ON FOLIAR HELIX OF *COCOS NUCIFERA* L.—SOME GEOMETRICAL CONSIDERATIONS

JOSE JOSEPH and N. NEELAKANDAN
Department of Botany, University of Calicut, Calicut 673 635, India.

ABSTRACT

The paper deals with the initiation and development of leaves in coconut palm. Contrary to the well-known hypothesis regarding the five spiral leaf arrangement in coconut a single space curve or genetic spiral is proposed for the leaf initiation and development. This has to be so as morphological features as a response in terms of retinal images alone should not be taken as a criteria since morphogenesis of leaf and the leaf scars on the trunk are to be correlated with each other.

INTRODUCTION

WELL marked large leaf scars cover nearly the entire surface of the cylindrical trunk of a coconut tree. These scars appear to be disposed in a closely placed spiral. Davis and Mathai[1] have expressed that initiation and development of leaves in coconut palm are in spiral succession, maintaining that there exist five such spirals around the trunk, since coconut like other palms lack secondary growth. The trunk and leaf scars on it change little in size, shape and relative position and as such it is possible to measure accurately the linear as well as lateral displacement between the leaf scars and thus understand phyllotaxy. Since growth and enlargement of trunk at the shoot apex are nearly uniform the relative position of leaf scars on the mature trunk can be taken as a reliable indicator of leaf initiation at the shoot apex. The angle of divergence between successive leaf scars and the distance between them can be expressed mathematically to represent the three-dimensional phenomena which growth is, rather than a unidimensional spiral as suggested by Davis and Mathai[1].

MATERIALS AND METHODS

Twentyfive fully erect coconut trees of uniform age and growing under uniform conditions were selected. Five measurements were taken of each aspect, i.e. the distance between successive leaf scars (z) the angle in between (θ) and the circumferance (c) from each tree and the mean values presented. The leaf scars of coconut are crescent-shaped with a broader middle part and tapering ends (figure 1). The middle point on the convex basiscopic side of the scar was marked (A) and taken as the position of leaf. Using a water level a horizontal line is drawn on the trunk (A-C_1). The position of the leaf above is marked (C) similarly. A plumb line is dropped from this point and the point of intersection of vertical and horizontal lines is marked (C_1). Points A, C and C_1 are connected by straight lines to constitute a right-angled triangle. The angle theta (θ) CAC_1 was measured. The distance between C and C_1 and the circumference of the trunk were also recorded in each case.

RESULTS AND CONCLUSIONS

The results of measurements taken are given in table 1. When the right-angled triangle C_1AC is rotated along the surface of cylinder in the direction of its hypotenuse AC, the vertex A would generate a helix around the cylinder[2]. When a right-angled triangle CC_1A is marked on the surface of a right

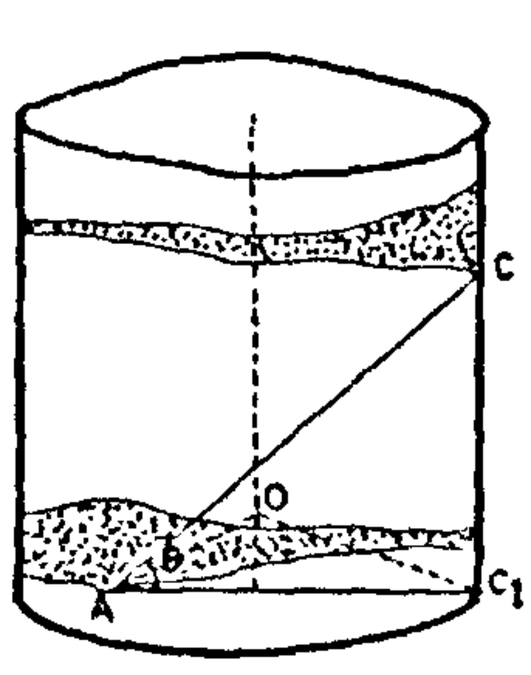

Figure 1. Trunk of coconut palm with leaf scars in relation to theta (θ) and divergence angle (t).

