Locking in finite-element analysis—from superstition to science

G. Prathap

The finite-element method offers the most effective approach to computational (digital) simulation of problems in engineering mechanics. However, very large errors are often encountered unless a set of rules are followed. This article discusses one such scheme, identified recently. This is an example of the epistemological conflict between the practical rules of finite-element modelling that can be described as art and those paradigms that allow a scientific basis to be invested in it and which can be admitted as science.

The principles of structural mechanics as a branch of mathematical physics are well founded and have a sound scientific basis. The analytical description of it has also a long history and is rigorously based on infinitesimal and variational calculus. Such descriptions lead to partial differential equations that describe the state of the basic variables defining the problem and, more often than not, are tractable only for a very limited range of structural loading and geometry. It has become expedient in recent years to overcome these limitations by turning to numerical modelling of the behaviour of structural systems. The most powerful method available to do this today is the *finite-element method*. It is eminently suited to carrying out the entire cycle of design and analysis of a structural configuration on a digital computer. At the heart of this procedure are the *mechanics algorithms*—the set of subroutines that capture the physics of the structural behaviour of the constituent parts of the structure in terms of matrices of discrete numbers relating forces to displacement at the nodes. It is therefore the discretized representation of the continuum-analytical description of structural behaviour that makes numerical computation possible.

All structural regions can be described by a set of subdomains called finite elements. Each element is designed in such a way that it captures the essential elastomechanical behaviour of the region it represents, replacing the differential equations of infinitesimal calculus with a discrete relationship. This article deals with one of the four principles that establish this procedure in a scientifically acceptable manner. The four principles are *continuity*, *completeness*, *consistency* and *correctness*. From this conceptual framework, I show how *consistency* permits the locking problem to be explained and resolved.

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The main step in the finite-element method is a procedure, called the *discretization process*, by which the continuum structural behaviour is replaced by a discretized description so that these can be coded as mechanics algorithms. The early efforts to do this were founded on engineering principles proceeding mostly from heuristic judgement (art). However, considerable critical and analytical studies in recent years have shown that this approach can lead to inexplicably large errors if applied blindly. Studies by me and my colleagues have shown that it is possible to establish further rules that allow this aspect to be rationalized on a more scientific basis.

Conventional finite-element modelling

We can appreciate that finite-element modelling would have some errors related to the fact that an exact analytical description over a large structural region is replaced by approximate polynomial functions over small regions (elements), taking care to ensure that the elements are connected correctly and smoothly to cover the whole region. These errors are often studied in the traditional way as truncation or discretization errors[1]. In this measure, if a one-dimensional region, as in a beam say, of length L, is divided into N equal subregions of length l, and linear polynomial functions are used as the interpolation functions, errors can be expected to be of the order of $(l/L)^2$ or less. Thus, with 10 elements, one should get errors of less than a few per cent. While this was the case with a very large class of problems in solid and structural mechanics, there soon appeared instances where the errors were inexplicably large, errors of 99% or more being typical of such cases. It was clear that the conventional rules of *completeness* (i.e. the basis of interpolation functions must cover the terms that ensure strain-free rigid-body motion and states of constant strain) and *continuity* (i.e. the variables are continuous across element boundaries and within the element domain) were insufficient to form a complete basis for the formulation of finite elements.

The linear shear flexible beam element is perhaps the simplest and also the earliest example in which such problems were identified. I shall therefore use this example to illustrate the magnitude of errors possible and the importance of the concept of field consistency to resolve the impasse.

The classical two-noded beam element with two degrees of freedom at each node, transverse deflection w, and slope dw/dx based on elementary beam theory is very well known (see Figure 1). This requires cubic interpolation functions and is the simplest element that can be constructed for this theory. It soon appeared to be very tempting to formulate a similar two-noded beam element based on the shear flexible Timoshenko beam theory, the attractive aspect about this being that the two degrees of freedom at each node follow from a transverse displacement w and a section rotation θ which are independent field variables. Such an element can be based on simple linear interpolations. Such formulations also make the finite-element modelling of nonlinear problems very simple, extend the range of applicability from thin beams to moderately thick beams, and are very attractive from the point of view of the general-purpose finite-element analysis packages as the elements can now be defined in terms of the six basic engineering degrees of freedom, namely the three translations and the three rotations.

