Harish-Chandra and his mathematical work

It is now a little more than six months since Harish-Chandra passed away so suddenly Our grief and pain are still very much with us. A vital part of the lives of the members of his family is gone while his friends and colleagues, especially from the younger generation, miss his inspiring presence. The Institute has lost a jewel, and India has lost one of her greatest sons It is a privilege, although a very sad one, to be asked to say a few words in his memory on this occasion. I hope that my thoughts will find some resonance in you so that they may be of help in replacing our sense of loss with something more affirmative, perhaps a sense of fulfillment in having been associated with a truly remarkable, and one might say, even heroic life.

It was T. S Eliot who said that after a great poet had lived certain things will have been said once and for all and cannot be repeated again. I feel that this is true of all great creative personalities, and so certainly true of Harish-Chandra The originality and depth of his work will compel later generations to confer on him that luminous distinction reserved only for the most exalted figures of our science. I do not believe that any of us here will ever again come across someone quite like him. In the austere simplicity and uncompromising nature of his approach to life, in his preference for solitary and profound reflection, and in his awesome capacity to discern and persevere after distant goals, he resembled the legendary figures from his country's ancient past. And like them, he came to be quite detached about his achievements as well as his failures. This detachment was not a false modesty; like many great men Harish-Chandra was fully conscious of his gifts and what he could do with them. It was rather a deeper humility, whose origin lay in a conviction that science was a collective endeavour and that any single life is but a fragment in a larger fabric.

Harish-Chandra grew to manhood in the plains of Northern India. This region, fed by the great rivers Ganga and Yamuna, had always been a vital part of Indian life and consciousness. Several of our most famous ancient cities and kingdoms were located there and many of India's greatest men and women had emerged from its soil. In the years following Mahatma Gandhi's call for the British to quit India, the country shook off its stupor and there was a cultural and intellectual flowering in the entire nation. The rise of modern Indian science dates back to that era and inspiring figures like Ramanujan and Raman filled the horizon Harish-Chandra grew to maturity in this milieu during the 1940s. His undergraduate studies were in the University of Allahabad, the modern city founded by the Moghuls in the sixteenth century near the ancient holy city of Prayag at the confluence of the rivers Ganga and Yamuna. His interest was at first in physics, and he came under the influence of K. S. Krishnan who was the professor of physics there at that time. Krishnan was a physicist of great distinction, at home in both theory and experiment, whose work on electromagnetic aspects of crystal physics had already won for him and his students international recognition. In his younger days Krishnan had collaborated with Raman in a series of experiments on the molecular scattering of light that had led to the discovery of the Raman effect in 1928 Cultured, gentle, with deep and wide interests, Krishnan had a real feeling for and understanding of the mathematical aspects of modern physics, and was the central figure in Harish-Chandra's early development. Harish-Chandra would later recall with genuine affection the influence that Krishnan, both as a man and as a scientist, had on him.

Harish-Chandra continued his studies in physics, first at the Indian Institute of Science in Bangalore in Southern India under H. J. Bhabha, and later, at Cambridge in England under P. A. M. Dirac. It was in Cambridge that he changed over from physics to mathematics. That was the time, before renormalization, when quantum electrodynamics was plagued by divergent

integrals; and Harish-Chandra began to have serious doubts whether what he was doing had any link with physical reality, doubts that eventually drove him to seek comfort in the security of pure mathematics.

The influence of Dirac would stay with him forever. He would often openly and unreservedly give expression to the awe and reverence he had for Dirac. Although his work was not directly related to Dirac's, his life-long preoccupation with group representations began with Dirac's suggestion that he investigate the infinite dimensional representations of the Lorentz group. Between him and Dirac there was a subtle bond, an affinity of souls; like Dirac, Harish-Chandra would go his own way in science and nothing could deflect him.

This is not the occasion to go into detail about his scientific work. However, mathematics was such an overwhelming part of his life that it is also impossible not to say something about it. One of the major classical discoveries in the theory of group representations was the notion, due to Frobenius in the nineteenth century, of the character of a finite dimensional representation. In the 1920s Hermann Weyl had discovered a beautiful formula for the characters of finite dimensional representations of semi-simple groups It was Harish-Chandra who significantly enlarged the scope of the concept of the character and discovered an infinite dimensional generalization of Weyl's formula that was equally beautiful. The theory of harmonic analysis on semi-simple groups that he erected based on his character theory is a wonderful generalization of classical harmonic analysis, and is destined to play, for reasons that I cannot go into here, a pivotal role in the great questions of analysis and arithmetic. He himself was unceasingly at work treating these questions until the end of his life In his last years, when his energies began to diminish, it became evident that only his inner strength kept him going. The vast amount of unpublished work that he has left behind, much of it in the form of meticulously handwritten manuscripts, is a moving testimony to the essentially spiritual nature of his quest

