It is interesting to note that even though approximate (mixed symmetry contributions having been included), this $a_3(\alpha)$ is nevertheless consistent with the two known exact results, i.e. $a_3(\alpha)$ does not have contribution of $0(\alpha)$ and further $a_3^F \equiv a_3(\alpha=1) = \lambda_T^4/36 = a_3^B$.

Concluding remarks

The partition function of a noninteracting anyon gas still remains one of the most challenging, unsolved problems in statistical mechanics mainly because of the nontrivial braiding effects. Unless this problem is solved we would not know if the realistic approximate calculations are reliable or not. I hope I have been able to convince you that the physical laws in 'flatland' could be even more complex, nontrivial and hence interesting than in our real world!

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RESEARCH COMMUNICATIONS

Transfer functions related to zeroth order modified Bessel equation

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We obtain the solution of the zeroth-order modified Bessel equation with the help of double-point space-dynamic Green's functions. The expressions showing the correspondence between transfer functions and Ber, Bei, Ker, Kei functions are derived with the help of this method. We discuss the application of the solution in the case of the distribution of alternating currents in wires.

THE double-point transfer function known as Green's function is the most powerful tool for solving differential equations[1-4]. Here we present the double-point transfer function solution for the zeroth-order modified Bessel equation

$$x\,D^2 y + D y - ip^2 x y = 0, \tag{1}$$

where $D = (d/dx)$. The traditional solution of this equation is given by[5-8],

$$y = A\, I_0 (px\sqrt{i}) + B\, K_0 (px\sqrt{i}), \tag{2}$$

where the complex functions $I_0(px\sqrt{i})$ and $K_0(px\sqrt{i})$ can be expressed in terms of Bessel real. (Ber), Bessel imaginary (Bei), Ker and Kei functions in the form

$$I_0(px\sqrt{i}) = \mathrm{Ber}(px) + i\,\mathrm{Bei}(px), \tag{3a}$$

$$K_0(px\sqrt{i}) = \mathrm{Ker}(px) + i\,\mathrm{Kei}(px). \tag{3b}$$

Let us consider that there is a vanishing force[9] which governs equation (1) in the form

$$D_1^2 y + \eta^{-1} D_1 y - y = -f_0(\eta, \eta_0). \tag{4}$$

In this equation we have taken $px i = \eta$ and $D_1 = d/d\eta$. The force $f_0(\eta, \eta_0)$ is known as self-destroying force of c form

$$f_0(\eta,\eta_0)=\lim_{\substack{t\to\infty\\ x\to 0\\ \alpha\to xt}} \sin xt/\alpha, \tag{5a}$$

ıd

$$0\leqslant \lim_{\substack{\alpha\to t\\ x\to 0\\ t\to\infty}} \alpha^{-1}\sin xt\leqslant 1. \tag{5b}$$

The formal Green's function solution for the uation (4) which is analogical to

$$(D_1^2+\eta^{-1}D_1-1)G(\eta,\eta_0)=-\delta(\eta,\eta_0), \tag{6}$$

given by[1]

$$y=\int f(\eta,\eta_0)G(\eta,\eta_0)\,d\eta. \tag{7}$$

re $\delta(\eta,\eta_0)$ is the Dirac-delta function and $G(\eta,\eta_0)$ Green's function for zeroth-order modified Bessel lation. To obtain the exact expression for $G(\eta,\eta_0)$ we ine the Fourier transform $G(\omega)$ of it as

$$G(\eta,\eta_0)=(2\pi)^{-1}\int_{-\infty}^{\infty} G(\omega)\exp[-i\omega(\eta-\eta_0)]\,d\omega. \tag{8}$$

