displacement field is antisymmetric about the origin. The coseismic and postseismic displacements are of the same sign. The coseismic displacement is discontinuous at the origin for source I and is continuous for the other three sources at $x_2 = 0$ and vanishes at that point. The postseismic displacement is continuous at $x_2 = 0$ for source I also and vanishes there.

Conclusions

We have analysed strike-slip faulting in the lithosphere for a long fault of arbitrary dip and finite width. Static elastic analysis is used to model the coseismic deformation. The correspondence principle of linear visco-elasticity yields the quasi-static response when the asthenosphere is assumed to be Maxwell-viscoelastic. The initial response of the Maxwell asthenosphere coincides with the elastic response. The postseismic field is obtained by subtracting the static response from the quasi-static response. The coseismic and postseismic fields are found to depend markedly on the depth of the fault and its dip. For a surface-breaking fault, the ratio of the absolute displacement of the hanging wall

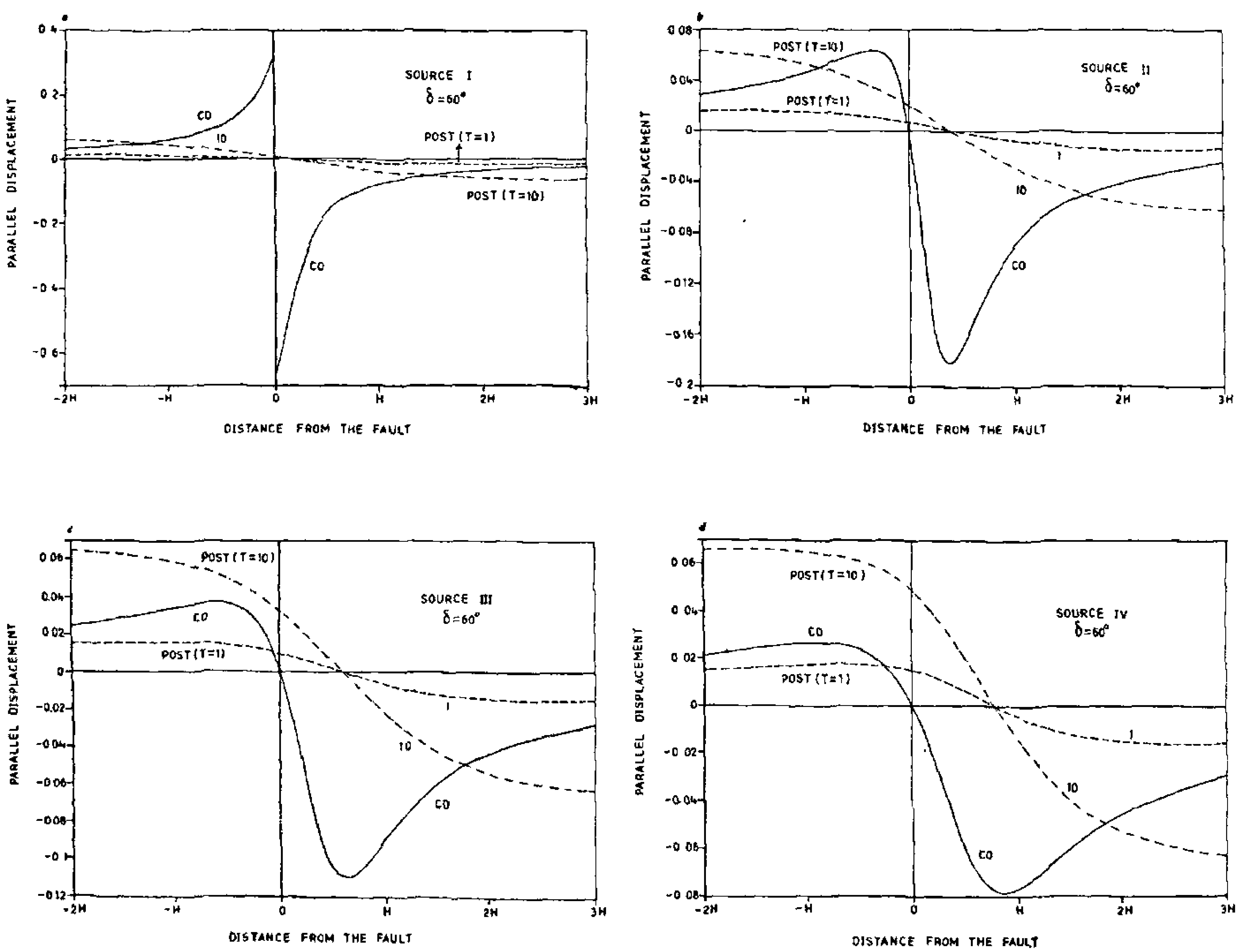

Figure 2. Variation of the coseismic and postseismic parallel surface displacement in units of the slip b with the distance from the fault (x_2) assuming $\delta = 60°$, $\mu_1 = \mu_2$, for (*a*) Source I, (*b*) Source II, (*c*) Source III, (*d*) Source IV. CO indicates coseismic (static) displacement. POST indicates postseismic (quasi-static minus static) displacement and is shown for two values of the dimensionless time $T = t/\tau$, where τ is the relaxation time.

