Source correlation and its relevance in optical measurements

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Statistical correlations of the light emitted by a partially coherent source can produce frequency shifts in its spectrum observed in the far field if the correlation function of the emitted radiation does not satisfy a certain scaling law. It has been shown that the error introduced owing to such source correlation in the detector plane is significant and depends on the optical set-up used in the experiment.

ONE of the most significant discoveries in modern optics in the last decade is the discovery of 'noncosmological redshift'. Wolf and his collaborators created a sensation by propounding a theory for a novel mechanism of spectral shift due to source correlation[1,2], a phenomenon quite different from Doppler or gravitational shift. The new concept has challenged traditional tenets of modern physics and may have repercussions in many areas of science. It also suggests that our view of the universe may need amending[2].

It has been known for decades that light of different spectral patterns, characteristic of the nature and chemical composition of the source, is emanated from different sources. Until recently it was taken for granted that the emitted spectrum remained unchanged on propagation in free space provided there is no relative motion between the source and the observer or no effect of gravity. The spectral shift (known to astronomers) due to Doppler shift or gravitational effects could not resolve many riddles of astronomical observations. While providing an explanation for the 'redshift controversy' concerning astronomical objects, Wolf suggested that source correlation may be responsible for the part of the observed spectral shift that could not be explained on the basis of Doppler or gravitational shift.

Basic theory

Source correlation is the spatial coherence possessed by the source points or developed between them during propagation of radiation in free space. Quantitatively source correlation is defined in terms of the cross-spectral density which is also a measure of the source or field spectrum.

To illustrate and bring out the essential features of source correlation, we consider the simplest radiating system of two small radiating scalar sources located in the neighbourhood of points P_1 and P_2. All the sources fluctuate randomly in time and hence can be represented by appropriate statistical ensembles. One may characterize the fluctuations in terms of ensembles of frequency-dependent realizations[3], say $(Q_1(w))$ and $(Q_2(w))$. The fluctuating field which the two sources generate at a point P in space is then given by

$$U(P,w)=Q_1(w)\frac{e^{ikR_1}}{R_1}+Q_2(w)\frac{e^{ikR_2}}{R_2}, \tag{1}$$

where $k=w/c$, w is the frequency and c the velocity of light, and R_1 and R_2 are the distances of the point P from the two sources.

The spectrum of the field at P is given by

$$S_u(P,w)=\langle U^*(P,w)\,U(P,w)\rangle, \tag{2}$$

where the asterisk denotes the complex conjugate and the angular brackets denote the ensemble average. On substituting $U(P,w)$ from eqn (1) in eqn (2), we get

$$S_u(P,w)=S_Q(w)\left[\frac{1}{R_1^2}+\frac{1}{R_2^2}\right]+\left[W_{12}(w)\frac{e^{ik(R_2-R_1)}}{R_1R_2}+\text{cc;}\right] \tag{3}$$

In this expression $S_Q(w)$ represents the spectrum of the two sources at P_1 and P_2 assumed to be the same, i.e.

$$S_Q(w)=\langle Q_1^*(w)\,Q_1(w)\rangle=\langle Q_2^*(w)\,Q_2(w)\rangle. \tag{4}$$

and

$$W_{12}(w)=\langle Q_1^*(w)\,Q_2(w)\rangle \tag{5}$$

is called cross-spectral density, which characterizes the correlation between the fluctuations of the two sources at frequency w. The abbreviation cc in eqn (3) stands for the complex conjugate of the term in square brackets.

Equation (4) shows that the spectrum of the field at a

The authors are in the Standards Division, National Physical Laboratory, New Delhi 110 012 point P depends not only on the spectra $S_Q(w)$ of the two sources, but also on the correlation between their fluctuations characterized by $W_{12}(w)$. Only in the simplest case, when $W_{12}(w)=0$, i.e. when the sources are mutually uncorrelated (incoherent sources), will the spectrum at every field point be proportional to the source spectrum, as is evident from eqn (4). If the cross-spectral density varies appreciably with frequency over the effective spectral range of each source, the field spectrum may be very different from the source spectrum. The spectral changes may, for example, occur in the form of narrowing, broadening or shifting of a spectral line, or even generation of multiple lines from a single line. Several examples of such spectral changes have been analysed theoretically[1,2,4] and have also been demonstrated experimentally[5-10].