: substitution of equation (8) into equation (6) yields Green's function after the contour integration as

$$,\eta)=(\pi^2-1)^{-1/2}\theta(\eta-\eta_0)[\cos\pi(\eta-\eta_0)+ i\sin\pi(\eta-\eta_0)]\sin[(\pi^2-1)^{1/2}(\eta-\eta_0)], \tag{9a}$$

$$,x_0)=(\pi^2-1)^{-1/2}\theta(x-x_0)[\cos\pi p\sqrt{i}(x-x_0)+$$

$$\pi p\sqrt{i}(x-x_0)]\sin[(\pi^2-1)^{1/2}\pi p\sqrt{i}(x-x_0)]. \tag{9b}$$

$$=\mathrm{Re}\ G(x,x_0)+i\ \mathrm{Im}\ G(x,x_0), \tag{9c}$$

re $\theta(x-x_0)$ is the Heaviside unit step function and

$$\begin{aligned}\mathrm{Re}\ G(x,x_0)=&N_1\theta(x-x_0)\exp[-N_2p(x-x_0)]\\ &\times\sin[N_3p(x-x_0)]\\ &-\exp[-N_4p(x-x_0)]\sin[N_2p(x-x_0)],\end{aligned} \tag{10a}$$

$$\begin{aligned}G(x,x_0)=&N_1\theta(x-x_0)\exp[-N_2p(x-x_0)]\\ &\{-\cos[N_3p(x-x_0)]\\ &+\exp[-N_4p(x-x_0)]\cos(N_2p(x-x_0)]\},\end{aligned} \tag{10b}$$

$$N_1=1.4890941,\ N_2=7.1348124,\ N_3=11.5776953,\ N_4=18.712508 \tag{10c}$$

use of equation (10) in equation (7) yields the tion of zeroth-order modified Bessel equation in the form

$$\begin{aligned}y(x,x_0)=&\int f(x,x_0)\ \mathrm{Re}\ G(x,x_0)\,dx+i\int f(x,x_0)\ \mathrm{Im}\ G(x,x_0)\,dx\\ =&A\,I_0(px\sqrt{i})+BK_0(px\sqrt{i}).\end{aligned} \tag{11}$$

Evidently, from equations (2), (3) and (11) we get

$$A_1\ \mathrm{Ber}\ (px)+A_2\ \mathrm{Ker}\ (px)=\int dx\,f(x,x_0)\ \mathrm{Re}\ G(x,x_0), \tag{12a}$$

and

$$B_1\ \mathrm{Bei}\ (px)+B_2\ \mathrm{Kei}\ (px)=\int dx\,f(x,x_0)\ \mathrm{Im}\ G(x,x_0). \tag{12b}$$

The values of real and imaginary Green's functions for various values of $(x-x_0)$ and p are shown in Table 1. It is evident from these data that the Green's functions (both real and imaginary parts) have their values well below unity and fluctuate negatively and positively, showing gradual damping. These solutions are helpful in determination of the distribution of alternating currents in wire of circular cross-section.

Let us consider that an a.c. generator is connected with the metallic wire of length l and radius r. If we neglect the fringing effects the current and the fields (electric and magnetic) will have axial symmetry and will vary only as functions of time and radius r. The relations existing between the fields present are expressible by Maxwell's field equations in cylindrical coordinates in the form

$$D_rE=i\omega\mu H \tag{13}$$

and

$$r^{-1}D_r(rH)=(\sigma+i\omega\varepsilon)E, \tag{14}$$

and ε, μ and σ are the permittivity, permeability and electrical conductivity of the material respectively. If the wire is a good conductor of electricity, the electrical conductivity $\sigma\gg\omega\varepsilon$ at even the highest attainable frequency, which resorts the electric field equation (14) in the form

$$D_r^2E+r^{-1}D_rE-i\omega\mu\sigma E=0. \tag{15}$$

Equation (15) is similar to equation (1) with $p^2=\omega\mu\sigma$ and $x=r$. The electric field can easily be obtained from equation (11) by appropriate substitution of p and r and the driving force can be replaced by a.c. generator $V=V_0\exp i\omega t$. The magnetic field can then be obtained from equation (13), which can yield the current density, alternating currents and impedance per unit length Z in the wires of circular cross-section from the relations $J=\sigma E$, $I=2\pi rH$, $Z=E/I$. For equations (13) and (14) the propagation constant $\gamma^2=-i\omega\mu(\sigma+i\omega\varepsilon)$ has real part as attenuation factor and imaginary part as phase constant, while equation (15) reveals the attenuation-

Table 1. Real and imaginary parts of modified Bessel Green's function.

