A comparative study of maximum ground level concentration of air pollutants using different plume rise formulae and dispersion parameters

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The maximum ground level concentration of air pollutants downwind of an elevated point source has been computed, incorporating commonly used plume rise formulae and dispersion parameters in the Gaussian plume model. Utilizing the relevant data in respect of the thermal power plant at Dahanu in Maharashtra, a comparison has been made of the results given by the different models and an optimum one has been suggested.

Many a time the decision of clearing projects from the environmental angle is delicately balanced over the maximum ground level concentration ($\chi_{\text{max}}$) of the main pollutant and the distance of its occurrence. The Gaussian plume model is widely used for calculating the ground level concentrations downwind from an elevated point source. The ground level concentration along the plume centre line is given by

$$\chi(0,0) = Q/(\pi u \sigma_x \sigma_z) \exp\left(-H_e^2/2 \sigma_z^2\right),$$

where $Q$ is the emission rate of the pollutant, $\sigma_x$ and $\sigma_z$ are the lateral and vertical dispersion parameters and are functions of downwind distance ($x$) and atmospheric stability. $H_e$ is the effective stack or plume height. It is the total elevation of the plume centre line relative to ground level and is equal to the sum of the physical height of stack ($H_s$) and the plume rise ($\Delta h$). $u$ is the wind speed at stack level. The ground level concentration becomes maximum at the point $x_{\text{max}}$ where $\chi_{\text{max}}$ is strongly dependent on the plume rise for elevated sources, as it is approximately proportional to the inverse square of $H_e$. $\chi_{\text{max}}$ is also dependent on the set of dispersion parameters used in the computation. Location of occurrence of $\chi_{\text{max}}$ is evidently dependent on vertical diffusion parameter ($\sigma_z$) and $H_e$. Increased turbulence reduces the value of $\chi_{\text{max}}$. The magnitude of $\chi_{\text{max}}$ depends on the ratio of $\sigma_z$ and $\sigma_x$ as well as the values of $H_e$ and $u$. The ratio of $\sigma_z$ and $\sigma_x$ decreases with increasing stability and this reduces the value of $\chi_{\text{max}}$. Increase in $H_e$ leads to a large decrease in $\chi_{\text{max}}$, but this does not hold at large distances ($x$) beyond the location of $x_{\text{max}}$.

In the literature there are several plume rise formulae as well as several semi-empirical schemes for calculating diffusion coefficients. All these formulae give varied results and no two of them agree with each other. The choice of plume rise equation and diffusion coefficients used in a model can make a lot of difference in the actual value of $\chi_{\text{max}}$ and predicted $\chi_{\text{max}}$ by the model. Therefore, there is a need to know the sensitivity of $\chi_{\text{max}}$ to the different formulae used. Plant characteristics of a thermal power plant in Dahanu, Maharashtra, form the data source for this study.

**Source characteristics**

- Stack height = 275 m
- Stack diameter = 5.8 m
- Stack gas exit velocity = 20 m/sec
- Emission rate of $\text{SO}_2$ from the stack = $4.3 \times 10^8 \mu g$/s
- Stack gas temperature = 413 K
- Stack level air temperature = 300 K
- Mean wind speed = 1.5 m s$^{-1}$

**Plume rise formulae used**

The plume rise $\Delta h$ is the elevation of the plume centre line above the stack outlet and is a function of distance downwind of the stack. Plume rise depends on the stack dimensions, the effluent and the prevailing meteorological conditions. Plume rise is greater if the pollutant is released at a high velocity and at a temperature much above the ambient temperature so that it possesses buoyancy. Increase in wind speed leads to decrease in plume rise. Stability conditions are also important as instability increases upward movement whereas stability produces a restraining influence. After the initial rise the form of the plume downwind depends on the prevailing structure of turbulence in the atmosphere.

$$\Delta h = \text{plume rise (m)}$$
$$V_s = \text{stack gas exit velocity (m/sec)}$$
$$T_s = \text{stack gas temperature (K)}$$
$$T_a = \text{ambient air temperature at stack level (K)}$$
$$d = \text{inside stack diameter (m)}$$
$$p = \text{atmospheric pressure (mb)}$$
$$F = \text{buoyancy flux parameter (m^4/sec^3)}$$
$$Q_M = \text{heat emission (cal/sec)}$$
$$g = \text{acceleration due to gravity (m/sec^2)}$$
$$C_p = \text{specific heat of air at constant pressure (cal/g/K)}$$
$$\rho = \text{density of air (g/m^3)}$$
$$\theta = \text{potential temperature at stack level (K)}$$
$$Q_H = \text{heat emission (MW)}$$
$$u_s = \text{wind speed at 1.5} H_s \text{ (m/sec)}$$

Holland's equation$^1$ was developed for large sources and involves many parameters

$$\Delta h = [V_s d/u_s] \left[1.5 + 2.68 \times 10^{-3} \frac{p(T_s - T_a)}{T_s d}\right]. \quad (1)$$

The two terms in the equation separately account for momentum and buoyancy. Various studies have shown
that the equation underestimates plume rise. A value 1.2 times the \( \Delta h \) is used for unstable conditions and 0.8 times the \( \Delta h \) is used for stable conditions.

