The geometrical theory of diffraction is a very convenient and easy method of calculating diffraction patterns, and an elegant approach to the problems of Fresnel diffraction at apertures and obstacles. In spite of its long and chequered history, it has not found sufficient emphasis in the standard literature on optical diffraction. Between Young (1802) and Keller (1962), the Indian school (1917–45) led by Raman was active in the field.

In recent years, there has been a revival of interest in the so-called geometrical theory of the diffraction of light, consequent on the systematic work of Keller, who in the late 1950s developed the detailed methodology for this approach. This concept of treating optical diffraction using geometrical ideas was first introduced by Thomas Young in 1802. In the intervening years, much progress was made and many salient features of the recent ideas in geometric theory were anticipated during the three decades ending in 1945 by the active school led by C. V. Raman. Unfortunately, this work has gone unnoticed in the subsequent, modern literature. It is our intention in this article to give a connected account of the development of the geometrical theory bridging the gap between Young and Keller by presenting the work of Gouy, Sommerfeld, Rubinowicz and Raman.

The Helmholtz–Kirchhoff theory of scalar waves presents many computational difficulties in the theoretical calculation of a general diffraction pattern. The procedures become inordinately complex even in the cases of apertures and obstacles having standard geometrical shapes. Over the years, attempts at improving this technique have met with very limited success. It is in this context that the oldest and perhaps the simplest theory, viz. the geometrical theory becomes relevant.

Diffraction at edges and apertures

Thomas Young was the first to propose that when light falls on a straight edge, the edge 'reflects' the light into space and the associated interference between the 'edge wave' and the geometrically transmitted wave gives rise to the observed diffraction effect. Gouy in 1886 gave reality to Young's edge waves when he observed that the sharp metallic edge held in a pencil of light appears luminous and the strongly polarized light is diffracted through large angles. Maggi later elaborated Young's model and showed mathematically that the diffraction integral over an aperture can be reduced to a line integral on the boundary of the aperture and a contribution due to the geometrically transmitted light. Sommerfeld, who was apparently unaware of this work, independently solved the problem exactly for a straight edge. This theory of electromagnetic diffraction at a straight edge made of perfectly conducting material leads to an interesting result. The field at any point can be looked upon as a sum of the transmitted wave and the wave that appears to emanate from the edge. This edge wave is given by the asymptotic formula

\[ u(r, \phi) = v(r, \phi - \phi_o) \pm v(r, \phi + \phi_o), \]  

where

\[ v(r, \theta) = \left[ \frac{1 + i}{4 \sqrt{k}} \right] \left[ \frac{e^{i kr}}{r} \right] \left[ \frac{1}{\cos \theta/2} \right]. \]

The + or − sign is taken according as the electric vector is parallel or perpendicular to the edge and \( \phi_o \) is the angle of the incident ray and \( \phi \) that of the diffracted ray as measured from the plane of the diffracting screen (see Figure 1). Along the shadow boundary \( v \) diverges since \( \theta = \pi \). (The so-called uniform geometrical theory of diffraction overcomes this lacuna, but we do not discuss it further in this article.)

The geometrical theory resolved, for the first time, the apparent puzzle associated with the concept of edge
diffraction, that the intensity of the bright fringe
remains almost constant as the observational plane
recedes from the edge. Although the edge wave is
cylindrical, its amplitude is dependent on the angle of
diffraction. The amplitude increases as the shadow
boundary is approached (by a factor sec θ/2) and the
1/√r law for amplitude diminution exactly compensates
this.

Sommerfeld's theory also agrees with the Fresnel
scalar theory of diffraction for a straight edge. But, it
fails to account for the observed diffraction pattern by
metallic edges. Raman and Krishnan⁹ pointed out that
this failure is due to the assumption that the material is
perfectly conducting. Instead, by incorporating the
complex metallic reflection coefficient in the second
term of eq. (1) these authors neatly accounted for the
experimentally observed features.⁹,¹⁰

Rubinowicz¹¹ many years later rediscovered Maggi's
result that the Kirchhoff's surface integral over the
diffracting aperture in the limit of short wavelengths for
an incident spherical wave could be reduced to a line
integral over the aperture. He also obtained the result
that the diffracted field at any point is made up of two
components—(i) the familiar geometrical optical field,
and (ii) a wave emitted by the boundary of the aperture.
Laue¹² showed that in Fraunhofer diffraction also, a
similar transformation from surface integral to line
integral along the boundary is possible. Raman¹³
showed the integral transformation to be a far simpler
procedure if one makes the justifiable approximation of
ignoring the obliquity factor. It must be remarked that
all these procedures are valid only when the size of the
diffracting object is large compared to the wavelength
of light.

