THE NATURE OF THE FOUR-FERMION INTERACTION (*)

E. C. G. Sudarshan (**) & R. E. Marshak

University of Rochester

Rochester, New York

ABSTRACT

The near equality of the effective coupling constants in the processes of beta decay, muon decay and muon capture has led to the postulation of a Universal Fermi Interaction between the pairs of spinor fields np, $\mu\nu$, $e\nu$. The weak decays of the strange particles are consistent with an effective coupling constant of the same order of magnitude and can be understood by adding the Λ^o-p pair to the above pairs of spinors. The possibility of a universal four-fermion interaction has been reexamined in the light of the recent experimental results on the nonconservation of parity and charge conjugation in weak interactions. From measurements on beta, muon, pion and kaon decay and the assumptions of the two-component neutrino and the law of conservation of leptons, it is concluded that the only possible universal four-fermion interaction is an equal admixture of vector plus axial vector interaction. Several experiments appear to contradict this hypothesis and their status is reexamined.

1. INTRODUCTION

The near equality of the effective coupling constants in the process of β decay, μ decay and μ capture led various authors [1] to postulate a Universal Fermi Interaction between the pairs of spinor fields np, $\mu\nu$, $e\nu$. The observed lifetimes for the several decay modes of strange particles are consistent with an effective coupling constant of the same order of

(*) This work was supported in part by the Atomic Energy Commission.

(**) On leave of absence from Tata Institute of Fundamental Research, Bombay, India.

magnitude. A detailed study (2) of the strange particle decays indicates that these can be understood as being brought about by a (postulated) interaction of the (Λ^o-p) pair to the above pairs of fields (3). We have investigated the question of a Universal Interaction in this extended framework.

An analysis is made of the relevant information from β decay experiments, as among the weak interactions this is the best studied coupling. There is a wealth of pertinent data, most of which are comparatively recent, including some which are contradictory and require remeasurement. The simplest and most satisfactory assignments of coupling types are ST or AV. A choice between the two possibilities requires arguments based on analogies with other weak interactions.

The examination of other weak interactions favors strongly the assignment of the AV combination on the basis of the muon decay data in conjunction with the law of Conservation of Leptons. Additional support for this assignment comes from the decay modes and lifetime of the pion and to some extent from the kaon decay modes. We are thus led to postulate a universal four-fermion interaction which is a mixture of vector plus axial vector interaction. In this manner a natural explanation of parity breakdown in non-lepton decays of kaons and hyperons is also achieved.

In Section 2 we review briefly the β decay data, and in Section 3 the data on the decay of other particles. The specific statement of the Universal Interaction hypothesis is given in Section 4.

2. EVIDENCE FROM THE β DECAY

The more important evidence on the nature of the couplings in β decay may be summarized as follows:

(I) The electron-neutrino angular correlation. The correlation parameter λ has the values -1, +1, +1/3, -1/3, for S, V, T, A interactions. Measurements have been made for He^6, Ne^{19}, neutron and A^{35}. The pure G-T transition in He^6 indicates a strong T interaction (4) and the dominant Fermi transition in A^{35} indicates a strong V interaction (5). The measurements on the mixed transitions Ne^{19}, neutron are not very decisive but they agree best with a VA or ST combination (6). Since the angular correlations in allowed spectra depend only on the absolute squares of the coupling constants (apart from the Coulomb effect), these conclusions are independent of the validity of time-reversal invariance. Also, since the correlation itself is a scalar, the parity conserving and nonconserving parts contribute independently and the details of parity violation do not affect the above conclusions.

(II) The measurement of pseudoscalars in pure G-T transitions. The emission asymmetry in the decay of polarized β emitters and the longitudinal polarization of electrons from β decay, being pseudoscalars, involve the interference between the parity conserving and parity nonconserving interactions. The two-component theory of the neutrino ([7]) brings about the parity violations in a natural fashion and leads to a maximum value for this asymmetry. Experimental results on Co^{60} are consistent with this prediction ([8]). The interference effect depends on the relative phase factors of the parity conserving and nonconserving terms. In the two-component theory, these phases are always real (i.e. + 1) and the relative sign is equivalent to the sign of the longitudinal polarization of the emitted neutrino. However, the experimental quantity measured is the electron polarization (or electron emission asymmetry); the interference as to the type of interaction depends on the assumption of one or the other polarization for the neutral particle emitted. For a $J \rightarrow J-1$ transition, the measurements of longitudinal polarization and emission asymmetry give essentially the same quantity, namely $\pm (|A|^2 - |T|^2)/(|A|^2 + |T|^2)$. The measurements on Co^{60} indicate A or T according as the neutral particle emitted is right-handed or left-handed. One easily verifies that the relative phase factor between A and T is irrelevant in this connection.