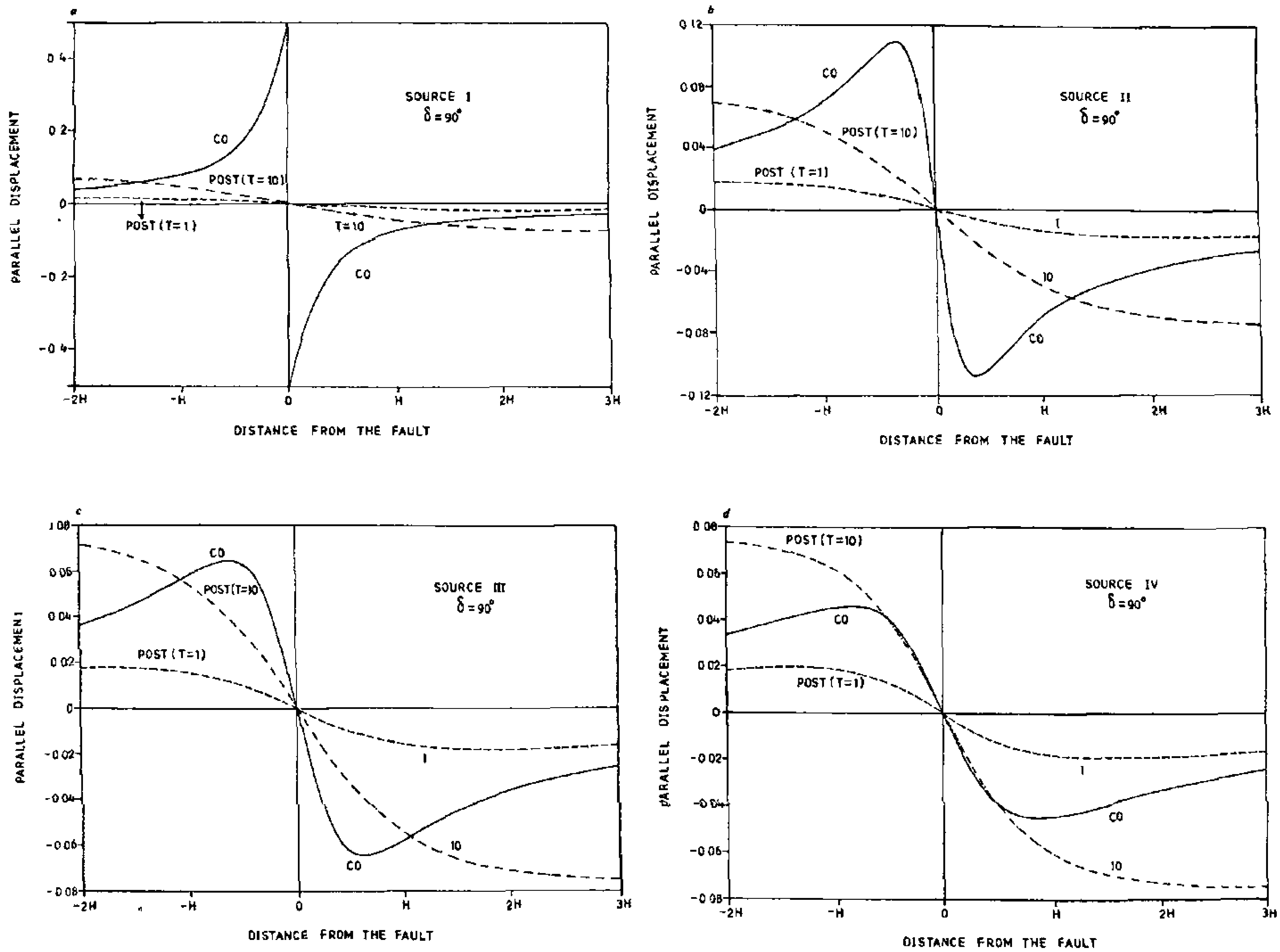

Figure 3. Variation of the coseismic and postseismic parallel surface displacement in units of the slip b with the distance from the fault (x_2), assuming $\mu_1 = \mu_2$, $\delta = 90°$ for (*a*) Source I, (*b*) Source II, (*c*) Source III, (*d*) Source IV.

to that of the foot wall is a sensitive indication of the dip angle. Similarly, the degree of asymmetry of the surface displacement field depends upon the dip angle.

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