Norbert Wiener by Pest R. Masani. Vita Mathematica 5, Birkhäuser Verlag, P.O. Box No. 151, CH 4106 Thunwil, Basel, Switzerland. 1990. 416 pp Price: SFR 118.

Wiener's most fundamental contribution to mathematics is his rigorous construction of a mathematical model for Brownian motion as a stochastic process. At about 1905, Einstein and Smoluchowski, independently of each other, proposed a physical theory of the erratic or random motion of small particles suspended in liquids. Their theory can be summarized as follows: (i) For simplicity, one can confine one's attention to a single co-ordinate of the Brownian particle and speak of the one-dimensional Brownian motion, (ii) The motion is Markovian and homogeneous in time so that the transition probabilities obey the Smoluchowski-Chapman-Kolmogorov equations, or, equivalently, generate a Markov semigroup, (iii) The free Brownian particle has transition probability density

\[ P(y | x; t, s) = \frac{1}{4\pi D(t-s)^{1/2}} \exp \left[ -\frac{(y-x)^2}{4D(t-s)} \right] \]  \hspace{1cm} (1)

where \( x \) denotes the position at time \( s \), \( y \) at time \( t \), \( s < t \) and \( D \) is the diffusion coefficient depending on the size of the particle and the viscosity and temperature of the liquid.

If one observes a large number of free Brownian particles during the same time interval of length \( t \) and equates the sample mean square deviation of the coordinate under consideration to the theoretically predicted value \( 2Dt \) one can obtain an estimate of the Avogadro number. The successful determination of the Avogadro number from experiments on Brownian motion established the atomic theory of matter on a firm footing.

When Wiener was an instructor at the MIT, 1 Barnett of the University of Cincinnati drew his attention to the interest in generalizing 'the concept of probability to cover probabilities where the various occurrences being studied were not represented by points or dots in a plane or in space but by something of the nature of path curves in space'. In the words of Mark Kac, 'this is enormously surprising that, in 1919, when probability theory was not even thought of as branch of pure mathematics, two young men should have contemplated problems of such degree of sophistication'.

The problem of Brownian motion was to provide the subject of Wiener's first major mathematical work. He saw the situation in which particles describe not only curves but statistical assemblages of curves. It was an ideal proving ground for his Gibbsian ideas in the context of a Lebesgue integral in a space of curves. Whereas the papers of Einstein and Smoluchowski concerned what was happening to any given particle at a fixed point of time, or long-time statistics of many particles Wiener began to think in terms of the mathematical properties of the whole trajectory of a single Brownian particle. 'Here what the French physicist J. Perrin wrote in 1913 was music to Wiener's ears':

'Those who hear of curves without tangents or of functions without derivatives often think at first that Nature presents no such complications nor even suggests them. The contrary, however, is true and the logic of the mathematicians has kept them nearer to reality than the practical representations employed by physicists'.

In a series of papers published during 1920–23 and nearly a decade before Kolmogorov presented his famous axioms for probability theory in the form of his triple \( (\Omega, \mathcal{F}, P) \) consisting of an \( \Omega \) of sample points, \( \sigma \)-algebra \( \mathcal{F} \) of events and a countably additive probability measure on \( \mathcal{F} \), Wiener had constructed it for describing his Brownian motion with \( \Omega \) as the space of all continuous functions on the time axis \((0, \infty)\) with value \( \theta \) at time 0, \( \mathcal{F} \) as the smallest \( \sigma \)-algebra generated by the topology of uniform convergence on bounded intervals and \( P \) as the unique probability measure satisfying

\[
P(x | \alpha < x(t_1) < \beta_1, \ldots, \alpha < x(t_n) < \beta_n) \]

\[
= \int \cdots \int p(x_1, t_1; 0, 0) p(x_2, t_2; x_1, t_1) \cdots p(x_n, t_n; x_{n-1}, t_{n-1}) \, dx_1 \cdots dx_n
\]

for all \(-\infty < \alpha < \beta < \infty, 0 < t_1 < t_2 < \cdots < t_n < \infty, n = 1, 2, \ldots\), where \( p(., .) \) is given by equation (1)