Another important aspect of Rubinowicz's work is
that the contour integral can be reduced by the
stationary phase method to contributions from a finite
number of points on the boundary whose locations
depend on the point of observation in the diffraction
field. Further, these special points for normal incidence
can be easily obtained by drawing perpendiculars to the
diffracting boundary from the point of observation. At
these points, the incident light ray and the diffracted ray
reaching the point of observation satisfy a reflection
condition. According to this, when the incident rays are
parallel and normal to the diffracting edge, the
diffracted light rays reaching any given point of interest
are also normal to the edge. Clearly, this answer yields
the cylindrical boundary waves for straight edges as
shown in Figure 2.b. The feet of the perpendiculars
mentioned earlier are the special points which seem to
be the source of radiation. Based on Raman's model,
Ramachandran¹⁴,¹⁵ also obtained the same result.

From a given point of observation only these
points—poles—should be visible and this was experi-
mentally demonstrated by Raman¹³,¹⁶, who showed

![Figure 2. Diffracted rays can be obtained from the Rubinowicz 'reflection' condition. a, Diffracted rays will be on a cone symmetric about the edge for an oblique incidence. b, Diffracted rays will be on a disc perpendicular to the edge for normal incidence.](image)

that only a finite number of luminous points are visible
on the boundary when viewed from the shadow region.
For these special points the total optical path from the
source to the point of observation via these points is an
extremum. This principle is very reminiscent of the well
known Fermat's principle in geometrical optics. For
this reason, it has been referred to as Fermat's principle
for edge diffraction by Keller. When the incident light
rays are parallel but are incident on the edge at an
angle then Fermat's principle of diffraction will result in
diffracted rays travelling on a cone symmetrical about
the local tangent to the edge. Thus, one gets diffraction
wavefronts to be parallel cones with the edge as their
common axis. This has been depicted in Figure 2.a.

Kathavate¹⁷ stated that, when dealing with sharp
corners of apertures and obstacles, the sharp corners
should be taken as additional point sources of light
emitting spherical waves. These are in addition to the
corners already considered. A decade later Keller¹⁸ also
suggested the same procedure. However, these workers
did not work out the diffraction coefficient. Independ-
ently, around the same year, Miyamoto and Wolf¹⁹ not
only came to the same conclusion but also worked out
the corner-diffraction coefficient.

**Diffraction within the shadow**

The geometrical theory of diffraction clearly indicates
that the shadow region of an obstacle gets light only
from the edge wave. These edge waves will have to be
added at any point within the shadow to get the net
optical field there. From what has been said in the
previous section it follows that we need to take only
two types of contributions: (i) from the poles obtained
with respect to the point of observation, and (ii) from
the corners. The question of the phase of the radiation
from the pole was considered by Ramachandran¹⁴. He
showed using the Cornu spiral method that the
radiation received at the point of observation from the
regions neighbouring the special points (poles) resulted in a phase advance or a phase lag of $\pi/4$ depending upon whether it is one of maximum or minimum optical path from the source to the point of observation via the edge. The same result was obtained many years later by Miyamoto and Wolf.

Surface diffraction

So far we have considered diffraction only at sharp edges. But in reality edges are never perfectly sharp, but are rounded. An extreme example of this was considered by Raman and Krishnan—the Fresnel diffraction by a spherical object. One might naively regard this as equivalent to diffraction by a circular disc. However, in the case of diffraction around metallic spheres, they noticed that the intensity of the central bright spot is always lower than what one observes for a circular disc of equal diameter. Also, the intensity of the central spot is found to be a very sensitive function of the distance from the centre of the sphere. In fact, it exponentially decays below the intensity of the disc spot as the point of observation approaches the sphere. They accounted for this by suggesting that light actually creeps over the spherical surface and the light reaching any point of observation emanates from the circular boundary along the tangent cone drawn to the sphere from the point of observation. They used the exponential law derived by Riemann–Weber for electromagnetic wave propagation around the earth and got a beautiful fit with experimentally observed data.