(III) The measurement of pseudoscalars in pure Fermi transitions. The quantity measured is $2\,\mathrm{Re}\,(SS'^* - VV'^*) / |S|^2 + |S'|^2 + |V|^2 + |V'|^2)$. The only decisive experiment available ([9]) has been done on the β transition in Ga^{66}. The longitudinal polarization has the sign expected for a V or S interaction according as the neutral particle emitted is right handed or left handed. The absolute value of the asymmetry is large and is consistent with the maximum expected $\sim V/C$. This latter result would require the Fermi coupling to be all pure scalar with a left handed particle being emitted, or all pure vector, with a right handed particle being emitted.

(IV) The electron polarization in mixed transitions. The nuclides Sc^{46} and Au^{198} have been measured

(Au^{198} is first forbidden) ([10]). The combination of coupling constants measured here is (χ is the ratio of G-T to Fermi matrix element):

$$\frac{\mathrm{Re}\quad (SS'^* - VV'^*) + \chi^2\,(TT'^* - AA'^*)}{(|S|^2 + |S'|^2 + |V|^2 + |V'|^2) + \chi^2\,(|T|^2 + |T'|^2 + |A|^2 + |A'|^2)}$$

In both cases the latest experimental results give a very large effect, which is consistent with the maximum value ~ v/c. In other words, if the neutral particles are of the same spirality for both Fermi and G-T couplings, the coupling types have to be AV or ST.

(V) The measurements of pseudoscalar interference terms between Fermi and G-T interactions. The emission asymmetry as well as the β-polarized γ correlations in mixed β transitions followed by γ emission lead (in favorable cases) to an estimate of the combination

$$\frac{\mathrm{Re}\quad \chi\left\{ (ST'^* + S'\,T^*) - (VA'^* + V'\,A^*) \right\}}{(|S|^2 + |S'|^2 + |V|^2 + |V'|^2) + \chi^2\,(|T|^2 + |T'|^2 + |A|^2 + |A'|^2)}$$

The Co^{58} measurement was originally interpreted to mean that this combination was nearly zero ([11]); but the Sc^{46} measurements ([12]) give a non-vanishing value for this quantity, implying that both the assignments VT and SA are incompatible. The absolute value of the effect is large and is not inconsistent with the possible maximum value. Unlike the electron polarization, these interference terms are not ~ (v/c) [but more nearly v/2c]. However, a measurement of the same quantity on the oriented neutron decay ([13]) seems to give a much smaller value.

One verifies immediately that there is no choice of coupling types consistent with all the experimental data. For example, the measurement of angular correlations in mixed transitions as well as the measurements of emission asymmetry or β-γ correlations in mixed transitions favor a VA or ST combination; but the measurements of angular correlation in He^6 and A^{35} give T, V respectively. Nevertheless, the fact that in practically all the measurements of pseudoscalars the values obtained are consistent with the maximum value possible must be highly significant.

The present β decay data, while still somewhat contradictory from an experimental point of view, seem to suggest some definite choices for the coupling types. Since in most experiments, the measured asymmetry is large and is consistent with the maximum possible value predicted by a two-component theory of the neutrino, the simplest interference would be that the β decay coupling is either AV or ST. While in most of the experiments one cannot establish that the absolute value is equal to the maximum, there are some precise experiments (like the Co^{60} asymmetry) which come quite close to it; and one can expect that this conclusion will be put to rigorous experimental test in the near future.