Wiener showed that for any \( \varepsilon > 0 \) this measure \( P \) is concentrated in the subset of \( \frac{1}{2} - \varepsilon \) Hölder continuous functions on \( \mathbb{R} \), but the probability of a Brownian path being Hölder continuous of order \( \frac{1}{2} + \varepsilon \) at some point of time is 0. In particular, with probability one, a sample Brownian path is nowhere differentiable. This Nature seems to produce in abundance what Weierstrass presented to the Berlin Academy in 1872 by his famous example of a nowhere differentiable continuous function in an interval. In his work with Paley, Wiener gave an explicit expression of the Brownian path in the interval \([0, \pi]\) as a random superposition of sinusoids.

\[
B(t) = C_0 + \sum_{n=1}^{\infty} \sum_{k=2^n}^{2^{n+1}} G_k \left( \frac{2}{\pi} \sin \frac{kr}{k} \right)^{1/2}
\]

where \( C_0, G_1, \ldots \) are independent and identically distributed standard Gaussian random variables and the infinite series on the right-hand side converges uniformly in \( t \) with probability one.

Wiener tried to construct the paths of an arbitrary ergodic stationary process with finite second moments from Brownian motion and to this end he introduced the enormously fruitful notion of multiple stochastic integrals and founded the theory of Wiener chaos. In the wake of this incomplete attempt he has left for the posterity several open problems in the theory of nonlinear filtering.

Thanks to his rigorous construction of Brownian motion, we have today a flourishing industry of stochastic differential equations and path integral methods with many beautiful formulae like the Feynman-Kac formula, Kakutani's formula for the solution of Dirichlet problem in a domain etc. Wiener introduced the idea of a generalized solution for the Dirichlet problem in a domain and this, according to Laurent Schwartz, anticipates the theory of distributions or generalized functions.

Today we know several interesting connections between Brownian motion and free-field theories in quantum mechanics. Wiener started an interesting trail of thought in collaboration with Max Born and observed a rather curious relation between a Gaussian random field and quantum correlations. If \( L^2(\mathbb{R}^n) \) is the Hilbert space of wave functions for a system with \( n \) degrees of freedom then the correlation between two observables \( A \) and \( B \) in the pure state \( \psi \) can be expressed as

\[
\langle \psi, (A \otimes B) \psi \rangle = \langle \Omega \rangle \langle \Omega \rangle - \langle \psi, A \psi \rangle \langle \psi, B \psi \rangle.
\]  \hspace{1cm} (2)

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where

\[ f_{A,w}(\omega) = \langle 4 \psi | (T \psi)(\omega) \rangle. \]

\( T \) being the \( n \)-dimensional homogeneous complex Gaussian chaos. The left-hand side of equation (2) is a quantum correlation but the right-hand side is a classical expectation of the product of two complex valued Gaussian random variables. If \( P \) is a projection in \( L^2(\mathbb{R}^n) \) the probability of the event \( P \) in the pure state \( \psi \) can be expressed as

\[ \langle \psi, P \psi \rangle = P_{\psi} \langle f_{P,\psi}^* | f_{T,\psi} \rangle. \quad (3) \]

This is alright when \( \psi \) is a pure state but Wiener has left the matter open in the case of mixed states. The right-hand side of equation (3) is the probability of a rather artificial classical event in a huge probability space but Masani wonders whether this has something to do with hidden variable theories. Wiener's weakness for looking at everything through his power spectral language of communication theory can be noticed in equations (2) and (3).