The above phenomenon is not restricted to radiation from a simple system consisting of two small sources, but also applies to radiations from extended sources. For example, the normalized spectrum $S(\bar{u}, w)$ of the field at a point P in the far zone in a direction specified by a unit vector $\bar{u}$, produced by a planar, secondary, quasihomogeneous source, is given by[11]

$$S(\bar{u}, w)=\frac{k^2 S^0(w)\,\tilde{\mu}(k\bar{u}_{\perp}, w)}{\int_0^{\infty} k^2 S^0(w)\,\tilde{\mu}(ku_{\perp}, w)\mathrm{d}w}. \qquad (6)$$

Here $S^0(w)$ is the spectrum of the field in the source plane specified by a position vector ρ assumed (for simplicity) to be the same at every source point.

$$\tilde{\mu}(\mathbf{f}, w)=\frac{1}{(2\pi)^2}\int \mu^0(\rho, w)\,\mathrm{e}^{\mathrm{i}\mathbf{f}\cdot\rho}\,\mathrm{d}^2\rho \qquad (7)$$

is the two-dimensional spatial Fourier transform of the degree of spatial coherence $\mu^0(\rho, w)$ of the field in the plane of the source and $\bar{\mathbf{u}}_{\perp}$ is the projection of the two-dimensional unit vector $\bar{\mathbf{u}}$.

An immediate consequence of eqn (6) is that the normalized spectrum $S(\mathbf{u}, w)$ of the light u in the far zone in a direction specified by the unit vector differs, in general, from the normalized source spectrum

$$S(w)=\frac{S^0(w)}{\int_0^{\infty} S^0(w)\mathrm{d}w} \qquad (8)$$

in two respects (i) the proportionality factor k^2 and (ii) the presence of the factor $\tilde{\mu}(k\bar{\mathbf{u}}_{\perp}, w)$ which depends on the coherence properties of the light in the source plane.

Wolf[11] has shown that a sufficiency condition for the normalized source spectrum of light produced by a planar, secondary, quasihomogeneous source, which has the same normalized spectrum at every source point and throughout the far zone, is that the degree of spatial coherence $\mu^0(\rho_2-\rho_1, w)$ of the light at any two points P_1 and P_2 in the source plane should have the functional form

$$\mu^0(\rho_2-\rho_1, w)=h(k|\rho_2-\rho_1|). \qquad (9)$$

The degree of coherence represented by eqn (9) is known as the scaling law.

All planar, quasihomogenous, lambertian sources satisfy the scaling law because such sources are known to have a degree of spatial coherence given by the expression[12].

$$\mu^0(\rho, w)=\frac{\sin k\rho}{k\rho}, \qquad (10)$$

which satisfies the scaling law. As the normalized spectrum of light which such sources generate throughout the far zone is the same as that of the normalized source spectrum, it was until recently believed that the spectrum of light remains invariant during propagation.

Wolf[12] showed that, if the spatial degree of coherence does not obey the scaling law, e.g. if it becomes independent of wavelength, the spectral energy may get redistributed and may cause a shift in the peak wavelength. The shift may occur towards either the red or the blue end of the source spectrum. The scaling law can be violated by generating a secondary source whose spectrum is the same at every source point and whose degree of spectral coherence is independent of wavelength. Spectral shift may also occur if the radiation is scattered by a medium whose constitutive parameters, like dielectric susceptibility, dipole moment, etc., are independent of time but are random functions of position with appropriate correlation property[13,14]. James and Wolf[15] have shown that frequency shift of any magnitude could be caused by dynamic scattering.

In optical measurements, the degree of coherence is related to the parameters of the experimental set-up, e.g. the size of the apertures used, focal length of the lens or mirror system, wavelength of the light. This can be shown by taking a simple example.

Consider a circular (or rectangular), spatially incoherent, polychromatic, planar source O of radius a_s situated at the front focal plane of a thin lens L of focal length f, as shown in Figure 1. The source is assumed to