The AV (or ST) combination has the added merit that the neutral particle emitted in electron decays is then right-handed (or left handed) both for the Fermi and the G-T interactions. This is to be contrasted with the VT or SA combinations where not only would neutral particles of apposite spiralities have to be coupled in the β interaction but these couplings would also be incompatible with the findings listed under (V) above. In the case of both AV and ST, the Fierz interference terms in allowed spectra and first forbidden spectra vanish identically.

The small value for the combination discussed in (V) for the neutron decay if taken literally requires that the Fermi and G-T terms have a complex relative phase factor. This conclusion applies equally for the ST and AV combinations. It is quite possible that the asymmetry from the neutron decay (13) has been underestimated experimentally.

The choice between AV and ST thus hinges essentially on the electron-neutrino angular correlations or equivalently on the determination of the spirality of the neutral particle emitted in β decay. As regards the electron-neutrino angular correlations, this implies a choice between the A^{35} and He^6 experiments. It seems fair to suggest that both the A^{35} and He^6 experiments ought to be redone (13a). The determination of the neutrino spirality directly is impossible, but the consideration of Universal Fermi interaction suggest that it should be right-handed (see below). In the next section we examine the other weak interactions in the light of such a principle.

3. THE EVIDENCE FROM OTHER WEAK INTERACTIONS.

For muon decay, since two of the emitted particles cannot be detected directly, an angular correlation experiment is not possible. Although the spectrum shape (measured by the Michel parameter [14]) depends on the coupling types the Michel parameter by itself yields no very precise information. The emission asymmetry for muons from pion decay at rest gives the coupling combination VA. Since the argument establishing this conclusion is somewhat elusive, we shall briefly sketch it below:

The observed positron asymmetry from the decay of stopped μ^+ (originating from π^+ decay at rest) is negative [15] and has an absolute value close to the maximum expected (1/3). Provided we assume that the G-T *interaction* in Co^{60} is tensor and that the Law of Conservation of Leptons [16] is valid, the μ^+ has positive longitudinal polarization. The explicit relation for the asymmetry for the positrons for a Hamiltonian which conserves leptons gives the coupling types. If on the contrary, we assume that the β transition, in Co^{60} takes place via the axial vector interaction, the μ^+ has negative longitudinal polarization. However, the interaction Hamiltonian for μ decay is now somewhat different, since the spirality of the neutrino field is altered to conform to the observed Co^{60} asymmetry. In fact, the new interaction Hamiltonian is obtained by replacing $(1 \pm \gamma_5)$ by $(1 \mp \gamma_5)$. Since pseudoscalars thereby change sign, one can now verify that the coupling type is again determined to be A, V.

The muon decay data thus suggests A, V interaction irrespective of the spirality of the neutrino field. The latter can be unambiguously determined if one measures the longitudinal polarization of the positrons from μ^+ decay. The positrons would be expected to be right - or left - polarized, according as the Co^{60} transition proceeds via axial vector or tensor interactions, provided the Law of Conservation of Leptons is valid.

The experimental value [17] of the Michel parameter, 0.68, is in good agreement with the value 0.75 expected for the case of one right-handed and one left-handed particle in the final state (especially if one takes account of the radiative corrections [18], which change the effective value of the parameter from 0.75 to 0.71). Since the T interaction vanishes identically in such a case (and since the A, V interactions are equivalent), one can equally well assign a VA, VT or AT combination for the interaction in μ decay.

Additional information on the coupling types is obtained from the decay lifetimes of π and K mesons, considered as taking place through Fermi interactions superposed on the strong Yukawa couplings. The branching ratios in these decays are the quantities of interest; and, to this extent, the validity of either time-reversal invariance or the details of parity violation may be disregarded. From the pion decay we conclude: (a) The comparative rarity [19] ($\lesssim 10^{-4}$) of the electron mode [20] $\pi^+ \to e^+ + \nu$ indicates that the pseudoscalar coupling of the $e\nu$, np fields is small compared with the axial vector (or pseudoscalar) coupling of the $\mu\nu$, np fields. (b) The rarity [21] ($\lesssim 10^{-5}$) of the radiative mode $\pi^+ \to e^+ + \nu + \gamma$ indicates that the V, T couplings of the $e\nu$, np fields cannot be much larger than the A, P couplings of the $\mu\nu$, np fields. (c) The pion lifetime is consistent with an axial vector coupling constant $\sim 10^{-49}$ erg cm^3.