The more recent work on the geometrical theory of diffraction at smooth surfaces is based on essentially the same mechanism. The detailed theory gives a series expansion for the attenuation coefficient which turns out to be different for the electric vector parallel or perpendicular to the surface. Raman and Krishnan's theory has only the leading term of this series. This is sufficient to account for the experimental data. But one feature which is important in this process of creeping is that the attenuation coefficient is a complex number. Hence when the waves interfere after creeping they have additional phase differences over and above that due to the actual path travelled by light. Keller invokes the generalized Fermat's principle, whereby the actual path from the source to the observer via the surface should be an extremum. This is only possible when light 'creeps' on the surface, travelling along a geodesic on the surface. In fact, for oblique incidence on a cylindrical surface the light creeps along a helix.

Implications of the theory

Diffraction caustics

An important implication of the geometrical theory of diffraction was stressed by Raman as early as 1919. He argued that for normal incidence the diffracted 'rays' will be predominantly proceeding in the direction of the local normals to the edge of the aperture. Hence, there will be a concentration of light along the evolute (the envelope of the normals to a given curve is defined as its evolute) to the diffracting boundary. Raman also demonstrated this experimentally and called these the diffraction caustics (Figure 3). Of course, the diffraction caustic degenerates into a point in the case of a circular disc, leading to the familiar Poisson spot. A few years later, Coulson and Becknell (for a disc) and Nienhuis (for an aperture) did similar experiments and obtained the same results.

Slits and gratings

In the case of multiple straight edges as in a slit or an array of slits, the standard procedure is to employ the Fresnel integral or Cornu spiral to work out the diffraction pattern. In the geometrical theory of diffraction, as Raman showed, the diffraction pattern can be obtained by adding the various cylindrical edge waves. This model leads to the well-known answers for a slit or a grating in the Fraunhofer diffraction limit. Keller's recipe to deal with these situations is also essentially the same.

Semitransparent edge

Anathanarayanan studied the diffraction at straight edges of thin films of metals coated on glass. When the metallic coatings were thin enough, he saw fringes of high visibility in the shadow region behind the metallic film. But when the film was thick, he observed in this region, the familiar gradual decay in intensity. He explained this fringe system in the shadow region as a consequence of the interference between the cylindrical edge wave and the wave weakly transmitted by the thin metallic film. When both these waves are of nearly comparable amplitudes the fringe system had a high visibility or contrast.

Figure 3. Diffraction caustic of an elliptical aperture. (After Raman)
Fraunhofer diffraction

A special mention may be made of Raman’s studies on Fraunhofer diffraction by triangular and semicircular apertures. Here, the boundary of the diffracting object is replaced by a set of points. Fraunhofer diffraction of an equilateral triangular aperture has a six-fold symmetry and is obtainable from an interference of radiation from three point sources placed at the vertices of the triangle. In the case of a semicircular aperture, Raman argued that in effect we can replace the boundary by three points. One lies on the curved edge its position given by the foot of the perpendicular from the point of observation to the curved edge, and two more respectively at the two corners. This leads to the observed higher symmetry in the Fraunhofer pattern than that of the object.

Again in all their studies on apertures, Raman’s school made a special experimental study of pattern transformation as one went from the Fresnel diffraction limit to the Fraunhofer diffraction limit. This is important in view of the fact that Fraunhofer diffraction is centrosymmetric whereas Fresnel diffraction is not. In Figure 4 we have shown this phenomenon.