Briggs equation\(^2\) for unstable and neutral atmospheric conditions is

\[
\Delta h = 1.6 \ F^{1.3} (3.5x^{2.3}/u),
\]

where \( F = g \ Q_w'/(n \ C_p \rho T) \) is buoyancy flux parameter.

\( x^* = 14 \ F^{5.8} \) when \( F < 55 \ m^4 \ s^{-3} \)
\( = 34 \ F^{3.5} \) when \( F > 55 \ m^4 \ s^{-3}. \)

The Briggs equation for stable conditions\(^3\) is

\[
\Delta h = 2.4 \ (F/(u s))^{1/3},
\]

where \( s = g/T_e \rho_0 \partial \rho/\partial z \) is the stability parameter.

Moore's formula\(^4\) is used extensively in the UK. The distinctive aspect of this formula is that it takes into account the fact that the plume often breaks up during the plume rise phase into interconnected blobs. Therefore, unlike the Briggs formula which implies two-dimensional mixing with the environment, this formula implies three-dimensional mixing. In Moore's formula, rise is proportional to \( Q^{1.4} \) rather than \( Q^{1/3} \). Moore's formula is more empirical than Briggs\(^5\). According to Moore's equation

\[
\Delta h = A \ Q_w^{1.4} x_a^{2/3} / u_a.
\]

\( A = 2.4 - 0.007 (120 - H_s) \), \( H_s < 120 \)
\( = 2.4, \ H_s > 120 \)
\( x_a = x / x_i (x_i^2 + x)^2 \)
\( x_i = x_0 / (1 + \partial \rho / \partial z \{ x_0 / 120 u_s \}^{2/3})^2 \)
\( x_0 = 1920 + 19.2 \text{Min}(120, H_s) \),

where Min(120, \( H_s \)) indicates that the lower value out of 120 m and \( H_s \) is to be taken.

Lucas et al.\(^6\) developed a plume rise equation for unstable and neutral conditions,

\[
\Delta h = [(60 + 5H_s)/u] \ Q_w^{2.5}.
\]

Padmanabhamurthy et al.\(^7\) gave a modified Lucas formula for stable conditions,

\[
\Delta h = (116/u) \ Q_w^{2.5} \text{ (stable and low windspeed)}
\]
\( = (160/u) \ Q_w^{2.5} \text{ (stable and high windspeed)}. \)

Dispersion parameters used

P-G coefficients describe the rate of plume dilution of specifying the horizontal extent \( \sigma_x \) and \( \sigma_z \) of the plume versus the downwind distance of the source under different meteorological conditions characterized by six atmospheric classes A through F. They were derived from Pasquill's\(^8\) data for low level sources in rural type open country for smooth surfaces. Gifford\(^9\) converted this plume spreading data into families of curves of the \( \sigma_x \) and \( \sigma_z \) of the plume concentration distribution. In the present study, a modified power law representation of the P-G dispersion coefficients given by Davidson\(^10\) has been used.

Briggs\(^1\) gave a series of interpolation formulae for \( \sigma_x \) and \( \sigma_z \) which agree with P-G coefficients in the range 100 m < x < 1 km. Briggs formulae are mainly intended for use in computing ground level concentrations, especially \( \chi_{\text{max}} \) for pollutants from elevated sources.

The comparative study of \( \chi_{\text{max}} \) using different combinations of plume rise formulae and dispersion coefficients (Table 1) gives widely varying results for each model. Holland's equation is known to underestimate plume rise and therefore gives very high values of \( \chi_{\text{max}} \) for both, the model using Briggs interpolation formulae as well as the one using P-G diffusion coefficients. The Briggs formula, on the other hand, overestimates plume rise and therefore gives low values of \( \chi_{\text{max}} \) and large values of \( \chi_{\text{max}} \) (Table 2). It gives especially large \( \chi_{\text{max}} \) under neutral conditions for the model using Briggs interpolation formulae and under stable conditions for the model using P-G coefficients.

The Lucas formula highly overestimates the plume rise under unstable and neutral conditions (Table 3). This formula is a function of the physical stack height, and for highly elevated sources like thermal power plants, this formula gives very high values of plume rise under low wind conditions. The Moore's formula seems to give reasonable values of plume rise.