Among the strange particle decays, the most informative in this connection are the lepton modes $K_{\mu 2}$ and K_{e2}, the latter being comparatively rare ($< 10^{-2}$). The relative parities of Λ^oNK are not known but if we assume that the K meson is γ_5 coupled, the rarity of the electron mode shows that P coupling of the Λ^op, $e\nu$ fields is small compared to the axial vector (or pseudoscalar) coupling of the Λ^op, $\mu\nu$ fields. (For scalar-coupled K mesons, A and P are to be replaced by V, S respectively.) The decay modes $K_{\mu 3}$, K_{e3} as well as the pion modes θ, τ are not very informative as far as coupling types are concerned.

4. A SCHEME OF UNIVERSAL FERMI INTERACTIONS.

The analysis of the decay of pions and kaons thus seems to point unequivocally to a dominant A interaction among those operative in the decay process. The muon decay data is consistent with VT, AT or AV. Among these three possible assignments, the only one involving the A interaction is AV. Since this is the only assignment consistent with β decay data and pion (as well as kaon) decay data, the only possibility for a Universal Fermi Interaction is to choose a vector + axial vector coupling between every two of the pairs of fields $\mu\nu$, $e\nu$, np, Λ^op. This scheme provides a natural mechanism for parity violating coupling of $\Lambda^0 p$, np leading to the τ and θ modes of the K meson. As an added merit, one can mention that the AV combination behaves in the same fashion for the decay of scalar and pseudoscalar mesons. This is important if present indications that the kaon is scalar (21a) turns out to be correct.

In the framework of our hypothesis, the β decay interaction is defined uniquely by the sign of the electron asymmetry in the decay of oriented Co^{60}. This unique form is

$$g \overline{P} \gamma_\mu (1 + \gamma_5) N \overline{e} \gamma_\mu (1 + \gamma_5) \nu + \text{h.c.}$$

The hypothesis of Universal Interaction generalizes this β coupling to a coupling of four Dirac fields A, B, C, D in the form

$$g \overline{A} \gamma_\mu (1 + \gamma_5) B \overline{C} \gamma_\mu (1 + \gamma_5) D$$

provided the reaction

$$B + D \rightarrow A + C$$

or equivalently, the reaction

$$B \rightarrow A + C + \overline{D}$$

satisfies the conservation laws of electric charge, lepton number and baryon number. Since γ_5 and γ_μ anticommute, one can rewrite the interaction of the four fields A, B, C, D in the form

$$g \overline{A} \gamma_\mu (1+\gamma_5) B \; C \gamma_\mu (1+\gamma_5) D =$$

$$= 1/4 \; g \; \overline{A} (1-\gamma_5)\gamma_\mu (1+\gamma_5) B \overline{C} (1-\gamma_5)\gamma_\mu (1+\gamma_5) D =$$

$$= \quad g \; \overline{A}' \gamma_\mu B' \; \overline{C}' \gamma_\mu D'$$

where A', B', C', D' are the "two-component" fields

$$A' = (1/\sqrt{2}) (1 + \gamma_5) A \quad \overline{A}' = A'^{*}B = (1/\sqrt{2}) \overline{A} (1-\gamma_5) \text{etc.}$$

Now the "two-component" field $(1/\sqrt{2})(1 \pm \gamma_5)$ A is an eigenstate of the chirality operator (22) with eigenvalue ± 1. Thus the Universal Fermi Interaction, while not preserving parity, preserves chirality and the maximal violation of parity is brought about by the requirement of chirality invariance. This is an elegant formal principle, which can now replace the Lee-Yang requirement of a two-component neutrino field coupling (or equivalently the Salam postulate of vanishing base mass and self mass for the neutrino).

Chirality is invariant under strong reflection (CTP). Vector and axial vector covariants bilinear in any Dirac field connect only eigenstates of the same eigenvalue for the chirality. But S, T, P couple fields of opposite chirality and have zero diagonal matrix elements in a representation diagonalizing the chirality operator. Thus our scheme of Fermi interactions is such that if one switches off all mesonic interactions, the gauge-invariant electromagnetic interactions (with Pauli couplings omitted) and Fermi couplings retain chirality as a good quantum number; and, surprisingly enough, chirality conservation would become a superselection rule. While at the moment these arguments are purely formal, they may not be entirely devoid of significance.