Except for a couple of visits of a few months' duration he never went back to India. He was somewhat unhappy about this for he felt that he could have done something valuable for Indian mathematics had he been able to make longer visits. He would often express his admiration for Takagi who went back to Japan and played an important role in the creation of the influential modern Japanese school in number theory. But with increasing uncertainties in his health, any return to India became out of the question; and his dreams were to remain just that. This circumstance often lent a sense of poignancy to this heart-felt concern for India, Indian science, and Indian scientists.

Although he was well aware of what was going on in mathematics he would often say that too much learning hindered his originality. This is, of course, an over-simplified description of his attitude toward the creative process and so, to give you a better feeling for his thinking, I shall read you a brief passage from a talk he gave, a few months before his death, on the occasion of the eightieth birthday celebration of Dirac: 'I have often pondered over the roles of knowledge or experience on the one hand and imagination or intuition on the other, in the process of discovery. I believe that there is a certain fundamental conflict between the two; and knowledge, by advocating caution, tends to inhibit the flight of imagination. Therefore a certain naiveté, unburdened by conventional wisdom, can sometimes be a positive asset.' He made these comments while discussing Dirac's discovery of the equations of motion of the electron; but they apply equally well to his own work and capture its spirit completely. He could even laugh at the fact that he was, in his opinion of course, not as well-informed as he should be; his favourite description of himself was as a man who had a deficiency of iron in his blood but for whom the only available remedy was to swallow a pound of nails every day.

His view of science was a noble one and there was no place for ego in it. There was no doubt in his mind that any one who had a chance to contribute to the flow of science should regard himself as a very fortunate person. He was critical of scientists, even, great ones, who were unable to distinguish between the progress of science and the progress of their own work. Throughout his life his goal was to do as much as he could, for as long as he had the strength to work. This attitude allowed him to view the world around him with remarkable equanimity. It is my firm belief that he was, in the most demanding sense of that Sanskrit word, a *Sthithapragna* – one with a steady inner gaze, having conquered both disappointment and exultation.

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Some recollections of Harish-Chandra

I met Harish-Chandra for the first time in Summer 1949, in Zurich. He had come there to study German and maybe also to see W. Pauli, whom he had known while still a physicist. His conversion from theoretical physics to pure mathematics was fairly recent, so that he would still be asked for its reason. One answer, as I heard at the time, was that now he could believe in what he was doing. This should not be viewed as a disparaging comment on theoretical physics, quite the contrary. He had a very high regard for theoretical physics, but felt that a very special intuition, not mathematical, some sort of 'sixth sense', was needed to make progress in it. Later I heard him on various occasions stating that 'no mathematician has ever made a dent in physics' (this is a quote), which, I think, is too sweeping a statement, in parti-

cular unfair to H. Weyl. He could still dabble in physics, though. While we were walking along the lake around sunset, the reflection of light struck him as somewhat peculiar. The following morning, when I picked him up in his hotel room, I saw a sheet or two of paper full of computations, with some drawings of reflected light rays.

At that time he had already published some papers on Lie groups and Lie algebras. Since my main interests were Lie groups and topology there was a considerable overlap and it was natural for us to talk shop.

I still remember these contacts rather vividly since he had such a striking personality. I had hardly ever met someone thinking and speaking so fast, although later I realized that this feature is not so uncommon among my Indian friends, or anyone so intense, with such a technical proficiency and concentration power. His working habits, as he described them to me, were also somewhat out of the ordinary: he would usually get up rather late, then have

breakfast, read the newspaper, so that he would really get going around ten or eleven, but then it would be 'straight to midnight'. In fact, it seems the habit of getting up rather late had been pretty much a constant in his life. Lily once told me that when he was living with his family, people would often sleep out of doors. In the morning everyone would get up and prepare for the daily work, but not Harish, who instead would go inside the house to sleep some more. His intense work, straight to midnight, may already have been taking on him since he felt the need to stay away from it for about three months a year. However, contrary to getting up late, this habit which, I presume, would have been later welcomed by his family, was not maintained. The self-imposed pressure from his work became greater and greater, and the time allotted for vacation shrank to practically nothing, until overwork took its toll and the doctor prescribed one month of vacation per year. His visit to Zurich was indeed part of a trip, he was coming from Paris