

RESEARCH ITEMS.

Modular Functions and Dirichlet's Series

Eulerian Products.—Hecke (*Math. Ann.*, 144, pp. 1-28) has investigated the generalization of the product formula of Dirichlet's series corresponding to various modular functions. In a previous article in the same journal he had discussed the number of linear independent Dirichlet's series which had certain functional properties and were characterised by functional equation. The Euler-product arguments that he determines here is not deductive in the ordinary sense but it is a product matrix ring connected with the totality of modular functions belonging to a certain class. It is interesting to note that many important results in the theory of quadratic forms with integral coefficients of even order can be derived from his results. He proposes to devote another article to these questions which we await with interest.

He considers functions $f(\omega_1, \omega_2)$ analytic in ω_1 and ω_2 , which possess the following properties:

in $\omega_2 \neq 0$, $R(\tau/i) > 0$, where $\omega_1/\omega_2 = \tau$, f is valued and regular in τ except for poles; $f(\lambda\omega_1, \lambda\omega_2) = \lambda^{-k} f(\omega_1, \omega_2)$; f is invariant for all substitutions

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \equiv \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \pmod{Q}.$$

Q is any integer. [This group is denoted (Q)]. If M is any substitution out of $G(1, Q)$, $\phi = \omega_2^{-k} f[M(\omega_1, \omega_2)]$ is a periodic function with period Q . \therefore we will have

$$\phi = \sum_n a_n e^{2\pi i \tau n / Q}.$$

It is further supposed (4) that this series has only a finite number of negative terms.

denote $\omega_2^{-k} f(\omega_1, \omega_2) = f(\tau)$, then $f(\tau)$ will be regular with respect to $z = e^{2\pi i \tau / Q}$ and at the rational corners. Functions satisfying these properties are designated as modular forms by him of the type $(-k, Q)$.

Among these that vanish at the corners are called a corner form. In this article he considers mainly the properties of functions of type $(-k, 1)$ and the investigation for any Q is connected with quadratic forms with integral coefficients of even order is to be discussed by him in another article. The proofs are rather technical and his main results are as follows.

[If $\phi(s) = \sum \frac{a_n}{n^s}$, then $\phi(s)$ is called a corresponding Dirichlet's series.]

To every complete system of linearly independent Modular forms of type $(-k, 1)$, where k is a number, there exist K matrices of degree k with numerical coefficients which are commutative with one another and form a complete matrix ring such that corresponding Dirichlet's series

$$\phi^p(s) = \sum_{n=1}^{\infty} a^p(n) n^{-s} \text{ form a matrix}$$

$$\Phi(s) = \sum_{v=1}^k \phi^v(s) B^v = \sum_{n=1}^{\infty} \lambda(n) \eta^{-s}$$

is a matrix] which has an Euler product of the following type:

$$\Phi(s) = \prod_p [E - \lambda(p) p^{-s} + p^{k-1-2s} E]^{-1}$$

for $R(s) > k$, where E is the unit matrix of order k .

The Dirichlet's series considered here can be made independent of the modular forms by being characterised as follows:

(1) $(s-k)\phi(s)$ is an integral function of finite order;

(2) $R(s) = (-1)^{k/2} R(k-s)$ where $R(s) = (2\pi)^{-s} \Gamma(s)\phi(s)$.

[In an earlier article it was shown by him by utilising the theory of modular forms that there exist only a finite number of such Dirichlet's series of that type which are linearly independent.] A Dirichlet's series which satisfies these conditions is $\zeta(s)\zeta(s-k) = \sum \frac{\sigma_{k-1}(\eta)}{n^s}$ where $\sigma_{k-1}(\eta)$ is the sum of the $(k-1)$ th powers of the divisors of n . He has shown also how a conjecture of Ramanujan which was proved by Mordell can be included in these general results.

K. V. I.

Inter-atomic Distances in Molecule.—The inter-atomic distances in simple organic molecules are constants, which depend only upon the nature of the bonds. A large number of organic molecules can, however, exist in more than one state, which are all in resonance with each other. In such molecules each inter-atomic distance assumes an intermediate value between the various possible limits. About two years ago, Pauling and Brockway pointed out that if an empirical curve is drawn connecting up the expected constant values with the actually observed distances for some well-known cases, then this curve can be used to estimate the degree of resonance in any new case from the observed deviations from expected values. As the precision of this method largely depends upon this empirical curve, Pauling and Brockway have recently redetermined (*Journ. Amer. Chem. Soc.*, p. 1223), the carbon-carbon distances in thirteen simple non-resonating type of hydrocarbons by the electron-diffraction method. It is found that the C-C distance is remarkably constant in all these cases at 1.54 Å, the hitherto accepted value. But the C=C distance constant requires a slight modification to 1.34 Å, being 0.04 Å less than the value given in the literature.