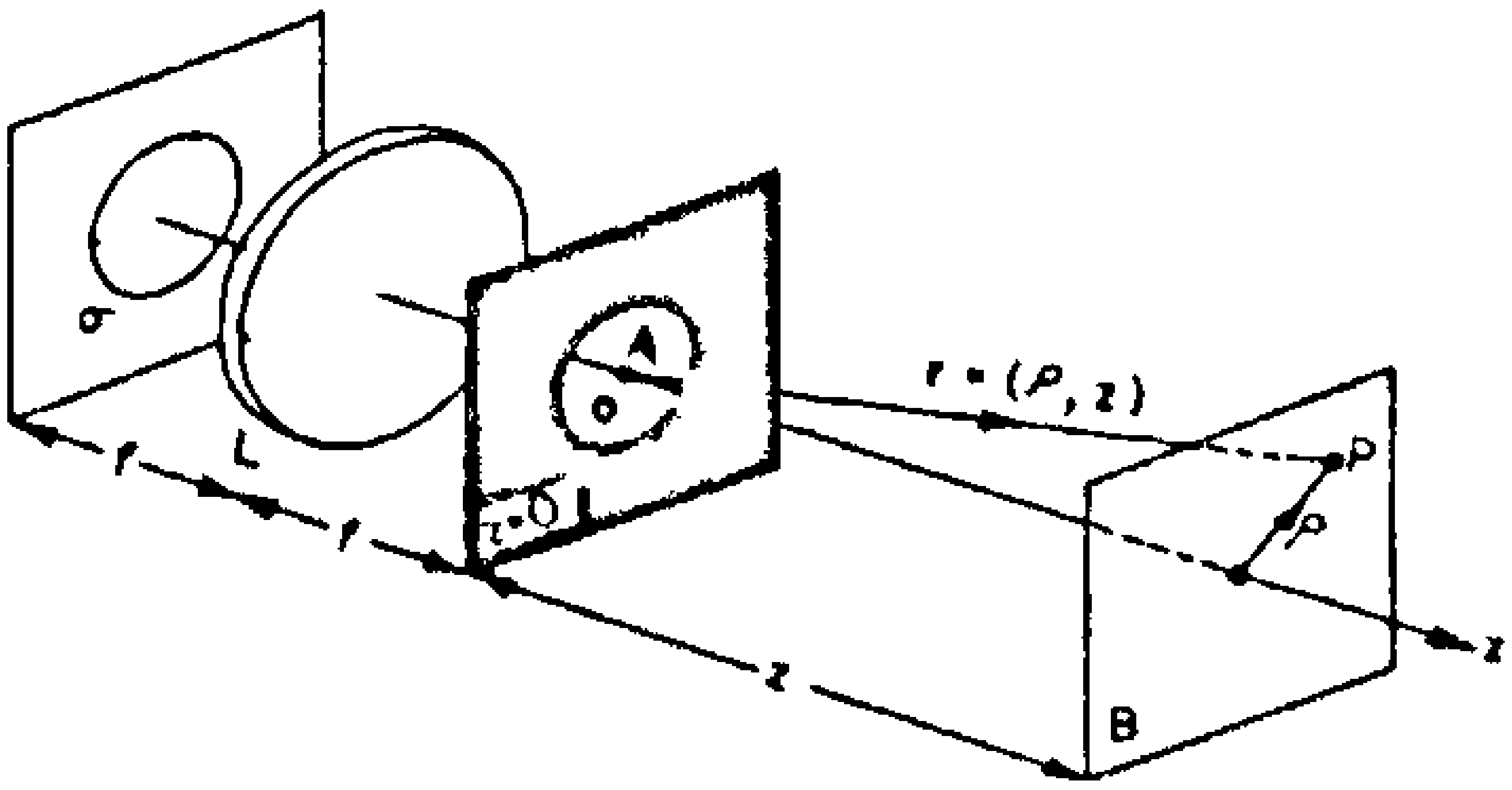

Figure 1. The optical system

be uniform, i.e. its spectrum $S(w)$ is the same at all source points. A circular or a rectangular aperture A is placed in the back focal plane ($\mathbf{Z}=0$ plane) of the lens. The point P on the on-axis position at a distance Z denotes an observation point in the far zone of the aperture. In order to determine $S(\mathbf{Z}, w)$, the spectrum at point P, the cross-spectral density of light in the aperture should be known. Morris and Faklis[5] have shown that, for a thin-lens focusing system under paraxial ray approximation, the cross-spectral density of light of P_1 and P_2 in A is given by

$$W^0(\rho_1, \rho_2; w) = S^0(w)\,\mu^0(\rho_1 \rho_2; w), \tag{11}$$

where

$$S^0(w) = K S(w) \tag{12}$$

and

$$\mu^0(\rho_1 \cdot \rho_2; w) = \frac{2J_1(k|\rho_2 - \rho_1|a_s/f)}{k|\rho_2 - \rho_1|a_s/f}. \tag{13}$$

Here K is constant and J_1 is the first-order Bessel function. It follows from eqn (12) that for a fixed w, μ^0 decreases steadily from unity (when $|\rho_2 - \rho_1| = 0$) to zero when this distance is equal to $3.832f/ka_s$ and oscillates with decreasing amplitude for larger values of $|\rho_2 - \rho_1|$. The length $3.832 f/ka_s = L(w)$ is referred to as effective correlation length of light of frequency w at the aperture. The spectrum of light at an on-axis observation point in the far zone of the aperture is related to the cross-spectral density of light at the aperture by

$$S(Z, w) = \left(\frac{k}{2\pi Z}\right)^2 \int d^2\rho_1\, d^2\rho_2\, W^0(\rho_1 \rho_2; w). \tag{14}$$

From equations (11), (13) and (14), it can be shown[16] that

$$S(Z, w) = \left(\frac{af}{a_s z}\right)^2 M(w)\, S^0(w), \tag{15}$$

where

$$M(w) = 1 - J_0^2(x) - J_1^2(x).$$

Here J_0 is the Bessel function of zero order and $x = (3.832\,a/L(w))$. Since $M(w)$ is a frequency-dependent factor, the spectrum in the far field would in general be modified, as is evident from eqn (15).

Experimental evidences

Morris and Faklis[5] were the first to confirm the theory of source correlation by generating a source whose spectral degree of coherence violated the scaling law. They used a Fourier achromat to achromatize wavelengths from 450 to 670 nm at its focal plane. But, owing to the complexities of this experimental set-up and also the inability of the system to produce varying degree of spectral coherence, far-reaching consequences of the phenomenon of spectral shift in optical measurements could not be perceived. A similar set-up has been used by Indebetouw[10] to demonstrate the phenomenon of spectral shift.