Table 1 *Comparison of actual and theoretically calculated values between successive leaf positions*

Circumference	σ	height	arc AP	$z=(AP\times\tan\theta)$
74 5	12	5 5	28 454	6.048
61 5	20	8 25	23.490	8.550
76 0	25	9 0	29 028	13.539
70 0	15	6.75	26.736	7.164
82.5	9	6.0	31 510	4.991
84 75	31	16 0	32.370	19.450
68 5	18	5.75	26.163	8.501
80 0	22	10 0	30.556	12.345
60 0	15	7.0	22.917	6.141
59 75	18	6 5	22.821	7.415
64.75	18	9 0	24.731	8 036
66.25	22	10.0	24.304	10.223
78 75	15	8 0	20 078	8 059
62 50	19	8.25	23.872	8.220
79.50	23	13 0	30.365	12 889
66 0	16	7.2	25.208	7.228
67.0	19	9 0	25.590	8 811
67 0	10	6.0	25 590	4 512
78.0	23	11.0	29.791	12 646
70 0	19	10.0	26.736	9.206
71.25	22	11.0	27.214	10.995
69 75	18	9.0	26 641	8 656
70 5	17	8 0	26.927	8 232
79 0	16	9 0	30.174	8.652
78 0	24	13 0	29.792	13 264

circular cylinder of radius a, whose axis coincides with z-axis (figure 2) in such a way the vertex A of the triangle lies at the point of intersection of the generator of the cylinder with x-axis, while the leg AC_1 lies in the xy-plane, then the hypotenuse AC if extended will generate on the cylinder a helix. If x, y and z denote the coordinate of the variable point C of the triangle and t denotes the angle AOC_1 (figure 2), then

$$x = a\cos t,$$

$$y = a\sin t, \text{ and}$$

$$z = CC_1 = AC_1\tan\theta,$$

where θ denotes the acute angle C_1AC of the triangle, AC_1 (arc AC_1)$=a\,t$, since AC_1 is an arc of the circle of radius a corresponding to the central angle t. Designating $\tan\theta$ in terms of C, the parametric equations of the helix $x=a\cos t$, $y=a\sin t$, $z=aC_1\,t$, can be derived. In the present study calculations were made only of z plane since this can be easily measured and compared with calculated theoretical value.

$$\text{radius} = \frac{\text{circumference}}{2\pi},$$

$$AOC_1 = t \text{ (divergence angle)}.$$

In palms the divergence angle of the successive leaves is approximately 137.5°. Since angular measurement on actual plant specimens or on camera lucida drawings, can never hope to give exact values. Divergence angle is taken as 137.5 by mathematical deduction for Fibonacci ratios converted to radian by equation

$$\frac{\pi\times 137.5}{180},$$

$$z = at\tan\theta,$$

$$= \frac{C}{2\pi}\times\frac{137.5\,\pi}{180}\times\tan\theta,$$

$$= \frac{137.5\,C\tan\theta}{360}.$$

Since growth and development of the coconut trunk is three-dimensional, space curve seems more appropriate to express leaf initiation and development rather than a unidimensional spiral[1]. Thus the equation for exponential spiral $r=\exp(a\theta)$ does not seem to correctly depict sequence of leaf initiation as currently maintained[1] since it does not take into account angle θ. Measurements of height between successive leaf positions recorded in the present study correspond closely with the theoretical values derived using the formulas proposed in the present study. A paired two tailed test was carried out and the results are found to be highly significant.

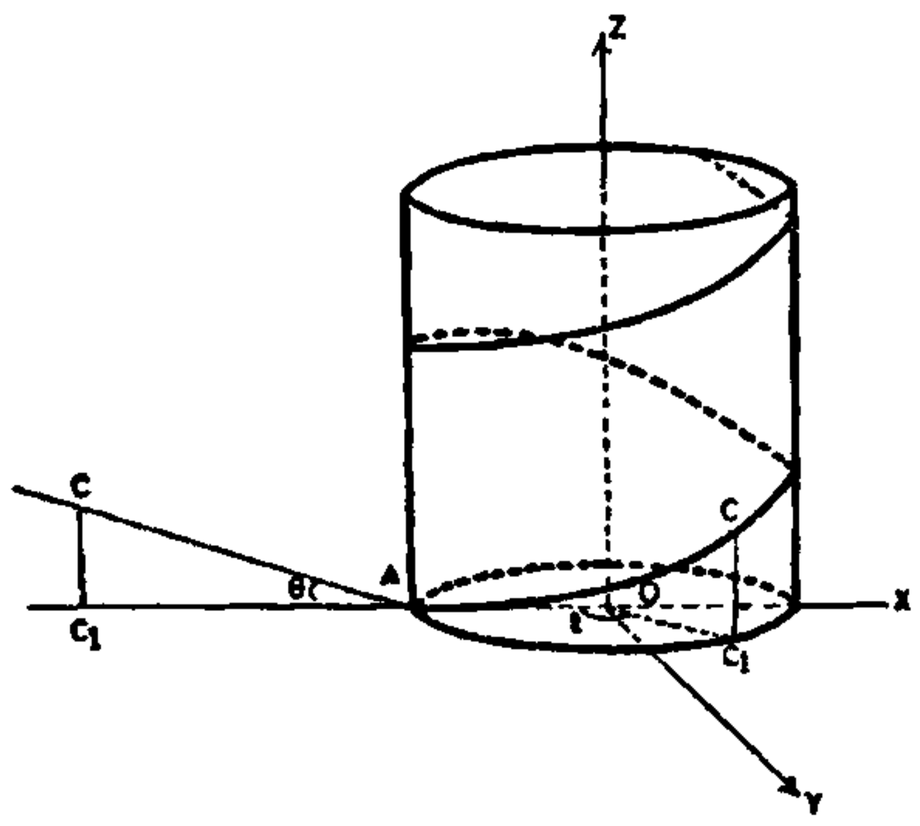

Figure 2. A right circular cylinder showing a helix wound on it. Theta (θ) and divergence angle (t) are represented in three-dimensional planes.

