NEW FACETS OF THE EINSTEIN-PODOLSKY-ROSEN PARADOX FROM ELEMENTARY PARTICLE PHYSICS

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ABSTRACT

It is pointed out that the $K^\circ \bar{K}^\circ$ and $B^\circ \bar{B}^\circ$ systems provide a new arena for exploring the intricacies of the Einstein-Podolsky-Rosen (EPR) type example. This also provides clues for experimental discrimination between quantum mechanics and local realism. Consideration of the effect of CP non-invariance in the context of such systems leads to the intriguing possibility, at least in principle, of an incompatibility between quantum mechanics and Einstein's locality condition at the statistical level. This calls for further investigations to clarify the subtleties involved.

1. INTRODUCTION

The peculiarities of non-local quantum correlation between space-like separated systems were highlighted by Einstein, Podolsky and Rosen (EPR) in their seminal paper[1] which evoked animated deliberations over the past 50 years. The well-known illustration of the EPR argument is given by Bohm's example[2] of a spin-zero system decaying into two spin-$\frac{1}{2}$ particles.

Apart from the epistemological implications, the EPR debate has stimulated studies on viable experimental tests of the non-separability of the two-particle quantum wavefunction and its incompatibility with various local realist models*. The photon-polarization correlation measurements using radiative atomic cascade transitions by Aspect *et al*[3] mark an important effort in this direction. However, the interpretation of these experimental results was subjected to vigorous controversy[4] due to the low efficiency of the photomultiplier detectors and uncertainties in the estimation of the background accidental coincidence counting rates. Specific examples of local realist models[5] have been proposed which reproduce the pertinent data of the atomic-cascade experiments equally well as quantum mechanics. One is, therefore, inclined to feel that the final verdict on the question of local realism has not yet been spelt.

A significant recent trend[6] has been to explore the possibility of new types of examples related to the EPR-type situation, which may be amenable to experimental studies concerning the incompatibility between quantum mechanics and local realism. One such interesting example is provided in the arena of particle physics by the decay of a $J^{PC} = 1^{--}$ vector meson into a pair of neutral pseudoscalar mesons. Lee and Yang[7] were the first to point out the EPR-type features for this case (pertaining to the pair of kaons K°–$\bar{K}^\circ$ resulting from the $p\bar{p}$ annihilation), followed by D'Espagnat[8], Six[9] and Selleri[10]

Here we seek to provide a critical analysis of the present status of this example and indicate possibilities of future investigations. We begin with a resume of the preliminary details of this example, considering specifically decay of the spin-1$\Phi(1020)$ resonance by strong interaction into a pair of neutral kaons K°–$\bar{K}^\circ$.

2. EPR-TYPE SITUATION FOR THE K°–$\bar{K}^\circ$ SYSTEM

Invoking charge conjugation invariance of the strong interaction, the wavefunction of the

*To put it concisely, 'local realism' implies the following notion: Individual measuremental results pertaining to the physical properties of a given system are assumed to be independent of the measurements performed on a spatially separated system with whom the given system may have interacted in the past but at present they are non-interacting.

K°–$\bar{K}^\circ$ pair at the time of production ($t = 0$) from the decay of the $J^{PC} = 1^{--}$ state is given by

$$|\Psi_0\rangle = [|K^\circ\rangle_L|\bar{K}^\circ\rangle_R - |\bar{K}^\circ\rangle_L|K^\circ\rangle_R]/\sqrt{2}, \quad (1)$$

where $L(R)$ refers to the left (right) hemisphere.

The subsequent time development of the K°–$\bar{K}^\circ$ pair is described in terms of the eigenstates of the effective Hamiltonian which includes weak interactions. For the situation under consideration, the weak interactions induce decays of both K°, $\bar{K}^\circ$ and also give rise to K°–$\bar{K}^\circ$ transitions.* The effective Hamiltonian is written as $H = M - i\Gamma/2$ where M and Γ are the hermitian mass and decay matrices respectively. The eigenstates of this H are $|K_L\rangle$, $|K_S\rangle$ with eigenvalues $\lambda_L = m_L - i\gamma_L/2$ and $\lambda_S = m_S - i\gamma_S/2$ respectively, where $m_L(m_S)$ and $\gamma_L(\gamma_S)$ are respectively the mass and the decay width of $|K_L\rangle$ ($|K_S\rangle$); $m_L - m_S = 0.53\times 10^{10}\,\hbar\,s^{-1}$ and $\gamma_S = 582\gamma_L = 1.12\times 10^{10}\,\hbar\,s^{-1}$. Throughout this section we assume CP invariance; the implications of CP violation in this type of example will be treated later in §4.