In the National Physical Laboratory (Optical Radiation Group), we have provided experimental support to the theory by using simple and intelligent experimental set-ups that are flexible enough to produce any desired degree of coherence[8]. We have also shown some of the potentially important implications of the phenomenon of spectral shift due to source correlation in spectroradiometry[7] and spectroscopy.

Spectroradiometric measurements

In spectral irradiance and radiance measurements a source of uniform radiance is generated through the use of a plane diffuser or an integrating sphere equipped with a precisely measured aperture mounted on the diffuser. The image of such a source is focused at the entrance slit of a monochromator using suitable focusing optics. A secondary source is thus created at the monochramator entrance slit. According to the van Cittert and Zernike theorem a certain degree of source correlation is developed between the field points when light propagates through free space. The secondary source thus generated would necessarily possess some degree of coherence. As some focusing optics is used between the primary source and the secondary source generated at the slit of the monochromator, source correlations would produce some spectral shift, and this spectral shift must be taken into consideration in all precision measurements. Here we report the errors caused in the spectral power measurements by the spectral shift. Alternative systems that would reduce such errors have been proposed

Experimental set-up

The experimental set-up consists of a tungsten halogen lamp (450 W spiral filament) as a primary source. The radiation from this source is passed through an interference filter and made incident on a plane diffuser or an integrating sphere coated with barium sulphate. Such an arrangement produces a source of uniform radiance. Light scattered from the diffuser or the integrating sphere is focused at the entrance slit of a monochromator. The focusing optics is either a concave mirror of focal length 50 cm or a convex lens of focal length 50 cm. The optics generates a secondary source at the entrance slit of the monochromator (its degree of spectral coherence can be given by eqn (13)). To avoid

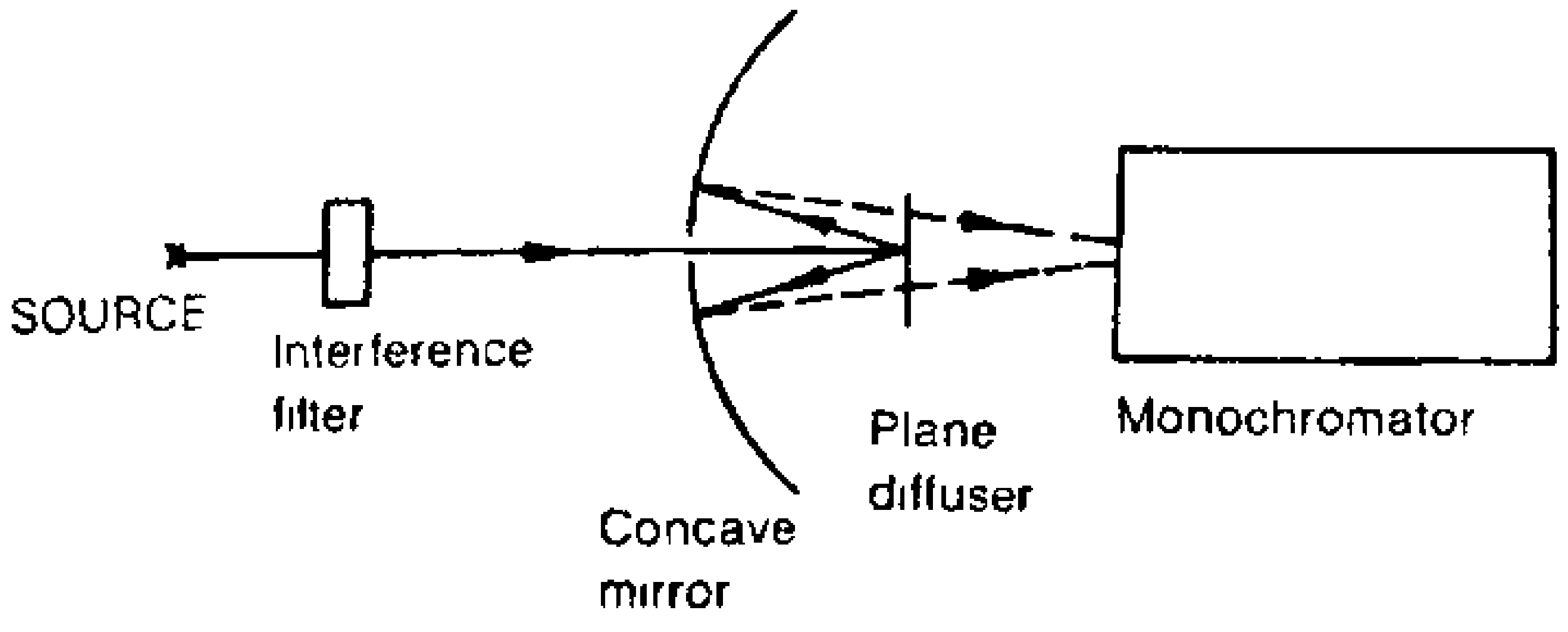

Figure 2. Schematic diagram for observing spectral shift due to a plane diffuser and mirror-focusing optics.