While it is clear that a mixture of vector and axial vector is the only universal four-fermion interaction which is possible and possesses many elegant features, it appears that one published and several unpublished experiments cannot be reconciled with this hypothesis. These experiments are:

(a) The electron neutrino angular correlation in He^6 (Rustad and Ruby (23)).

(b) The sign of the electron polarization from muon decay (Lederman et al (24)).

(c) The frequency of the electron mode in pion decay (Anderson and Lattes (20)).

(d) The asymmetry from polarized neutron decay (Telegdi et al (13)).

All of these experiments should be redone, particularly since some of them contradict the results of other recent experiments on the weak interactions. If any of the above four experiments stands, it will be necessary to abandon the hypothesis of a universal V + A four-fermion interaction or either or both of the assumptions of a two-component neutrino and/or the Law of Conservation of Leptons.

5. ACKNOWLEDGMENT

We are grateful to Professor M. Gell-Mann and Dr. B. Stech for valuable discussions.

FOOTNOTES

1. O. Klein: Nature 161, 897 (1948).
   G. Puppi: Nuovo Cimento 5, 587 (1948).
   Lee, Rosenbluth and Yang: Phys. Rev. 75, 905 (1949).
   J. Tiommo and J. Wheeler: Rev. Mod. Phys. 21, 153 (1949).
2. E. C. G. Sudarshan: *Ph.D. Thesis, University of* Rochester (1957).
3. M. Gell-Mann: Pisa Conference (1955).
   R. E. Marshak: Rochester Report NYO - 7138 (1955).
4. B. M. Rustad and S. L. Ruby: Phys. Rev. 89, 880 (1953).
5. Herrmannsfeldt etc.: to be published.
6. Maxson, Allen and Jentschke: Phys. Rev. 97, 109 (1955).
   Good and Lauer: Phys. Rev. 105, 213 (1957).
   Alford and Hamilton: Phys. Rev. 95, 1351 (1954).
   J. M. Robson: Phys. Rev. 100, 933 (1955).
7. Lee and Yang: Phys. Rev. 105, 254 (1957).
8. Wu et al: Phys. Rev. 105, 1413 (1957).
   Frauenfelder et al: Phy. Rev. 106, 386 (1957).
9. Dutch group: Private Communication from Dr. Stech.
10. Frauenfelder et al: to be published.
11. C. S. Wu: *Proceedings of the Seventh Rochester Conference* (1957).
12. Wapston and Boehm: to be published.
13. Novey and Telegdi: private communication from Dr. Stech.

13a. One possibility is for the ground state spin of He^6 to be 1

instead of 0. The transition would then become a mixed 1-1 transition and the angular correlation would then become consistent with the V A assignment. However, a spin of 1 for He^6 would violate our present views of nuclear structure.

14. L. Michel: Proc. Phy. Soc. A 63, 514 (1950).

15. Garwin, Ledermann and Weinrich: Phy. Rev. 105, 1415 (1957).

16. T. D. Lee: Proceedings of the Seventh Rochester Conference (1957).

17. W. H. Barkas: Proceedings of the Seventh Rochester Conference (1957).

18. Kinoshita and Sirlin: to be published.

19. Lokanathan and Steinberger: Nuovo Cimento 2, 151 (1955).

20. A recent experiment by Anderson and Lattes at Chicago gives an upper limit for the ratio of electron and muon decay modes of 10^{-5}. Pure AV couplings (without any P admixture) would yield an expected frequency of about 1 in 8000 for the electron mode. This experiment, if correct, would already constitute evidence against a pure AV coupling. Private Communication from Prof. Gell-Mann.

21. S. Lokanathan Ph.D. Thesis, Columbia University (1957). Cassels et al: to be published.

21a. Private communications from R. L. Walker and R. R. Wilson concerning the photoproductions of K. mesons.

22. S. Watanabe: Phys. Rev. 106, 1306 (1957).

23. B. M. Rustad and S. L. Ruby: Phy. Rev. 89, 880 (1959).

24. L. Ledermann: private communication to P. T. Matthews.