$$|K_L\rangle = [|K^\circ\rangle + |\bar{K}^\circ\rangle]/\sqrt{2}, \text{ and } |K_S\rangle = [|K^\circ\rangle - |\bar{K}^\circ\rangle]/\sqrt{2}.$$

They time evolve as

$$U(t, O)|K_L\rangle = |K_L\rangle \exp(-i\lambda_L t) + |\phi_L(t)\rangle, \quad (2)$$

and a similar equation for $|K_S\rangle$. Here $|\phi_L(t)\rangle$ ($|\phi_S(t)\rangle$) represents the decay products from $|K_L\rangle$ ($|K_S\rangle$); $|\phi_L\rangle$ ($|\phi_S\rangle$) is taken orthogonal to the states $|K_L\rangle$ and $|K_S\rangle$. CP invariance requires $\langle K_L|K_S\rangle = 0$.

In terms of the states $|K_L\rangle$, $|K_S\rangle$ the wavefunction $|\Psi_0\rangle$ given by (1) can be written as

$$|\Psi_0\rangle = [|K_S\rangle_L|K_L\rangle_R - |K_L\rangle_L|K_S\rangle_R]/\sqrt{2}. \quad (3)$$

Time evolution of the non-separable form of the two-particle wavefunction $|\Psi_0\rangle$ given by (1) and (3) correlates the oscillations between $|K^\circ\rangle$, $|\bar{K}^\circ\rangle$ states such that it carries the essence of non-local correlation, reminiscent of the EPR-type situation. If the left (right) kaon is observed to be a K° (strangeness $S = +1$) at a certain instant then the right (left) kaon can be predicted with certainty to be observed as a $\bar{K}^\circ$ ($S = -1$) at that same instant. Alternatively, if the left (right) kaon decays in the K_S mode (CP $= +1$) then the right (left) kaon is bound to decay as a K_L (CP $= -1$) at any future instant. It is to be noted that there is a subtle distinction between the K°–$\bar{K}^\circ$ and K_L–K_S correlations; while the former holds only for equal proper times, the latter is a time-independent consequence of the non-separable form of the wavefunction. This aspect has been clearly discussed by Selleri[10].

Six[9] suggested an experimental test of this EPR-type situation by measuring the joint probability $P_{--}(t_1, t_2)$ of a double $\bar{K}^\circ$ observation on two sides at times t_1 and t_2 on the left and right respectively. The quantum-mechanical prediction for $P_{--}(t_1, t_2)$ is given by

$$P_{--}(t_1, t_2) = |\langle\bar{K}^\circ_L\,\bar{K}^\circ_R|\Psi(t_1, t_2)\rangle|^2$$

where $|\Psi(t_1, t_2)\rangle$ is the state evolved from $|\Psi_0\rangle$ at $t = 0$:

$$|\Psi(t_1, t_2)\rangle = (1/\sqrt{2})\,\{|K_S\rangle_L|K_L\rangle_R \exp - i(\lambda_S t_1 + \lambda_L t_2) - |K_L\rangle_L|K_S\rangle_R \exp - i(\lambda_L t_1 + \lambda_S t_2)\}, \quad (4)$$

whence one obtains

$$P_{--}(t_1, t_2) = (1/8)\,\{\exp - (\gamma_S t_1 + \gamma_L t_2) + \exp - (\gamma_L t_1 + \gamma_S t_2) - 2\exp(-\gamma(t_1 + t_2)) \cos \Delta m(t_1 - t_2)\}, \quad (5)$$

where $\gamma = (\gamma_L + \gamma_S)/2$ and $\Delta m = m_L - m_S$.

Selleri derived an upper bound on $P_{--}(t_1, t_2)$ for the K°–$\bar{K}^\circ$ system ($P^u_{--}(t_1, t_2)$) using a general argument based on the notion of local realism:

$$P^u_{--}(t_1, t_2) = (1/8)\,\{\exp - (\gamma_S t_1 + \gamma_L t_2) + \exp - (\gamma_L t_1 + \gamma_S t_2)\}. \quad (6)$$

*A pure $|K^\circ\rangle$ ($|\bar{K}^\circ\rangle$) state evolving in time becomes a superposition of $|K^\circ\rangle$, $|\bar{K}^\circ\rangle$, and decay products. This gives rise to K°–$\bar{K}^\circ$ oscillations.