	$p=0.6$		$p=1.1$	
$(x-x_0)$	Re $G(x,x_0)$	Im $G(x,x_0)$	Re $G(x,x_0)$	Im $G(x,x_0)$
0.2	+0.5195 E+00	−0.3179 E+00	+0.2687 E+00	+0.1452 E+00
0.6	−0.2572 E−01	+0.1113 E+00	−0.1448 E−02	−0.1334 E−01
1.0	−0.1278 E−01	−0.1615 E−01	−0.4433 E−03	+0.3762 E−03
1.4	+0.3716 E−.02	+0.1959 E−04	+0.2426 E−04	+0.6752 E−05
1.8	−0.4216 E−03	+0.5214 E−03	−0.3474 E−06	−0.1034 E−05
2.2	−0.2595 E−04	−0.1182 E−03	−0.2862 E−07	+0.3758 E−07
2.6	+0.1958 E−04	+0.9665 E−05	+0.2043 E−08	+0.1132 E−09
3.0	−0.3568 E−05	+0.1669 E−05	−0.4558 E−10	−0.7599 E−10
3.4	+0.1670 E−06	−0.6909 E−06	−0.1617 E−11	+0.3481 E−11
3.8	+0.7853 E−07	+0.1014 E−06	+0.1641 E−12	−0.2661 E−13
4.2	−0.2314 E−07	−0.3660 E−09	−0.4942 E−14	−0.5237 E−14
4.6	+0.2660 E−08	−0.3219 E−08	−0.6768 E−16	+0.3044 E−15
5.0	+0.1538 E−09	+0.7377 E−09	+0.1256 E−16	−0.4971 E−17
5.4	−0.1213 E−09	−0.6147 E−10	−0.4834 E−18	−0.3295 E−18
5.8	+0.2233 E−10	−0.1016 E−10	−0.6654 E−22	+0.2534 E−19
6.2	−0.1086 E−11	+0.4292 E−11	+0.9101 E−21	−0.6134 E−21
6.6	−0.4228 E−12	−0.6367 E−12	−0.4411 E−22	−0.1773 E−22
7.0	+0.1441 E−12	+0.3797 E−14	+0.4362 E−24	+0.2012 E−23
7.4	−0.1678 E−13	+0.1987 E−13	+0.6154 E−25	−0.6454 E−25
7.8	−0.9095 E−15	−0.4604 E−14	−0.3809 E−26	−0.6383 E−27
8.2	+0.7512 E−12	+0.3908 E−15	+0.6968 E−28	+0.1521 E−27
8.6	−0.1397 E−15	+0.6181 E−16	+0.3760 E−29	−0.6194 E−29
9.0	+0.7041 E−17	−0.2666 E−16	−0.3134 E−30	+0.1572 E−31
9.4	+0.2962 E−17	+0.3997 E−17	+0.8081 E−32	+0.1086 E−31
10.0	−0.1524 E−18	−0.3495 E−18	+0.8386 E−34	+0.8933 E−34

free propagation because the propagation constant is purely imaginary.

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Non-existence of Maxwell fields in Bianchi type-V model in bimetric relativity

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Restricting to a particular type of the background metric it is observed that there is no contribution from Maxwell fields to the Bianchi type-V model in bimetric theory of gravitation.

To remove some of the unsatisfactory features of the general theory of relativity, Rosen[1] proposed the bimetric theory of relativity, in which there exist two metric tensors at each point of space-time—g_{ij}, which describes gravitation, and the background metric γ_{ij}, which enters into the field equations and interacts with g_{ij} but does not interact directly with matter. Accordingly, at each space-time point, one has two line elements

$$ds^2 = g_{ij}\,dx^i\,dx^j$$

and $$d\sigma^2 = \gamma_{ij}\,dx^i\,dx^j,$$

where ds is the interval between two neighbouring events as measured by a clock and a measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable. One can regard it as describing the geometry that exists if no matter were present.

This note concludes that the Bianchi type-V model of bimetric theory of gravitation does not accommodate electromagnetic fields. This inference, however, does differ from that of general relativity[2].

The Bianchi type-V model is

$$ds^2 = e^{2a}(dx^2 - dt^2) + e^{2x}(e^{2b}dy^2 + e^{2c}dz^2), \quad (1)$$

where a, b, c are functions of t alone.
The background metric of flat space-time is