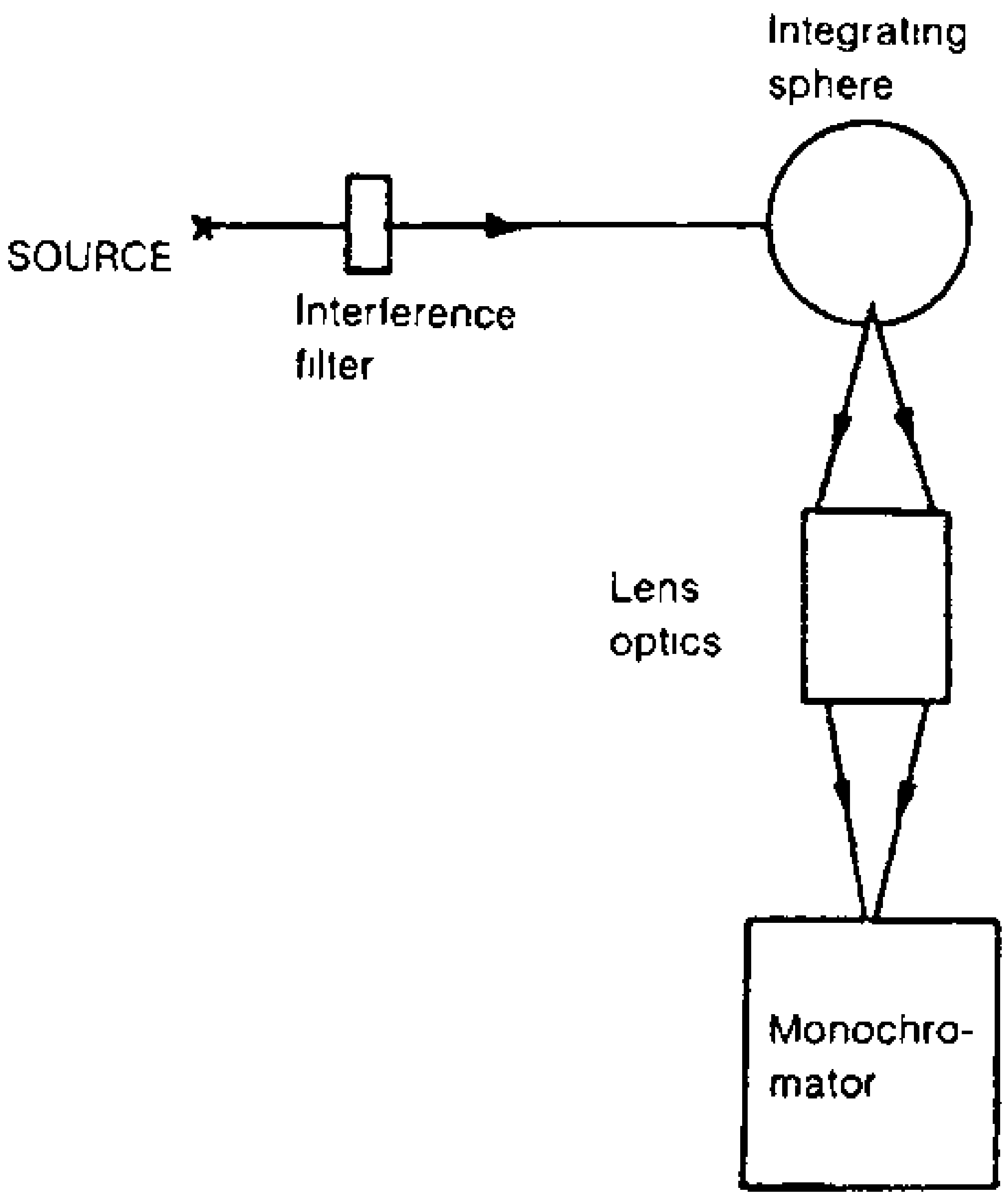

Figure 3. Schematic diagram for observing spectral shift due to an integrating sphere and lens-focusing optics.

stray light the monochromator used is a Spex 1404 model with focal length $f=0.85$ m and aperture $f/7.8$, and two gratings of 1200 lines/mm and 110 mm × 110 mm size. Thus the resolution achieved is 0.005 nm. The detector is a water-cooled photomultiplier tube. The monochromator and the photomultiplier are coupled with a DM1B computerized data acquisition and processing system. Spurious signals caused by fluctuations in the mains supply are avoided by using stabilized power supply and spike suppressor. Interference filters graded at different wavelengths between 380 and 750 nm and with bandwidth 10 nm were used in the experiment. Room temperature was maintained well within ± 1°C during the experiment. The experimental set-ups are shown in Figures 2 and 3.