Early experiments with such an element proved to be disastrous. With the advantage of hindsight, we can now understand why no record of such an element appeared until 1977, when a deceptively elementary 'trick', that of using a reduced integration of the shear strain energy, produced a remarkably accurate element[2]. The initial response was to dismiss this as a 'useful trick'. Implicit in this was the assumption that reduced integration introduced errors that compensated somehow for the other constraining errors. These early interpretations were based on an understanding in terms of the rank and singularity of the matrices corresponding to the penalty linked, i.e. the shear energy terms. These interpretations argued that the exact integration of the shear energy terms introduced too many constraints among the limited number of variables (degrees of freedom) available per element and that this was reflected in the very degraded 'locking' behaviour of the element.

Figure 2 shows a typical illustration of what one means by the very poor behaviour of the original unmodified element (FI, for field inconsistent) compared to the dramatic improvement in efficiency obtained by making it field-consistent (FC). It can be shown that over the practical range in which the Timoshenko beam theory is appropriate (i.e. say $L/t = 5$ to $L/t = 1000$), the FI models are virtually unpractical to use—needing as many as $100(L/t)$ elements to achieve the same accuracy as that obtained with 10 FC elements!

I now trace the path that led to the identification of the need of a new paradigm, i.e. a new way of looking at the problem, so that these unexpected problems arising when the conventional displacement-type formulation of the Timoshenko beam is used can be rationalized. I begin with a description of the variational statement of the conventional displacement-type problem. A stiffness matrix corresponding to this leads to numerical problems that defy intuition. It becomes clear then that the conventional paradigms are insufficient to explain the nature of the difficulty. I then proceed to identify the new condition and to verify that this is an accurate scientific basis and not just a 'trick'.

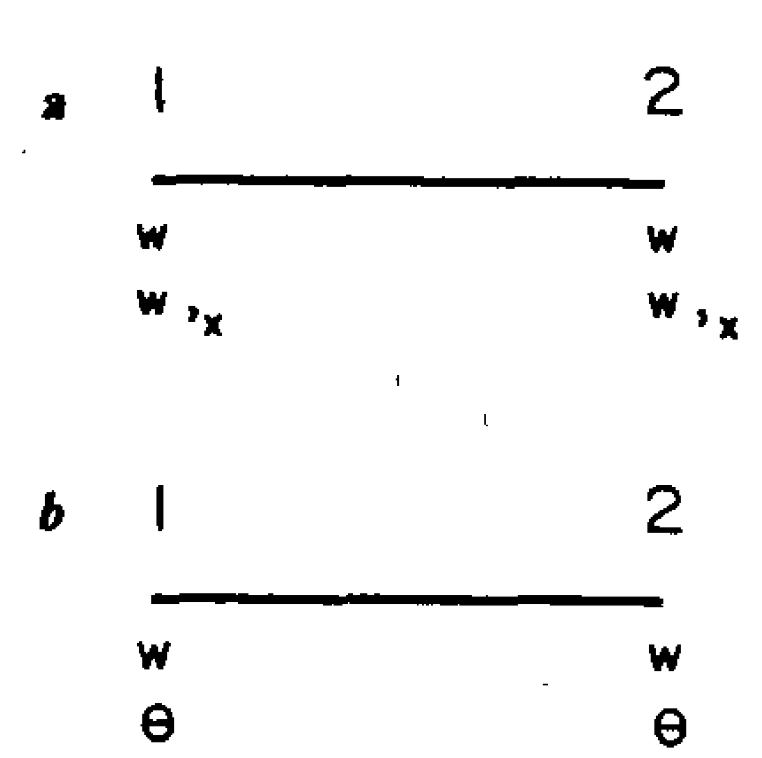

Figure 1. ***a***, Classical thin beam, and ***b***, Timoshenko beam elements.