During the Second World War Wiener was deeply involved in many defence projects on communication and control systems at the MIT. It was necessary 'to determine the position and direction of flight of an enemy aeroplane and extrapolate over the flight time of a projectile so that the projectile could be aimed so as to reach it'. Wiener formulated the problem as follows: consider a signal function \( f(t) = g(t) + n(t) \), where \( g(t) \) is the message and \( n(t) \) is noise. Predict \( g(t+h) \) from a knowledge of \( f(s) \) for \( s \leq t \). To achieve this choose \( K(s) \), \( s \geq 0 \) so that

\[ \lim_{T \to 27} T \int_0^T \left( g(t+h) - \int_0^t K(t-s) f(s) ds \right)^2 dt \]

is minimized. A crude application of the variational technique leads to the equation

\[ \int_0^T K(t-s) \varphi(s) ds = \chi(t+h), \quad (4) \]

where

\[ \varphi(t) = \lim_{T \to 27} T \int_0^T f(t+\tau) f(\tau) d\tau, \]

\[ \chi(t) = \lim_{T \to 27} T \int_0^T g(t+\tau) g(\tau) d\tau. \]

The function \( \varphi \) is the auto-correlation of the signal process \( f \) whereas \( \chi \) is the cross-correlation between signal and message. Imposing suitable hypotheses like stationarity and ergodicity on \( f, g \) one can determine \( \varphi \) and \( \chi \) from past experience via Birkhoff's individual ergodic theorem. Knowing \( \varphi \) and \( \chi \) one solves the Wiener-Hopf equation (4) for \( K \). This yields the required predictor

\[ l(t) = K(t-s) f(s) ds. \]

Here Wiener exploits a whole gamut of machinery developed by him in his generalized harmonic analysis, Fourier transform in the complex domain, causality and analyticity. Based on these ideas Wiener wrote several important monographs, which have now become standard references in the fields of electrical engineering and harmonic analysis.

Right from boyhood days Wiener was interested in languages, philosophy and logic. His doctoral thesis was on philosophy and he went to Cambridge in order to work under the guidance of Russell but, thanks to Russell's advice, took courses in mathematical analysis from Hardy and Littlewood. His work on prediction, filtering and Brownian motion on the one hand and his collaboration with physiologists on the other led him to formulate the ideas of his pet theory which he christened as cybernetics. According to him we live in a chaotic world of ever increasing entropy. 'We are swimming upstream against a great current of disorganization, which tends to reduce everything to the heat-death of equilibrium and sameness in the second law of thermodynamics. We live in a chaotic moral universe and our main obligation is to establish arbitrary enclaves of order and system' which again are impermanent. We constantly receive signals and our business is to filter the noise and see or feel good messages. For Michelangelo a marble rock was a signal, his chisel the filter and the hidden message a pieta or David.

During 1955-56 Wiener visited the Indian Statistical Institute at Calcutta and got interested in problems of economic development. I vaguely remember someone at the Institute having told me that when the late Mahalanobis initiated a conversation on economic planning Wiener suggested that it could be done best by an application of harmonic analysis! The study of stochastic processes in India began with Wiener's collaboration with Masani and Kailhanpur at Calcutta. Whereas Wiener's work with Masani on multivariate prediction theory was published in the Acta Mathematica and received wide recognition, his work with Kailhanpur first appeared as an MIT technical report and was later included in his book Nonlinear Problems in Random Theory without a reference to Kailhanpur.

There is plenty of autobiographical material from the pen of Wiener himself and, furthermore, there is a special issue of the Bulletin of American Mathematical Society (1966, vol. 72, no. 1, part II) which contains an analysis of Wiener's contributions to mathematics, communication engineering, biology and philosophy by a team of nine eminent scientists including Masani. Indeed, it is illuminating to read the volume under review along with the special issue of the Bulletin. Here the author has taken great pains 'to present the Wiener message after filtering the Wiener noise from the Wiener signals'. The author's comparative presentation of the personalities of Wiener and von Neumann in the context of matters pertaining to science, defence and the state makes fascinating reading. My only complaint is Masani's temptation to overload his prose with repetitive use of pet phrases like cognate, germane, propaedeutical, prothesis, phylogenetic and so on. A second reprinting of this volume calls for a thorough proof reading by competent editorial staff.

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The importance of public health has been recognized in the West, but it continues to be one of the less glamorous specialties of medicine. It is now realized that health has a social context and a disease context. Earlier public health specialists depended heavily upon drugs and vaccines to eradicate diseases of public health concern. Now it is realized that it is equally essential to give importance to