Under unstable conditions, the models using P-G coefficients and a particular plume rise equation give much higher values of \( \chi_{\text{max}} \) and lower values of \( \chi_{\text{max}} \) than models using Briggs interpolation formulae and the

Table 1. MGLC (µg/m²) computed by different models

<table>
<thead>
<tr>
<th>Model</th>
<th>Unstable</th>
<th>Neutral</th>
<th>Stable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>336.3</td>
<td>82.9</td>
<td>4 × 10⁻⁶</td>
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<tr>
<td>2</td>
<td>333.6</td>
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</tr>
<tr>
<td>3</td>
<td>402</td>
<td>7.5</td>
<td>8 × 10⁻⁷</td>
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<tr>
<td>4</td>
<td>113.1</td>
<td>2.7</td>
<td>3 × 10⁻⁴</td>
</tr>
<tr>
<td>5</td>
<td>84.2</td>
<td>33.4</td>
<td>5 × 10⁻³</td>
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<tr>
<td>6</td>
<td>225.1</td>
<td>22.4</td>
<td>9 × 10⁻³</td>
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<tr>
<td>7</td>
<td>12.3</td>
<td>2.0</td>
<td>2 × 10⁻⁸</td>
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<tr>
<td>8</td>
<td>46.6</td>
<td>0.2</td>
<td>8 × 10⁻²</td>
</tr>
</tbody>
</table>

Model 1 uses Holland's equation and Briggs interpolation formulae.
Model 2 uses Holland's equation and P-G diffusion coefficients.
Model 3 uses Briggs equation and Briggs interpolation formulae.
Model 4 uses Briggs equation and P-G diffusion coefficients.
Model 5 uses Moore's equation and Briggs interpolation formulae.
Model 6 uses Moore's equation and P-G diffusion coefficients.
Model 7 uses Lucas equation and Briggs interpolation formulae.
Model 8 uses Lucas equation and P-G diffusion coefficients.
Table 2. \( x_{\text{max}} \) (km) computed by different models

<table>
<thead>
<tr>
<th>Model</th>
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<th>Neutral</th>
<th>Stable</th>
</tr>
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<td>1.5</td>
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<tr>
<td>8</td>
<td>2.0</td>
<td>437.6</td>
<td>168.8</td>
</tr>
</tbody>
</table>

For description of models, see Table 1.

Table 3. \( \Delta h \) (m) computed by different models

<table>
<thead>
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<th>Model</th>
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<th>Stable</th>
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<td>234.6</td>
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<tr>
<td>8</td>
<td>1885.2</td>
<td>1764.3</td>
<td>102.4</td>
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</tbody>
</table>

For description of models, see Table 1.

Table 4. Dispersion parameters (m) at \( x_{\text{max}} \) computed by different models

<table>
<thead>
<tr>
<th>Model</th>
<th>( \sigma_x )</th>
<th>( \sigma_y )</th>
<th>( \sigma_z )</th>
<th>( \sigma_z' )</th>
<th>( \sigma_z'' )</th>
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<td>1090</td>
<td>213</td>
<td>2810</td>
<td>96</td>
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<tr>
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<tr>
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<td>3998</td>
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<tr>
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<td>376</td>
<td>1847</td>
<td>4560</td>
<td>793</td>
<td>3200</td>
<td>98</td>
<td></td>
</tr>
</tbody>
</table>

For description of models, see Table 1.

same plume rise equation. Under neutral conditions, the P–G coefficients give lower values of \( x_{\text{max}} \) and \( x_{\text{max}} \) than Briggs interpolation formulae. Under stable conditions, P–G coefficients give higher values of \( x_{\text{max}} \) and \( x_{\text{max}} \) than the models using Briggs interpolation formulae.

The maximum concentration has been computed for different stability conditions. The seasonal variations of maximum concentration will depend on the relative proportion of different stability conditions existing in the particular seasons. In Dahanu, Maharashtra in the winter season, surface winds blow from east in the morning hours and north in the evening hours. In the pre-monsoon season winds blow from south-east in the morning hours and west in the evening hours. During the monsoon, winds always blow from the west. In the post-monsoon season, winds blow from east in the morning and from north in the evening hours. In the boundary layer, as one goes up, winds usually vary with height and this veering is more over rough terrain. For rough terrain, this veering can be up to 10°–20° at the stack level and this affects the direction in which pollutants are transported. This will naturally reduce the centre line ground level concentrations.

In the present study the terrain has been taken as a flat one. In the case of a complex terrain the Gaussian equation has to be suitably modified. Land and sea breezes affect the transport of pollutants and their ground level concentrations. During daytime when sea breeze occurs, the on-shore winds transport pollutants towards land and this increases the concentrations. In the evening hours the off-shore winds due to land breeze transport pollutants away from land into the sea and this leads to a decrease in the concentration of pollutants over land.

There are no suitable observations in India against which the models can be tested. Model 5, which uses Moore's plume rise formula and Briggs interpolation formulae, seems to be the optimum formula under India conditions. Moore's formula has been widely tested in UK and found to be the most suitable one for moderately buoyant plumes. Most thermal power plants in India emit moderately buoyant plumes. Briggs plume rise formula is more suitable for highly buoyant plumes emitted by super thermal power plants. Briggs interpolation formulae have been recommended by US EPA for plumes emitted from elevated sources.

The model using Moore's plume rise and Briggs interpolation formulae (Model 5) seems to be the optimum one for calculating \( x_{\text{max}} \) under Indian conditions for an elevated source.

It is necessary to exercise caution in clearing the projects as one model would clear the project while another might not allow it to be set up at all. It is high time that experiments are undertaken to validate the models under Indian conditions to avoid any ambiguity in the computations and to have firm ground in taking decisions on environmental clearance of projects.


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