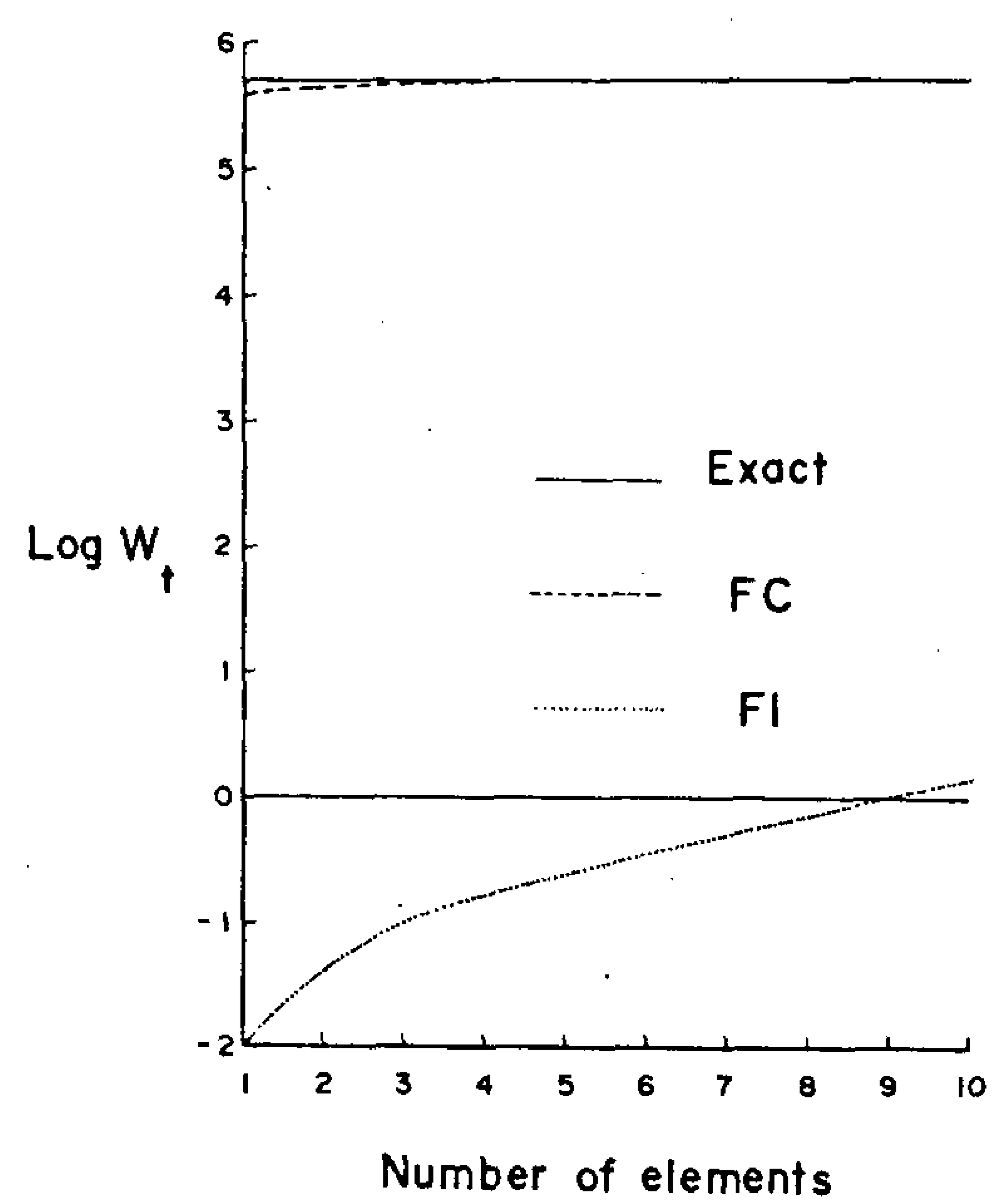

Figure 2. Convergence plot for tip deflection of a cantilever beam under tip shear load for $L/t = 100{,}000$.

The conventional formulation

The strain energy of a Timoshenko beam element of length $2l$ can be written as the sum of its bending and shear components as

$$\int (1/2\, EI \mathscr{K}^T \mathscr{K} + 1/2\, kGA\gamma^T \gamma)\, \mathrm{d}x, \qquad (1)$$

where

$$\mathscr{K} = \theta_{,x}, \qquad (2a)$$

$$\gamma = \theta - w_{,x}. \qquad (2b)$$

In equations (2a) and (2b), w is the transverse displacement and θ the section rotation. E and G are Young's and shear moduli and k the shear correction factor used in Timoshenko's theory. I and A are the moment of inertia and the area of cross-section.

Discretization—field-inconsistent element

Linear interpolations are chosen for the displacement field variables. This ensures that the element is capable of strain-free rigid-body motion and can recover a constant state of strain (the *completeness* requirement) and that the displacements are continuous within the element and across the element boundaries (the *continuity* requirement). The bending and shear strains are then computed directly from these interpolations using the strain gradient operators given in equations (2a) and (2b). These are then introduced into the strain energy computation in equation (1), and calculated in an analytically or numerically exact way. Of course, it turns out that the element performs very badly—the phenomenon known as shear locking—and gives wildly oscillating shear forces along the length of the element.

The linear isoparametric representation of the two field variables w and θ are based on the functions

$$N_1 = (1-\xi)/2, \qquad (3a)$$

$$N_2 = (1+\xi)/2, \qquad (3b)$$

where the dimensionless coordinate $\xi = x/l$ varies from -1 to $+1$ for an element of length $2l$. The strain energies in equation (1) are then directly computed, in an analytically or numerically exact (a two-point Gauss Legendre integration rule) way, using these interpolation functions in the expressions for the strain fields.