This local realist upper bound differs from the quantum-mechanical prediction (5) by the absence of the interference term. Quantum mechanics, therefore, leads to a prediction violating (6) whenever the interference term is positive, that is, whenever $\cos \Delta m(t_1 - t_2) < 0$. The maximum possible discrepancy is seen to be about 12% for $\gamma_S(t_2 - t_1) \simeq 5$. Some experimentalists* had shown interest in checking this case of incompatibility between quantum mechanics and local realism, but nothing significant on the experimental side has been yet reported.

It needs to be noted that the experimental study envisaged in the context of (5) and (6) has an intrinsic handicap that for meaningful results, t_1, t_2 must be shorter than the life-time of K_L, K_S; i.e. one requires $t_1, t_2 \lesssim 10^{-10}\ s^{-1}$. The uncertainties involved in ensuring observations at the specified instants t_1, t_2 will be quite appreciable within such small time interval. Such a difficulty may be circumvented by considering the time-integrated joint probabilities. This aspect has been recently probed[11] in the context of the B°–$\bar{B}^\circ$ system which is exactly similar to the K°–$\bar{K}^\circ$ system, with the only difference that $\gamma_L = \gamma_S (= \gamma)$** for the states of the B°–$\bar{B}^\circ$ system analogous to the $|K_L\rangle$, $|K_S\rangle$ states, which are denoted by $|B_H\rangle$, $|B_L\rangle$ corresponding to the masses m_H, m_L respectively $(m_H > m_L)$.

3. PROPOSAL FOR A NEW TEST USING THE B°–$\bar{B}^\circ$ SYSTEM

Of late, experimental studies concerning the decay of the spin-1 $\Upsilon(4s)$ vector meson into a pair of neutral pseudo-scalar mesons B°–$\bar{B}^\circ$ have attracted considerable attention in the context of the search for the evidence of B°–$\bar{B}^\circ$ mixing[12]. In this section, following the treatment by Datta and Home[11], we analyse the possibility of experimentally investigating the EPR-type quantum non-local correlations within the framework of the ongoing experiments on $\Upsilon(4s) \to B^\circ \bar{B}^\circ$. We focus our attention on the time-integrated joint probabilities remembering that B° and $\bar{B}^\circ$ can be identified by their characteristic semi-leptonic mode of decays: $B^\circ \to \ell^- \bar{\nu} X$; $\bar{B}^\circ \to \ell^+ \nu X$ where ℓ and X denote lepton and hadron respectively.

The present experimental arrangement to study $\Upsilon(4s) \to B^\circ \bar{B}^\circ$ is geared to measure the parameter R defined as follows:

$$R = N_{++} + N_{--}/N_{+-} + N_{-+}, \qquad (7)$$

where N_{++} = total number of double $\bar{B}^\circ$ decays (corresponding to the observation of double ℓ^+ decay products on both sides); N_{--} = total number of double B° decays (corresponding to the observation of double ℓ^- decay products on both sides); N_{+-} = total number of $\bar{B}^\circ$ decays on the left associated with B° decays on the right (corresponding to the observation of ℓ^+ decay products on the left associated with ℓ^- decay products on the right); N_{-+} = total number of B° decays on the left associated with $\bar{B}^\circ$ decays on the right (corresponding to the observation of ℓ^- decay products on the left associated with ℓ^+ decay products on the right). The parameter R is calculated by evaluating the quantities $N_{ij} (i, j = \pm)$. The general expression for N_{ij} is given by

$$N_{ij} = 2N_0 \lambda^2 \int_0^\infty dt_1 \int_{t_1}^\infty dt_2\, P_{ij}(t_1, t_2), \quad (8)$$

where $P_{ij}(t_1, t_2)$ is the joint probability for observing the decay products ℓ^i, ℓ^j on two sides at times t_1 and t_2 respectively, N_0 is the total number of $\Upsilon(4s)$ decays, and λ = the semi-leptonic decay width of B° decaying into a ℓ^- = the semi-leptonic decay width of $\bar{B}^\circ$ decaying into a ℓ^+. The quantum mechanical expressions for $P_{ij}(t_1, t_2)$ are given by (derived

*Such as O. Piccioni (University of California, San Diego), D. Jovanovic (Fermi Laboratory, Batavia), and S. Zenone (Concordia University, Montreal). Some relevant experimental data on the two-kaon state given by the wavefunction (1) were presented by R. Armenteros *et al* at the 1962 International Conference on High Energy Physics, CERN, Geneva; but they were not carefully analysed in the context of the EPR problem.