The results show further evidence of the extraordinary extent to which the tetrahedral carbon atom of van't Hoff and LeBel determine the structure of organic molecules. The values found for the angle between two single C-C bonds, are in all cases (excepting cyclopropane) within 2° of the tetrahedral angle 109° 28'. Another striking observation is that in cyclopropane, in spite of the strain due to distortion of bond angle from 109° 28' to 60°, the C-C bonds are not weakened and there is no consequent increase in the bond distance.

M. A. G.

Viscosity of Liquid or Plastic Monomolecular Films.—Myers and Hartins (*J. Chem. Phys.*, 1937, 5, 601) have described two instruments for measuring the viscosity of surface films. They call one of them the capillary slit surface viscometer; this is the two-dimensional analogue of the capillary tube viscometer. The second device for the measurement called the torsion ring surface viscometer, is based on the damping of the torsion pendulum. They have worked out the theory of the instruments and found a general agreement with experimental results. As a result of their investigations, they conclude that the viscosity of a surface film is much more dependent on the tightness of packing of the molecules in the film than upon the number of carbon atoms in the chain.

K. S. G. D.

Coal Conservation in India.—In a symposium on 'Coal Conservation in India' held in the early part of the year under the auspices of the Geological Mining and Metallurgical Society of India (Bull. No. 1, 1937) various aspects of coal industry have been discussed. Regarding the actual reserves of high grade coal suitable for metallurgical processes, a strong case has been made out, to get correct estimates of available coal to safeguard against possible shortage in future. Two important suggestions have been made—the formation of a 'Coal Issue Board' to control production and thus nationalise the industry,—the starting of a 'Fuel Research Board' for the proper utilisation of all grades of coal, the cost of maintaining these bodies being met by a small cess on coal mined annually. Several points regarding state legislation to prevent mine disasters, mine fires and underground labour have been discussed and valuable suggestions offered.

Fossils of the Lower Siwalik Beds.—In the first Yale North India Expedition a few fossils belonging to the Unionidae were identified in the lower Siwalik beds. In the course of a more recent visit to this area, the expedition has collected more fossil unionidae from the Chinji zone of the lower Siwaliks and some from the Tatrot and Dhok pathan zones. An account of these has recently been published by Prof. H. E. Vokes of Yale University (*Geol. Min. Met. Soc. Ind.*, Vol. 8, No. 4). The fossils are rather imperfectly preserved. Five new species have been identified, and described with illustrations.

Note on the Osteology of Palaeopropithecus.—Sometime back Prof. G. L. Sera of Naples published a monograph on the morphology of the extinct Lemur of Madagascar palaeopropithecus. Several inferences as to the primitive aquatic adaptations of Mammals were drawn

on the basis of this study. Alice Carleton criticises (*Proc. Zool. Soc. Lond.*, 1937, 107) some of the findings of Prof. Sera. The early workers like Standing and Grandidier, having studied the appendicular skeleton and fragments of skulls, distinguished forms like Megaladapsis, Bradylemur, Hadropithecus and the problematical Palaeopropithecus. Prof. Lambertson considerably added to our knowledge later. According to Carleton, the femur and tibia ascribed by Prof. Sera to Palaeopropithecus may rightly belong to Archeolemurinae and as regards the limb bones Prof. Sera discovered in the clavicle a powerful sternocleidomastoid and the lodgement in a hollow of the bone of an aerial sac. This has been now correctly identified as the fibula of Megaladapsis corresponding exactly to the description of Prof. Lambertson.

Maxillary Teeth and Blood Preferences of *Anopheles hyrcanus* var. *sinensis*.—The habits of the Chinese Anophiline, *A. hyrcanus* var. *sinensis* which appears to vary its food preferences according to the locality in which it is found were examined by T. L. Chang (*Lingnan Sci. Journ.*, 1937, 16, 435), with a view to determine the food preferences of this animal. The findings from precipitin reactions alone are not sufficient to distinguish the groups of anophelines in regard to their food habits. Examination of their maxillary indices should be carried on simultaneously. Two batches of *A. hyrcanus sinensis* collected from room-where examples of the various natural reservoirs of blood were assembled were examined and the results apparently show no correlation between food preferences and maxillary indices. A high maxillary index (as 17 in *A. maculipennis*) is said to indicate shortage of animal hosts and an overflow of such zoophilic anophelines into human habitations but in the present instance, though the maxillary index is as high as 17, the blood preferences appear to indicate that the mosquito is essentially zoophilic and only attacks man when severity of competition forces it to do so.

A New Boloceroidarian from Brackish Water near Madras.—N. Kesava Panikar (*Proc. Ind. Acad. Sci.*, 1937, B 5) describes in detail the morphology and asexual reproduction of *Boloceractis gopalai* n.g., n.sp. The mesenteries of the Actinian are not sharply differentiated into macrocnemes and microcnemes; the gonads are borne by the later cycles of imperfect mesenteries, the perfect ones being typically stellate. Asexual reproduction takes place in the anemone by a process of regeneration of deciduous tentacles, the sphinctered partitions at their bases playing an important rôle. The habits of the anemone are given and the systematics of the anemone are discussed in detail.