ON THE BEHAVIOUR OF VELOCITY PROFILE IN HYDROMAGNETIC FLOWS

S. K. RITOLIA and B. P. SINGH
Department of Mathematics, Banaras Hindu University, Varanasi 221 005, India.

ABSTRACT

Equilibria of a finitely conducting fluid are shown to exist in systems with cylindrical topology for which constant Bernoulli surface has an elliptic transverse cross-section with the eccentricity given by an arbitrary function of the axial distance. A change in the parameters characterizing the solution leads to a generalized class of hyperbolic equilibria for which the constant Bernoulli surface has an hyperbolic transverse section.

INTRODUCTION

Exploitation of geometric theories of curves and surfaces in fluid flow theories could furnish information of the flow structures. Suryanarayan[1] showed that Bernoulli's surface exists in the case of steady incompressible hydromagnetic flows when the magnetic field is in the fixed direction. It seems that the Bernoulli surface plays a key role in the investigation of the geometrical aspects of such flows. Several workers[2-10] have also tried to study the geometrical properties of hydromagnetic flows without giving due importance to the existence of Bernoulli surface.

The present investigation of geometrical properties of hydromagnetic flows considers Bernoulli surface following Suryanarayan[1]. For simplicity, we consider a steady, incompressible, infinitely conducting hydromagnetic flows and discuss the behaviour of velocity profile. Although the scope of the present paper is limited, the method can be applied to more general cases.

BASIC EQUATIONS

The basic equations governing the steady flow of finitely conducting fluid when the magnetic field is in the fixed direction is characterized by[1]

$$\operatorname{div}\bar{v} = 0 \tag{1}$$

$$\bar{w}\times\bar{v} = -\nabla(p+\tfrac{1}{2}\rho v^2+\tfrac{1}{2}\mu H^2) \tag{2}$$

$$\operatorname{curl}\bar{v} = \bar{w}, \tag{3}$$

where p is the pressure of the fluid, $P = p+\frac{1}{2}\mu H^2$ is the magnetic pressure, $\bar{v}$ the velocity vector, $\bar{w}$ the vorticity vector and $B(= P+\frac{1}{2}\rho v^2)$ the Bernoulli function. Equation (2) is useful for investigating the nature of the field of flow in specific class of MFD flows.

ANALYSIS

In a system of rectangular cartesian coordinates (x,y,z), let us prescribe the velocity field as

$$v_x = \phi_1[yf(z)],\quad v_y = \phi_2[xh(z)],\quad v_z = 0, \tag{4}$$

which identically satisfies (1). Here $f(z)$ and $h(z)$ are arbitrary functions of z. Using (4), equation (3) gives the vorticity[11]

$$w_x = -\phi_2' xh',\quad w_y = \phi_1' yf',$$
$$w_z = \phi_2' h - \phi_1' f, \tag{5}$$

where primes denote differentiation with respect to the argument. Using (4), equations (2) and (3) give

$$P/\rho = B(x,y)-\tfrac{1}{2}(\phi_1^2+\phi_2^2). \tag{6}$$

Applying (4), (5) and (6) in (2), we have

$$\frac{\partial B}{\partial x} = -\phi_2\phi_1' f,\quad \frac{\partial B}{\partial y} = \phi_1\phi_2' h, \tag{7}$$

from which, it is clear that one requires

$$\phi_1[yf(z)]\phi_2[xh(z)] = \tau(x,y). \tag{8}$$

From (7) the integrability condition for the existence of $B(x,y)$ is

$$\phi_1'' f^2/\phi_1 = \phi_2'' h^2/\phi_2 = k(z), \tag{9}$$

where $k(z)$ is an arbitrary function of z.

Let us differentiate the first equation in (9) with respect to y and z, and we get

$$f^2\phi_1'' = k\phi_1' \tag{10}$$

$$2ff'\phi_1''+yf^2\phi_1''' = kyf'\phi_1'+k'\phi_1. \tag{11}$$

Using (10), equation (11) becomes

$$2ff'\phi_1'' = k'\phi_1, \tag{12}$$

which is consistent with the first equation in (12). Similar deduction follows for the second equation in (9).

There exists two distinct classes of the equations according as $k \neq 0$ or $k = 0$. For $k \neq 0$, from (7) and (9), we have

$$P/\rho = -\phi_2'\phi_1' fh/k + \text{constant}. \qquad (13)$$

Using (13), equation (6) becomes

$$B = B_0 - \phi_2'\phi_1' fh/k + \tfrac{1}{2}(\phi_1^2 + \phi_2^2), \quad k \neq 0, \qquad (14)$$

B_0 being an arbitrary constant. For $k = 0$, (9) gives

$$\phi_1 = Gyf(z), \quad \phi_2 = Hxh(z), \qquad (15)$$

where G and H are arbitrary constants. From (15) and (8), we have

$$[f(z)h(z)]^{\frac{1}{2}} = \text{constant}. \qquad (16)$$

First let us choose this constant to be imaginary, say iC., where C is real.