**$|K_L\rangle$ has a longer lifetime compared to $|K_S\rangle$ because the phase space available in its principal decay mode $|K_L\rangle \to 3\pi$ is smaller than that available in the decay mode $|K_S\rangle \to 2\pi$. For the decay of the $|B_H\rangle$, $|B_L\rangle$ states, the phase space available is more-or-less the same. Hence their decay widths are taken to be identical.

from the non-separable form of the wavefunction (4)).

$$P_{++}(t_1,t_2) = P_{--}(t_1,t_2) = \{2\exp[-\gamma(t_1+t_2)] - 2\exp[-\gamma(t_1+t_2)] \cos\Delta m(t_2-t_1)\}/8, \quad (9)$$

$$P_{+-}(t_1,t_2) = P_{-+}(t_1,t_2) = \{2\exp[-\gamma(t_1+t_2)] + 2\exp[-\gamma(t_1+t_2)] \cos\Delta m(t_2-t_1)\}/8, \quad (10)$$

where $\Delta m = m_H - m_L$. Using (9) and (10) we obtain from (8) the following quantum mechanical values for N_{ij}:

$$N_{++} = N_{--} = (N_0\lambda^2)\,[(1/4\gamma^2)-(1/4\alpha^2)] \quad (11)$$

$$N_{+-} = N_{-+} = (N_0\lambda^2)\,[(1/4\gamma^2)+(1/4\alpha^2)], \quad (12)$$

where $\alpha^2 = \gamma^2 + (\Delta m)^2$. This leads to the following quantum mechanical prediction[13] for the parameter R defined by (7):

$$R_{QM} = x^2/(2+x^2), \quad (13)$$

where $x = \Delta m/\gamma$. The result given by (13) hinges on the quantum non-separability inbuilt into the wavefunction (1), assumed to be maintained even after the particles get well-separated in space. The experimental verification of (13) will, therefore, constitutes an interesting test for the quantum non-separability in this EPR-type situation.

In this connection it should be instructive to compare (13) with the corresponding prediction derived from the notion of local realism. As an example, let us consider Furry's hypothesis[14] in the following form: The wavefunction has the non-separable form (1) at the production of the $B^\circ\bar{B}^\circ$ pair, but after spatial separation between the two particles the wavefunction becomes an equal mixture (not superposition) of the two independent states $|B_H\rangle_L|B_L\rangle_R$ and $|B_L\rangle_L|B_H\rangle_R$.

One is tempted to envisage this hypothesis because it enables one to avoid the conceptual anomalies arising from the quantum non-separability presented by the EPR paradox. Einstein[15] himself favoured such a proposal. Bohm and Aharonov[16] analysed the tenability of Furry's hypothesis and pointed out the significance of testing whether this hypothesis leads to any conflict with the available experimental results.

In the present EPR-type example of the $B^\circ\bar{B}^\circ$ system, applying Furry's hypothesis to evaluate the general formula given by (8) we obtain:

$$P_{++}(t_1,t_2) = P_{--}(t_1,t_2) = P_{+-}(t_1,t_2) = P_{-+}(t_1,t_2) = (1/4)\exp[-\Gamma(t_1+t_2)],$$

whence,

$$N_{++} = N_{--} = N_{+-} = N_{-+} = (N\lambda^2)\,(1/4\Gamma^2), \quad (14)$$

which leads to the following prediction for the parameter R according to Furry's hypothesis:

$$R_F = 1. \quad (15)$$

Comparing (15) with (13) we observe that within the present experimental framework for the study of $\Upsilon(4s) \rightarrow B^\circ\bar{B}^\circ$ it is possible to discriminate between the predicted values of R_{QM} and R_F, unless $x^2 \gg 1$ (the case of maximal $B^\circ\bar{B}^\circ$ mixing corresponding to $\Delta m \gg \gamma$). In this context, it is interesting to note that the CLEO group has already furnished[17] an experimental upper bound on R given by $R < 0.3$. This may appear to rule out Furry's hypothesis. However, a word of caution is necessary. This upper bound involves certain theoretical model-dependent inputs (such as the use of the spectator model of mesonic decay). Nevertheless, with better statistics, it should be possible to set the empirical upper bound in a model-independent way.*