Results

To determine the peak transmitted wavelength of the filters, the spectra of the source through the set of interference filters are recorded without focusing optics. A secondary source is generated at the entrance slit of the monochromator using a barium sulphate-coated plane diffuser and a concave mirror in the path of light to produce a source of uniform radiance. The spectra are then recorded. Figures 4 and 5 show that the peak transmission wavelengths of the filters are shifted towards red or blue ends with the use of a barium sulphate surface and the focusing optics. The spectral profiles of the filters also change. When spectra are recorded with different diffuser surfaces coated with barium sulphate and magnesium carbonate, the magnitude and direction of the shift are the same with different diffusing surfaces. It is noted that the spectral bands (transmitted through the interference filters) lying in the blue region shift towards the red end and those

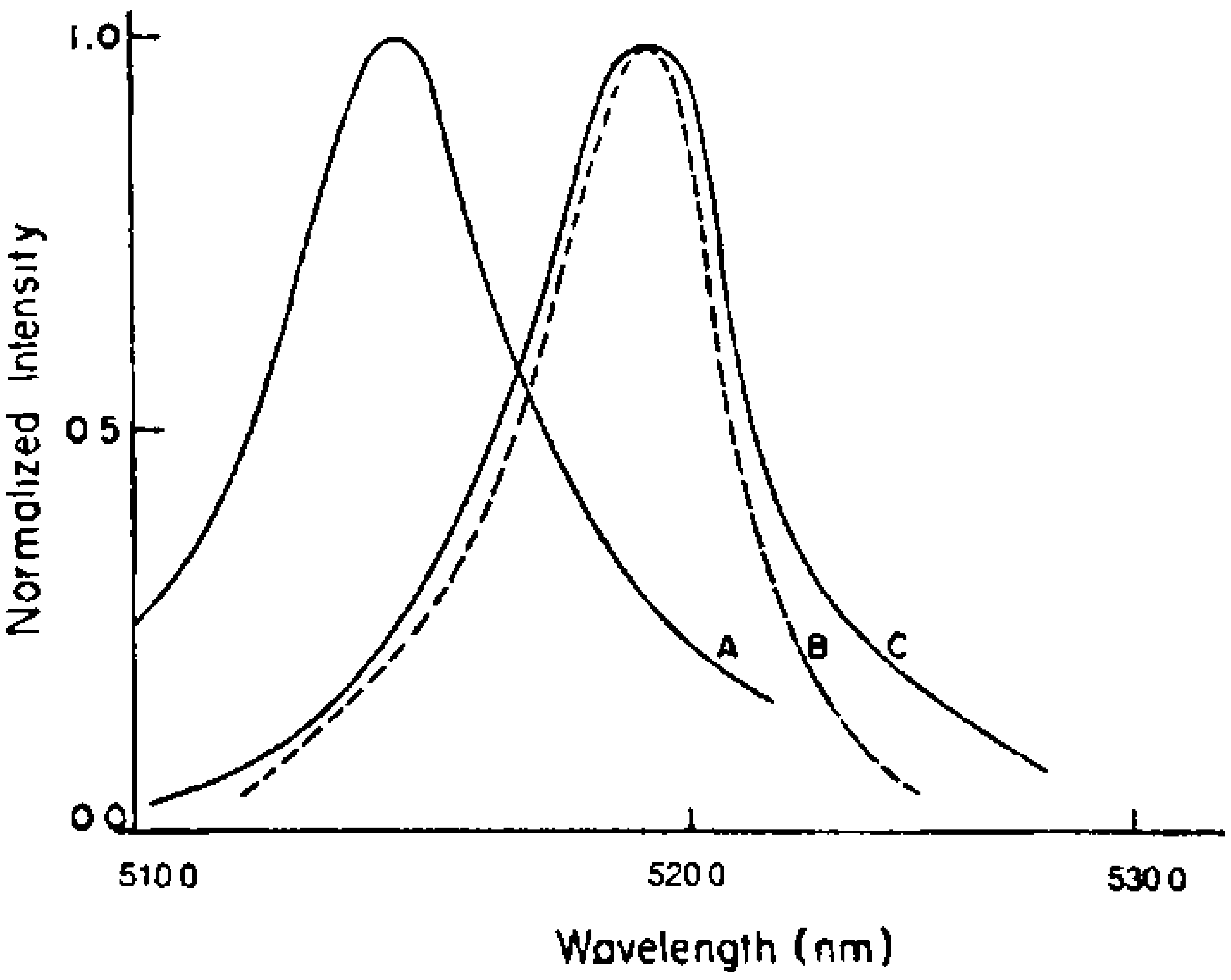

Figure 4. Spectral profile of 515.5-nm peak through the interference filter (A) without a diffuser, (B) with a $BaSO_4$ diffuser, and (C) with a $MgCO_3$ diffuser.