Following the concept put forward by Plantefol[3] it is often maintained that there are five phyllotaxic spirals in coconut[1,4,5]. The youngest leaf primordia of a helix series determines the foliar generative centre of the next leaf primordia of that helix. In coconut, leaf bases near the growing apex completely encircle the apex. As such foliar generative centres are to be conceived as induced across several concentric leaf bases if Plantefol's hypothesis is accepted. Leaf arrangement being a function of the growing point of the stem it is more appropriate to consider what leaves are arranged in a space curve as proposed here. Such a helix would correspond to genetic spiral also. Viewed from above the shoot apex of coconut appears to bear leaves in five spirals but this is due to perceptual grouping on account of proximity. These illusory spirals do not extend up to the Annuae initial' (figure 3) as they should have according to Plantefol's hypothesis. If perceptual grouping is to be depended upon the arrangement of leaves at the apex of the coconut palm can either be regarded as consisting of five clockwise spirals or 8 anti-clockwise spirals (figure 4) both of which would

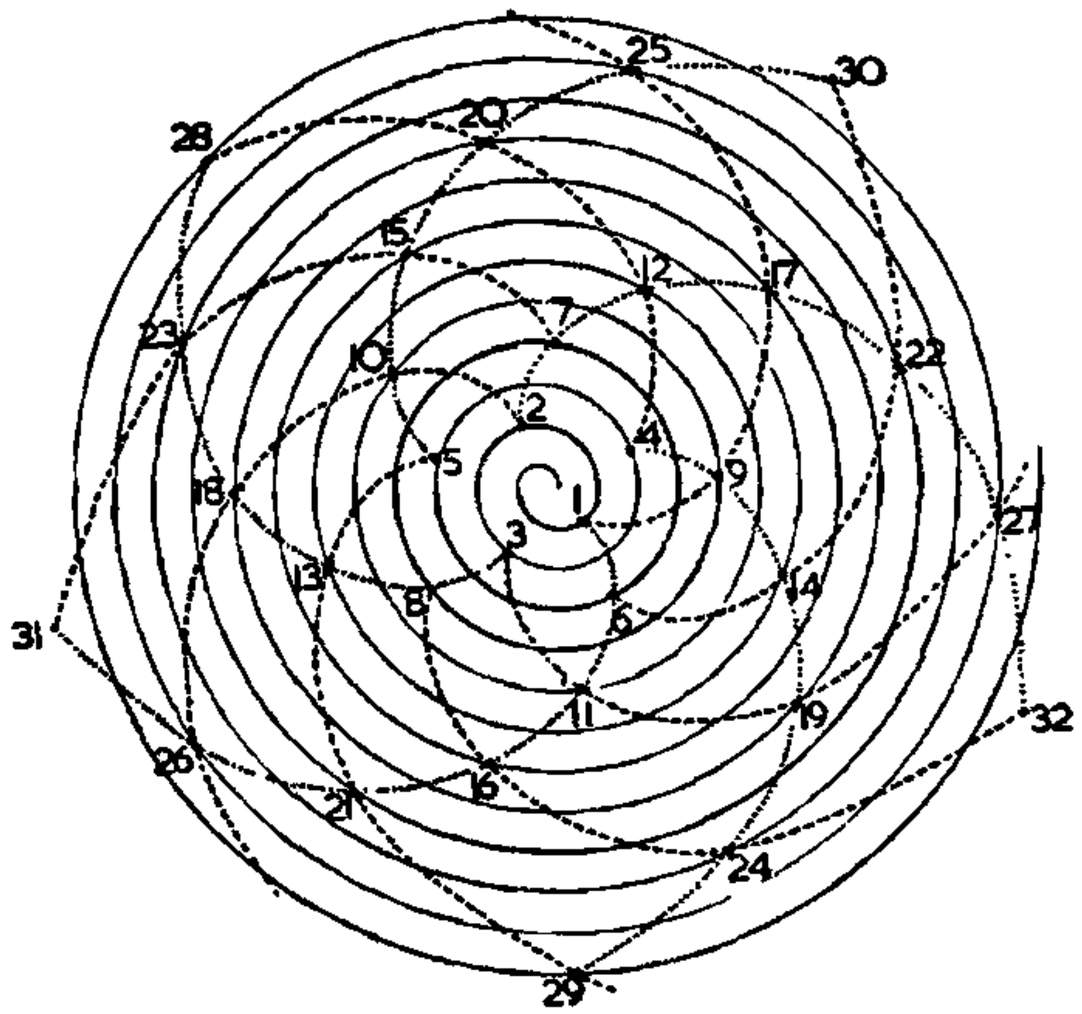

Figure 4. Almost the same configuration as figure 3. The clockwise winding geometric spiral and spirals on anticlockwise 8 spirals system also can be perceived through dotted lines.

be incorrect from the point of view of leaf initiation. On the other hand a single space curve as proposed here would explain leaf initiation and development in the coconut palm.

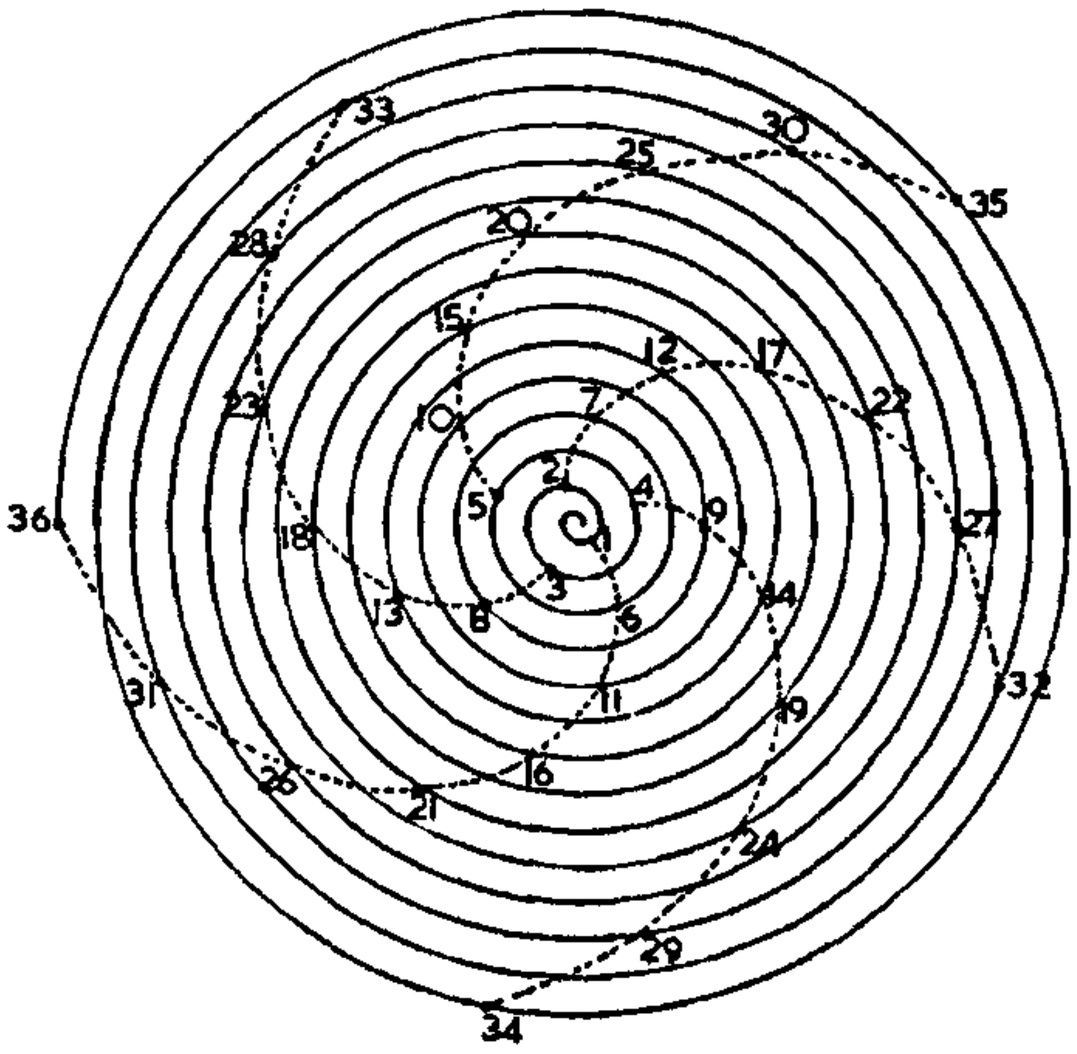

Figure 3. Geometric spiral showing how five separate spirals can be perceived on it due to proximity of leaves. But see all the five are not originating from the central point.

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