For the beam element shown in Figure 1, for a length $2l$ the stiffness matrix can be split into two parts, a bending-related part and a shear-related part[2]. This element was carefully studied[2] and the first numerical results reporting the locking effect obtained. I examine this to see how the locking effect can be quantified. Figure 3 shows a cantilever beam subjected to an end load. Two cases were considered: a deep beam and a thin beam. While the results for the deep beam were reasonable, those for the thin beam were dramatically in error. Table 1 shows the trend as the number of elements was increased. A curious trend can be noticed: the tip deflections obtained, which are several orders of magnitude lower than the correct answer, are directly related to the square of the number of elements used for the idealization. In other words, the discretization process has introduced an error so large that the resulting answer has a stiffness related to the inverse of N^2. This is clearly unrelated to the physics of the Timoshenko beam and also not the usual sort of discretization error encountered in the finite-element method.

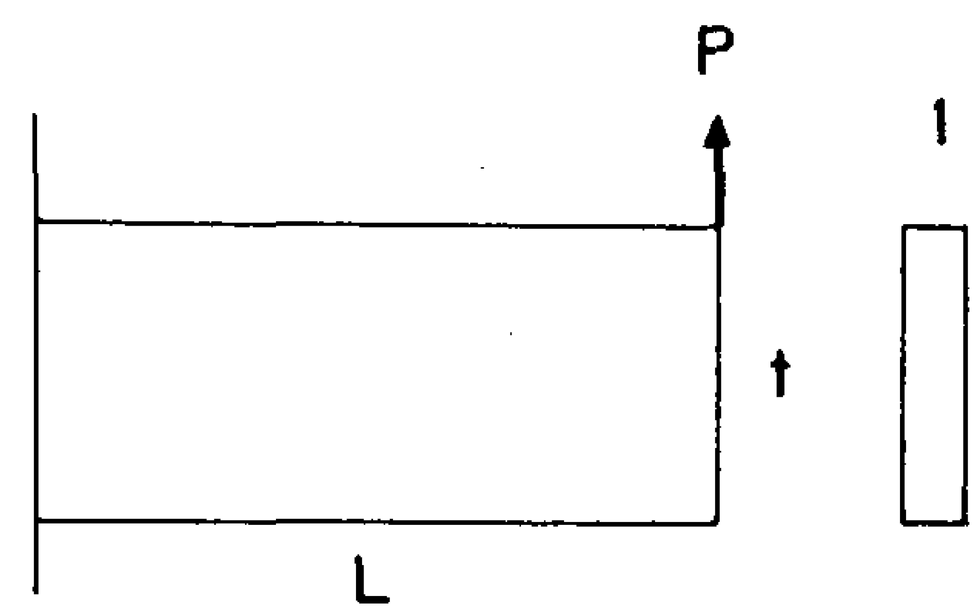

Figure 3. Cantilever beam subjected to end load.

It is also logical to argue that the error in each element must be related to the element length, and, therefore, when a beam of overall length L is divided into N equal-length elements, the additional stiffening introduced in each element owing to shear locking is proportional to l^2. Further, numerical experiments[3] showed that the locking stiffness progresses without limit as the element depth t decreases. Thus we have now to look for a mechanism that can explain how this spurious stiffness of $(l/t)^2$ can be accounted for by considering the mathematics of the discretization process.

The field-consistency paradigm

The above exercise makes it clear that the two paradigms introduced so far, namely *completeness* and *continuity*, which had for a long time been considered to be necessary and sufficient conditions for describing displacement interpolations, are really not enough. There is cause for concern here; one needs, first, to identify the class of problems where these two paradigms are insufficient, and then look for a paradigm

Table 1. Normalized tip displacements for the thin beam.

No. of elements	Tip displacement
1	0.200×10^{-4}
2	0.800×10^{-4}
4	0.320×10^{-3}
8	0.128×10^{-3}
16	0.512×10^{-3}

that can 'scientifically' explain the difficulties that appear in the displacement-type linear beam element. The main body of work done by me and my colleagues demonstrates that such difficulties occur in constrained-media problems. I also propose a requirement for a *consistent* interpolation of the constrained strain fields as the necessary paradigm.

If we start with a linear isoparametric representation of the two displacement field variables w and θ, as we did in equations (3) above, we can associate two generalized displacement constants with each of the interpolations in the following manner:

$$w = a_0 + a_1(x/l), \quad (4a)$$

$$\theta = b_0 + b_1(x/l). \quad (4b)$$

These generalized displacement constants can be related to the field variables obtaining in this element in a discretized sense; thus, $a_1 = w_{,x}$ at $x = 0$, $b_0 = \theta$ and $b_1 = \theta_{,x}$ at $x = 0$. This denotation would become useful when we try to explain how the discretization process can alter the infinitesimal description of the problem if the strain fields are not consistently defined.