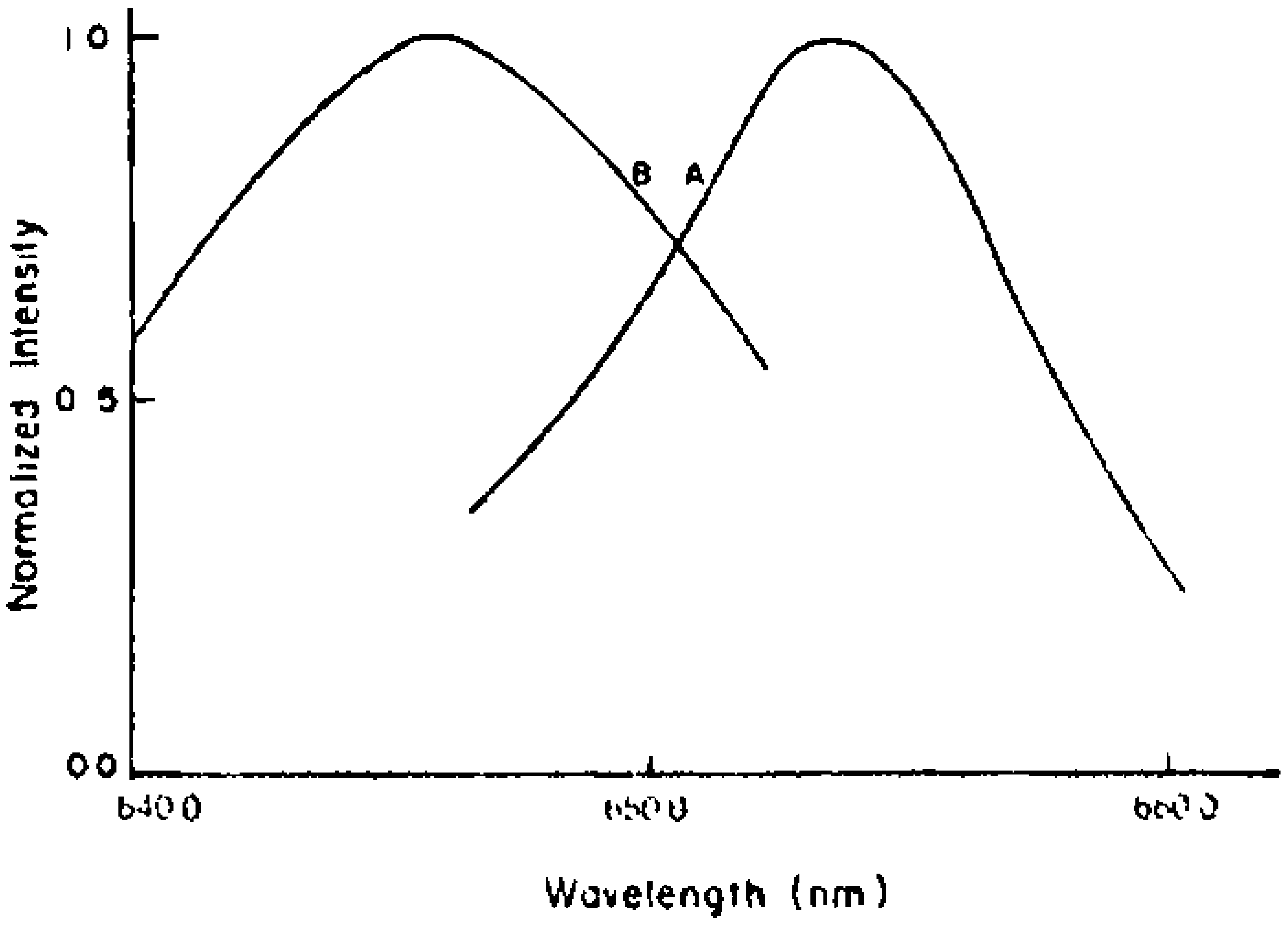

Figure 5. Spectral profile of 653.5-nm peak through the interference filter (A) without a diffuser and (B) with a $BaSO_4$ diffuser

lying towards the red region shift towards the blue end of the spectrum. The spectral profile of the band that is shifted towards red becomes narrower whereas the profile of the blue-shifted band becomes broader. The spectra through the filters were also recorded by replacing the plane diffuser with an integrating sphere diffuser using the same focusing optics and it was found that the magnitude of shift decreased. The spectra were also recorded with integrating sphere diffuser and lens-focusing optics. Table 1 gives the shifts in the peak transmission wavelengths of different interference filters. It is seen that the magnitude of shift with integrating sphere and lens-focusing optics is much smaller than that with the plane diffuser and mirror-focusing optics. With the integrating sphere, the magnitude of shift throughout the visible region is almost the same provided the half-bandwidths of the filters used are the same.