Using (7), (15) and (16), equation (6) becomes

$$B = B_0 + \frac{1}{2}\left[\left(\frac{C^2H}{Gf^2} + 1\right)\phi_1^2 + \left(\frac{C^2G}{Hh^2} + 1\right)\phi_2^2\right]. \qquad (17)$$

For $G = H = 1$, (17) becomes

$$B = B_0 + \frac{1}{2}C^2[1 + \alpha^2(z)]\left[\frac{x^2}{\alpha^2(z)} + y^2\right], \qquad (18)$$

where $\alpha(z) = f(z)/C$.

The intersection of the constant Bernoulli surface with a transverse plane $z =$ constant gives a family of nested ellipses.

$$(x^2/a^2) + (y^2/b^2) = 1. \qquad (19)$$

Within the bounding ellipse corresponding to $B = 0$. Here

$$a^2(z) = \frac{2(B - B_0)\alpha^2(z)}{C^2[1 + \alpha^2(z)]}, \quad b^2(z) = \frac{2(B - B_0)}{C^2[1 + \alpha^2(z)]}. \qquad (20)$$

The eccentricity σ of the loci $B =$ constant, in a given plane $z =$ constant is given by

$$\sigma(z) = \frac{b^2 - a^2}{b^2 + a^2} = \frac{1 - \alpha^2(z)}{1 + \alpha^2(z)}. \qquad (21)$$

Choosing the constant in (16) to be real, say A, (6) becomes

$$B = B_0 - \frac{1}{2}\left[\left(\frac{A^2H}{Gf^2} - 1\right)\phi_1^2 + \left(\frac{A^2G}{Hh^2} - 1\right)\phi_2^2\right]. \qquad (22)$$

Choosing $H = k_1G$, where k_1 is a positive constant, (22) reduces to

$$B = B_0 + \frac{1}{2}k_1G^2A^2[\beta^2(z - 1]\left[y^2 - \frac{x^2}{\beta^2(z)}\right] \qquad (23)$$

where $\beta(z) = f(z)/A\sqrt{k}$. The intersection of the constant Bernoulli surface with a transverse plane $z =$ constant gives a family of hyperbolae

$$(y^2/b^2) - (x^2/a^2) = 1, \qquad (24)$$

where

$$a^2(z) = \frac{2(B - B_0)\beta^2(z)}{k_1G^2A^2[\beta^2(z) - 1]},$$

$$b^2(z) = \frac{2(B - B_0)}{k_1G^2A^2[\beta^2(z) - 1]}.$$

The eccentricity ν of the loci $B =$ constant in a given plane $Z =$ constant is given by

$$\nu(z) = \frac{b^2 + a^2}{b^2 - a^2} = \frac{1 + \beta^2(z)}{1 - \beta^2(z)}. \qquad (25)$$

Now we consider the case $f(z) = h(z) = 1$ and then (5) gives

$$v_x = Gy, \quad Hv_y = kGx. \qquad (26)$$

Equation (26) represents a streamline configuration near a x type velocity neutral point that is relevant for the streamline reconnection. For this case, we have

$$w_x = w_y = 0, \quad w_z = G(k - 1),$$

so that it is necessary to have $k \neq 1$ in order to allow the streamline reconnection to take place.

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ANNOUNCEMENTS

INTERNATIONAL COLLOQUIUM ON MICROBIOLOGY IN POECILOTHERMS

The colloquium will be held at the CNRS, 15 Qui Anatole, France during 10–12th July 1989.

The aim of the colloquium is to resume the themes analysed in EXTER, enlarging them to the totality of poecilotherms and to the aquatic environment.

The program involves studies carried out in the laboratory, in natural environment as well as modelling tests: Microbial ecology: host-bacteria relationships; symbiosis, saprophytism; trophic role of the environmental flora . . . . ; microbial metabolism: digestion of complex glucids; production of growth factors; other specific metabolisms; participation of saprophytic bacteria to pathological syndromes; intestinal microorganisms housed by poecilotherms. This list of proposed topics is not restrictive and can include all the aspects concerning bacteria and animal-host relationships.

There will be plenary sessions with oral and poster presentations. Each session will be opened by an invited keynote speaker. Papers will be reviewed, discussed and edited by a selected editorial board. There will also be an opportunity for workshop to discuss specific problems. French and English will be the official languages of the International Colloquium.

Further particulars may be had from the Secretariat: CNERNA, Mme A. Hilaire, Microbiology and Poecilotherms, 75007 PARIS (France).

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