If the strain fields are now derived from the displacement fields given in equations (4a) and (4b) (call these the kinematically derived strain fields as they are derived by using the strain-gradient operators on the kinematically admissible displacement interpolations), we get

$$\mathscr{K} = (b_1/l), \quad (5a)$$

$$\gamma = (b_0 - a_1/l) + b_1(x/l). \quad (5b)$$

An exact evaluation of the strain energies for an element of length $2l$ will now yield the bending- and shear-strain energy as

$$U_B = 1/2(EI)(2l)[(b_1/l)]^2, \quad (6a)$$

$$U_S = 1/2(kGA)(2l)[(b_0 - a_1/l)^2 + 1/3b_1^2]. \quad (6b)$$

It is possible from this to see that in the constraining physical limit of a very thin beam modelled by elements of length $2l$ and depth t, the shear-strain energy in equation (6b) must vanish. An examination of the condition produced by this requirement shows that the following constraints would emerge in such a limit:

$$b_0 - a_1/l \to 0, \quad (7a)$$

$$b_1 \to 0. \quad (7b)$$

In our new terminology, constraint (7a) is field-consistent as it contains constants from both the contributing displacement interpolations relevant to the description of the shear-strain field. These constraints can then accommodate the true Kirchhoff constraints in a physically meaningful way, i.e., in an infinitesimal sense, this is equal to the condition $(\theta - w_{,x}) \to 0$ at the element centroid. In direct contrast, constraint (7b) contains only a term from the section rotation θ. A constraint imposed on this will lead to an undesired restriction on θ. In an infinitesimal sense, this is equal to the condition $\theta_{,x} \to 0$ at the element centroid (i.e. no bending is allowed to develop in the element region). This is the 'spurious constraint' that leads to shear locking and violent disturbances in the shear force prediction over the element, as we shall see presently.

A 'falsifiable' error model

It is now necessary to establish the scientific quality of the field-consistency paradigm that I have introduced here. For this, I borrow an idea from the philosophy of science, the falsifiability theorem of Karl Popper[4]. The idea here is that the discretized finite-element model will contain an error which can be recognized when digital computations made with these elements are compared with analytical solutions where available. I have offered the *consistency* requirement as the missing paradigm for the formulation of the constrained-media problems. Therefore, to establish the scientific validity of this conceptual scheme, it is necessary to devise auxiliary procedures that will trace the errors due to an inconsistent representation of the constrained strain field and obtain precise a priori measures for these, and then show by actual numerical experiments with the original elements that the errors are as projected by these a priori error models. This exercise, which will complete the scientific validation of the consistency paradigm, is made possible by a procedure called the functional reconstitution technique.

Functional reconstitution

We try to set up an error model for the error due to the spurious shear constraint when the inconsistent element is used to model a beam of length L and depth t. The strain energy for such a beam can be set up as

$$\Pi = \int_0^L [1/2EI\theta_{,x}^2 + 1/2kGA(\theta - w_{,x})^2]\,dx. \quad (8)$$

If an element of length $2l$ is isolated, the discretization process produces an energy for the element of the form given in equation (6). In this equation, the constants introduced owing to the discretization process can be replaced by the continuum (i.e. the infinitesimal) description. Thus we note that, in each element, the constants in equations (6a) and (6b) can be traced to the constants in equations (4a) and (4b) and can be replaced by the values of the field variations θ, $\theta_{,x}$ and $w_{,x}$ at the centroid of the element. Thus the strain energy of deformation in an element is

$$\pi_c = 1/2(EI)(2l)\,\theta_{,x}^2 + 1/2(kGA)(2l)(\theta - w_{,x})^2 + 1/6(kGAl^2)(\theta_{,x}^2). \quad (9)$$

Thus the constants in the discretized strain-energy functional have been reconstituted into an equivalent continuum or infinitesimal form. From this reconstituted functional, we can argue that an idealization of a beam region of length $2l$ into a linear displacement-type finite element would produce a modified strain-energy density within that region of

$$\bar{\pi}_c = 1/2(EI + kGAl^2/3)(\theta_{,x})^2 + 1/2(kGA)(\theta - w_{,x})^2. \quad (10)$$

This strain-energy density reflects the alteration of the original physical system introduced by the presence of the inconsistent term in the shear-strain field. Thus we can postulate that a beam of length L modelled by equal elements of length $2l$ will have a reconstituted functional

$$\overline{\Pi} = \int_0^L [1/2(EI + kGAl^2/3)(\theta_{,x})^2 + 1/2(kGA)(\theta - w_{,x})^2]\,dx. \quad (11)$$

We now understand that the discretized beam is stiffer in bending (i.e. its flexural rigidity) by the factor $kGAl^2/3EI$. For a thin beam this can be very large, and produces the additional stiffening effect described as *shear locking* in the literature.