To see whether the spectral shift observed is due to source correlation or due to systematic errors, the precautions suggested by Jones and Moore[17] and Moore[18] were also taken into consideration:

(i) *Stray light.* In the experiment a Spex double-grating monochromator was used whose stray-light performance (10^{-14} at $\Delta\lambda = 0.5$ nm) was also checked with the procedural details described in ref. 18. It has been found that the direct transmittance does not cause any wavelength change. However, change in intensity may affect the spectral power by almost 0.01%.

(ii) *Linearity of the detector.* The linearity of the detector is checked at all measurement points. Since the response of the photomultiplier is almost non-selective in the measurement range, the linearity is well maintained by the detector in the measurement range.

(iii) *Slit variation to maintain constancy of bandwidth:* This is the main factor responsible for the change in the wavelength. As has been shown by Foley[16], that the focusing optics produces a certain coherence length at the aperture (the slit of the monochromator in the present case), the ratio of the size of the aperture to that of coherence length determines the magnitude of spectral shift. Here the aperture size is changed to maintain constancy of bandwidth, which, however, makes the source globally coherent or incoherent. So long as the source formed at the aperture is globablly coherent spectral shift will occur. In the present study the slit width is put from 100 to 1000 μm and the coherence interval produced by the optics is about 50 μm. The condition for this source being globally coherent is maintained so the spectral shift is only due to source correlation and not due to any physical effect. Since the slit is changed at every step of wavelength, the magnitude of the shift may also change. It has also been shown that the spectral shift is proportional to the bandwidth of the filter. If one uses constant slit width the constancy of the spectral bandwidth of different interference filters has been found to give approximately constant spectral shift for an integrating sphere and lens combination.

(iv) *Constancy of temperature.* Calibration checks were performed at the beginning and at the end of the experiment. It was found that, with temperature variation of ± 1°C, the monochromator maintains the performance well within the wavelength accuracy of ±0.1 nm. The calibration is done without the use of any focusing optics using light from mercury cadmium, neon and hydrogen discharge lamps.

(v) *Error due to polarization.* The interaction of the polarization of the sources with the polarization of the monochromator is minimized since depolarization of radiation by the integrating sphere used is essentially complete. However, with a plane diffuser, which is not a good depolarizer, we may have large errors caused by polarization. The larger shift observed with the use of a plane diffuser compared to that with an integrating sphere may be due to polarization effects.

The above observations reveal that sources of systematic errors like stray light, linearity of detector, constancy of temperature and polarization do not cause a serious change in the spectral distribution. However,

Table 1. Filter peak transmission wavelength with and without plane diffuser and integrating sphere diffuser.

		Peak wavelength with		
		Plane diffuser	Integrating sphere	
Peak wavelength without diffuser (nm)	Half-bandwidth (nm)	With concave-mirror optics (nm)	With concave-mirror optics (nm)	With lens optics (nm)
422.0	9	427.1	424.6	421.0
484.1	9	488.6	486.6	483.6
515.1	8	519.2	517.8	514.6
585.1	8	588.6	587.1	584.6
653.5	10	646.0	652.8	654.0
685.6	10	678.1	683.7	686.2

change in spectral distribution occurs when the slit of the monochromator is changed for constancy of bandwidth. It is evident from these observations that spectral shift is mainly due to source correlation. The consequences of this are discussed below.

It is evident from the above that if the spectral shifts due to source correlation are not accounted for, they would introduce errors in spectroradiometric measurements. To give an idea of the magnitude of error introduced by source correlation: the spectral radiance of a black body at 2000 K would change by approximately 1.5% at 555 nm and by approximately 3% at 400 nm if a spectral shift of ~1 nm is introduced. At different black-body temperatures the magnitude of this variation would be different. It may also be highlighted that while calibrating any lamp for spectroradiometric measurements against a black-body cavity radiator, the correlation introduced by the intervening optical system used for the lamp and the cavity radiator would be different. This would cause a difference in spectral shifts and would make the calibration erroneous. The entire calibration procedure, therefore, needs a critical examination. The following are the suggestions made on the basis of the experimental findings:

(i) To reduce the magnitude of spectral shift in the context of the experimental system discussed above, interference filters with half-bandwidths as low as possible, say 1 nm, should be used. This will reduce the spectral shift as it is dependent on the half-bandwidth of the filter.

(ii) The spectral transmission of the filter must be measured *in situ* with and without the optics used in the experiment so that the magnitude of shift and the resulting errors can be estimated.

(iii) A combination of integrating sphere and lens-focusing optics should preferably be used (instead of plane diffuser and mirror) for all spectral measurements because the magnitude of spectral shift is not large and also the effect of polarization is minimized with such an experimental set-up.

In the light of the above, it has become necessary that the optics used while calibrating reference and secondary standard lamps for different radiometric parameters be very well defined. This would minimize the scatter that may otherwise be present in comparison of spectroradiometric scales maintained by different standards laboratories. While comparing these scales, if all the participating laboratories use identical optical set-ups, any deviation caused in these measurements by change in source correlation would be ruled out.

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