Shadow patterns

On the experimental side, Kathavate, using objects of a few millimetres and 10 to 50 hours of exposure, got beautiful and intricate diffraction patterns in the case of apertures and discs of various shapes. He came up with a simple and an elegant geometrical procedure, based on the geometrical theory of diffraction, to work out the positions of diffraction maxima (or minima). The whole geometrical construction is carried out on the plane of observation on to which we project the obstacle and the rays from the special points. It is easy to convince oneself that to get the projection of the poles, we just draw perpendiculars to the boundary of the shadow from the point observation. Light from the feet of these perpendiculars must be considered while calculating the positions of maxima or minima. In Figure 5, we show his theoretical calculation along with the observed diffraction pattern for the square disc. This work appears to have gone unnoticed in the literature. Recently English and George have reported the same result. The shadow patterns from elliptic discs of different eccentricities are shown in Figure 6.

Some new results

The Poisson spot

The bright central Poisson spot in the case of a circular disc is the brightest region of the diffraction pattern in the shadow and it is due to the constructive interference of radiation from the entire boundary. At other points of observation we have only two boundary waves emanating from diametrically opposite poles, leading to periodic weak maxima (and minima). Even in the case of other obstacles like elliptic, square, triangular and rectangular discs, we get such a central spot. It is easy to work out the features associated with this Poisson spot in the language of the geometrical theory of diffraction. For example, in the case of an elliptic disc, the centre of the pattern gets light from four poles—two poles at the ends of the major axis and two poles at the ends of the minor axis of the ellipse. Radiations from the poles of the major axis (or minor axis) are in phase at the centre. But radiations from a pole associated with the major axis may not always be in phase with the radiations from a pole associated with the minor axis. In fact, as we recede from the diffracting plane along the central axis these pole radiations will be successively in and out of phase, giving rise to a brightness fluctuation in the Poisson spot. For a square obstacle, the fluctuations are due to the pole and the corner radiation being in and out of phase. Since corner radiations are weaker as their intensity falls as \(1/r^2\), these fluctuations will not be prominent.

The Poisson spot associated with an elliptic disc is, in many ways, different from the one associated with a rectangular obstacle whose length and breadth are respectively equal to the major and minor diameters. If we ignore the corner radiation, then we get four poles
as in an elliptic disc. Yet the net intensity at the Poisson spot will be different for the elliptic disc due to the curvature at the boundary. This arises owing to the focusing effect of a curved wavefront emitted by a curved boundary. In fact, Keller shows from the geometrical theory of diffraction that a curved boundary contributes more than a straight boundary when the point of observation is towards the centre of curvature. Hence poles of the rectangle make a weaker contribution to the Poisson spot than the poles of an ellipse of 'equal' size. To our knowledge these interesting consequences of geometrical theory of diffraction have not been emphasized in the literature.

**Diffraction at a strip and at a cylinder**

In the shadow region at a finite distance from an opaque strip or a cylinder, experimentally one observes a fringe system. Careful investigations show that this fringe system is strictly not a set of equidistant bright and dark fringes. Nor is the visibility of the fringe system the same all over. In the language of the geometrical theory of diffraction the fringe pattern in the two situations is due to entirely different processes. For a strip it is the interference between the two cylindrical waves from the two straight edges. On the other hand, to reach any point in the shadow of a cylinder, light will have to creep from both sides along the boundary. Thus the interference pattern in general will be different in the two cases. The same arguments are valid even in the case of diffraction at a circular disc and a sphere. However, calculations of the diffraction pattern are easy for the case of a strip and cylinder. At extremely large distances there is very little creeping and the two patterns can be expected to be nearly the
Multiple-edge radiations

In another respect Keller improved the geometrical theory of diffraction. We shall illustrate Keller’s correction with the example of single-slit diffraction. It was argued earlier that in this case we have two cylindrical waves diverging from the two edges. A wave from one such edge will reach the other edge and will result in a second cylindrical wave. This process could go on endlessly, indicating that each edge gives rise to a multiplicity of edge rays. These have been termed by Keller as second, third, etc. diffracted edge rays. In principle, a complete solution must include the effect of these multiply diffracted rays. However, in practice, these extra effects do not appear to be all that significant, since the strength of the diffracted ray decreases considerably with increasing order.

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12. Laue, Von M., Berliner Sitzungsberichte, 1936, 89; see also Jame, R. W., The Optical Principles of the Diffraction of X-rays, G. Bell and Sons Ltd, 1967.