Numerical experiments

Our auxiliary procedure (to distinguish it from the actual finite-element procedure that yields the stiffness matrix) now provides an instrument for critical self-examination of the consistency paradigm. It indicates that an exactly integrated or field-inconsistent finite-element model tends to behave as a shear-flexible beam with a much stiffened flexural rigidity I'. This can be related to the original rigidity I of the system by comparing the expressions in equations (8) and (11):

$$I'/I = 1 + kGAl^2/3EI. \quad (12)$$

We must now show through a numerical experiment that this estimate for the error, which has been established entirely a priori, starting from the consistency paradigm and introducing the functional reconstitution technique, anticipates very accurately the behaviour of a field-inconsistent linearly interpolated shear-flexible element in an actual digital computation. Exact solutions are available for the static deflection W of a Timoshenko cantilever beam of length L and depth t under a vertical tip load. If W_{fem} is the result from a numerical experiment involving a finite-element digital computation using elements of length $2l$, the additional stiffening can be described by the parameter

$$e_{fem} = W/W_{fem} - 1. \quad (13)$$

From equation (12), we already have an a priori prediction for this factor:

$$e = I'/I - 1 = kGAl^2/3\,EI. \quad (14)$$

Figure 4 shows a variation of e with the structural parameter that denotes the penalty multiplier in this case, namely kGL^2/Et^2 for the case presented in Figure 3. The crosses indicate the additional stiffening parameter obtained from the finite-element experiment (equation (13)) and the solid line shows the variation predicted by the error model (equation (14)).

We have therefore succeeded in investing a scientific validity in the consistency paradigm through this exercise—note that the traditional procedures such as counting constraint indices or computing the rank or condition number of the stiffness matrices could offer only a heuristic picture of how and why locking sets in.

The reduced-integration formulation

A magic formula to overcome the locking seen for the linear beam element is the reduced-integration method[2]. The bending component of the strain energy of a Timoshenko beam element of length $2l$ shown in equation (1) is integrated with a one-point Gaussian rule as this is the minimum order of integration required for exact evaluation of this strain energy. However, a mathematically exact evaluation of the shear-strain energy will demand a two-point Gaussian integration rule. It is this rule that resulted in a non-singular shear stiffness matrix that locked. With a one-

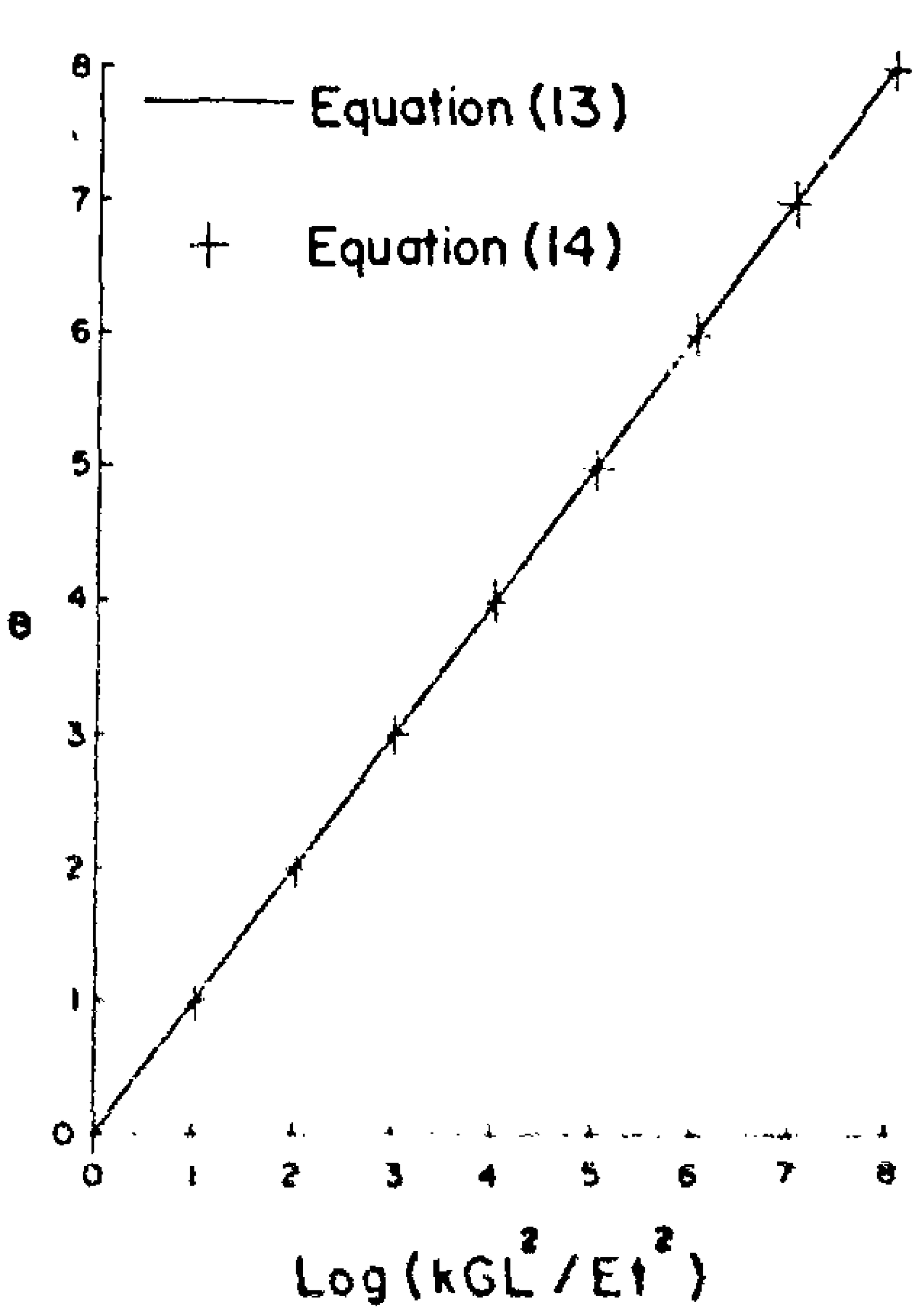

Figure 4. Error norm e as function of penalty multiplier for cantilever beam under tip shear force.

point integration of the shear-strain energy component the shear-related stiffness matrix changed to a singular one, as shown below. The performance of this element was extremely good, showing no signs of locking at all. I have proved[5] that reduced integration is effective because it makes the shear strain field-consistent and thereby removes locking completely.

A critique of the conventional wisdom

The conventional wisdom was to relate this singularity to the improved performance seen above. The argument proceeded thus. The functional of equation (1) becomes constrained when $kGAl^2 \gg EI$. This leads to finite-element equations of the form

$$(\mathbf{K}_1 + \alpha \mathbf{K}_2)\mathbf{a} + \mathbf{f} = 0, \tag{15}$$

where $\mathbf{a}$ is the displacement vector and $\mathbf{f}$ the load vector. $\mathbf{K}_1$ is the unconstrained part of the stiffness matrix (in this instance, that derived from the bending energy) and $\mathbf{K}_2$ the constrained part (here derived from the shear energy). The penalty parameter α (here, we know this is $kGAl^2/EI$) increases as the beam becomes thinner, and it is argued that equation (15) degenerates to

$$\mathbf{K}_2 \mathbf{a} = -\mathbf{f}/\alpha \rightarrow 0, \tag{16}$$

and $\mathbf{a} \rightarrow 0$ unless the matrix $\mathbf{K}_2$ is singular. In a conventional displacement-type formulation of constrained-media elasticity (as in the exactly integrated or field-inconsistent case), this singularity does not arise naturally. The reduced-integration strategy is therefore viewed as an artifice that can bring about the required singularity so that, in the penalty limit, equation (15) does not degenerate as seen above.

There are several weaknesses in this heuristically appealing argument. The first is that there is no certainty that a violation of the variational theorems has not taken place in this 'trick' of introducing singularity into the constrained matrix. Nor is there any suggestion arising from the argument that there is a unique way in which singularity must be achieved. Thirdly, there is no possibility of constructing a numerical experiment that can 'falsify' (verify) this paradigm and lead at the same time to a measure of error of the $kGAl^2/EI$ type that the field-consistency paradigm was seen to do. There are also instances where exact integration (with a non-singular constrained matrix) would not lead to locking (if by locking we mean the increase of stiffness without limit as the penalty parameter increases) as in the field-inconsistent quadratic beam element. In this case, the argument involving the degeneration of equation (15) to equation (16) is no longer valid.

There are other closely related arguments that have found their way to the textbooks but are no more scientifically valid than the singularity argument. One relates to the rank of the shear-stiffness matrix—that this must not be too high. Reduced integration helps to reduce this rank condition. Another very closely related paradigm concerns the number of constraints contained in the stiffness matrix, the so-called constraint counting procedure. Reduced integration lowers the constraint count as one can show quite easily that the number of constraints is directly linked to the number of integration points used to integrate the constrained strain energy. Another argument that was prevailing some time ago was that of relating locking to the spectral condition number: exactly integrated stiffness matrices always had a higher spectral condition number and this was linked to the locking effect. Note that these are all heuristic arguments, if not specious, more in the nature of a myth or superstition than a scientifically rigorous paradigm. Again, these are conditions that reflect the symptoms of the problem (locking is seen where there is a non-singular constrained matrix, or where the rank is too high, etc.) and not really the cause of the problem. Only the consistency paradigm traces the problem to the root and then argues forward to a falsifiable error estimate.

To close this critique of the conventional wisdom, it may be well to bring in a renowned theorem in the philosophy of science, called Occam's razor. Stated very simply, it recommends that the simplest explanation (i.e. the one making the least assumptions) is usually the best. The field-consistency paradigm is the simplest as it stipulates only one requirement, namely that the constrained strain field must be consistently interpolated. No further conditions are required, none on the singularity, or rank or spectral condition number of the stiffness matrix or the constraint-count index. Nor does it require the degeneration of equation (15) to equation (16) to explain why locking takes place.

Concluding remarks

The linear beam element serves as an example to demonstrate the principles involved in the finite-element modelling of a constrained-media problem. I have demonstrated that a conceptual framework that includes a condition that specifies that the strain fields to be constrained must satisfy a consistency criterion is able to provide a complete scientific basis for the problems encountered in conventional displacement-type modelling. I have also shown that a correctness criterion (which links the assumed strain variation of the displacement-type formulation to the mixed variational theorems) allows us to determine the consistent strain field interpolation in a unique and mathematically satisfying manner. Work done over the last decade confirms that the consistency paradigm is valid for a wide range of problems in structural mechanics, fluid dynamics and thermoelasticity.

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Computer aids in production engineering

N. Chandrasekaran

Solutions to several industrial production problems call for a systems approach involving integration of computer-based techniques for performing material flow analysis with press tool design. Software packages have been developed that perform the complex mathematical analyses required for optimizing design and system parameters. Computational tools are thus key to competitive production engineering.

Modern production engineering environment requires that a technological system is primarily controlled by means of extensive scientific inputs. The inputs are provided by mathematical formulations, which tend to be rather complex when in the field of solid mechanics. Solutions to the mechanics problems often entail making suitable approximations that will simplify the mathematics. The advent of powerful modern digital computers has dispensed with the need to accept inexact solutions. Most accurate solutions can be obtained using numerical treatments. Another feature is that these techniques allow the development of software packages in modular form. Still, the metal forming industry has been frustrated by the fact that the commercially available software packages are mostly irrelevant to their needs, because the formulations, though sound in fundamentals, often tend to ignore certain critical engineering aspects of the problem that are peculiar to the practitioner.

Whether it is forging, extrusion, rolling, deep drawing, flow turning or any other metal forming operation, the flow of metal within the body is responsible for bringing about permanent shape changes. Metal flow is associated with displacements, strains and stresses. Each of them is related to the other by material constitutive equations, flow rule, associated flow rule, displacement–strain relationships, etc.[1–4] Apart from the deformation behaviour of the work material, the machine tools (press, rolling mill, etc.) and the tooling (die, punch, etc.) involved in the production process significantly affect the overall system response. Suffice it to state at this juncture that numerous parameters representing the machine tool, production tooling and work material play equally important roles.

This leads us to the crux of the problem. Traditionally, academics and textbooks have treated metal flow and tooling problems in isolation, as evidenced by the published literature. The important corollary is that one can perform the metal flow (stress-distribution) analysis, but cannot use the information to design the tooling, or vice versa; in other words, that technology lacks scientific input. One reason for this trend may be the non-availability until recently of systems with enormous computing power at the desk-top level. Advancements in the field of computers have further brought powerful graphics engines with the systems, which facilitate problem definition.

It is generally accepted that no single software can solve a wide range of practical problems. In this article I describe the salient features of selected fundamental software packages that were found to be useful in the consultancy work performed by me for the American Dow Chemical Co., the Canadian Cosma International, Fabricated Steel Products Division of Indal Ltd, etc. Since the methodology essentially uses a short-term approach that derives benefit from long-term development work, I first briefly discuss the motivating factors.

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A parallel and a lesson

It may come as a great surprise to note that the realization that the material manufacturer (the steel