*Experimental studies reported at the XXIII International Conference on High Energy Physics (Berkeley, USA, 16–23 July, 1986) indicated the upper bound on R given by $R < 0.12$. It has been also observed that the pertinent empirical data can accommodate $R = 1$ only if a large departure (> 50%) from the spectator model predictions

As regards the quantum-mechanical prediction for R given by (13), we note that R_{QM} is model-dependent. Confining our attention within the ambit of the Glashow-Weinberg-Salam standard model of electro-weak interactions, we observe the following. There are two types of B° mesons: B°_d($b\bar{d}$ quark-antiquark bound state) and B°_s($b\bar{s}$ quark-antiquark bound state). $\Upsilon(4s)$ decays into $B^\circ_d \bar{B}^\circ_d$ system only ($B^\circ_s \bar{B}^\circ_s$ channel is forbidden by the kinematic considerations). For this case, the standard model predicts $\Delta m/\Gamma \leqslant 0.2\beta$ where β is estimated[18] to be within the range 0.33–1.5. It, therefore, follows that $R_{QM} \ll 1$ according to the standard model, suggesting that the experimental distinction between R_F and R_{QM} based on the standard model should be quite feasible.

To indicate further work along this direction we wish to point out that, apart from calculating the parameter R using the various local realist models (analogous to the types[5] used for analysing the EPR atomic-cascade experiments), it seems important to derive general bounds on R from local realism independent of the details of any particular model. This would enable decisive use of the ongoing experimental studies on R to test the notion of local realism against quantum mechanics. Such tests would constitute a valuable complement to the current EPR experiments. They would, incidentally, be also the first EPR test involving electro-weak interactions.

It is relevant to recall here that Selleri[10] derived an upper bound on $P_{--}(t_1, t_2)$ for the $K^\circ \bar{K}^\circ$ system using a general argument based on local realism; interestingly, this bound coincides with the value obtained from Furry's hypothesis (mere coincidence!). It needs to be explored whether Selleri s treatment can be extended for the $B^\circ \bar{B}^\circ$ example to obtain general local realist bounds on the parameter R Then the empirical probing can be concentrated on the domains of incompatibility between such bounds and the quantum mechanical prediction for R. This possibility is presently under study. It is also being investigated whether Bell-type inequality from the notion of local realism, involving the experimentally observable time-separated joint probabilities, can be formulated for the $K^\circ \bar{K}^\circ$ or $B^\circ \bar{B}^\circ$-type example of the EPR paradox.

for the B-mesonic decay is allowed (A. Bean *et al* CLEO-86-11, 1986). Other experimental results, such as the limits on B^+, B° meson lifetime, are consistent with the spectator model predictions and theoretical arguments suggest that the deviations from the spectator model, if any, should not be more than 30% [I. I. Bigi, *Phys. Lett.*, 1986, B169, 101]. The most recent comprehensive experimental investigation on this issue has provided evidence for substantial $B^\circ - \bar{B}^\circ$ mixing and the value of R is claimed to be 0.21 ± 0.08 [H. Albrecht *et al Phys. Lett.*, 1987, B192, 245].

4. QUANTUM NON-LOCALITY AND CP NONCONSERVATION: A CURIOUS GEDANKEN EXAMPLE

Recently Datta *et al*[19] (DHR) probed the effect of CP violation in the EPR-type gedanken example, considering a generalized situation of the K°–$\bar{K}^\circ$ or B°–$\bar{B}^\circ$ type systems. A crucial feature introduced by CP non-invariance is that the eigenstates of the effective weak interaction Hamiltonian, which exhibit exponential decay, are not mutually orthogonal. DHR argue that this aspect leads to an intriguing incompatibility of quantum mechanics with Einstein's locality condition at the statistical level, at least in the gedanken formulation. In the present section, we analyse the details of the DHR example.

Let us consider that a vector meson V with $J^{PC} = 1^{--}$ decays by strong interaction into a pair of spatially separated neutral pseudo-scalar mesons (one of which is the antiparticle of the other) denoted by M°–$\bar{M}^\circ$; typical examples being the decay of the $\Phi(1020)$ resonance into K°–$\bar{K}^\circ$ or $\Upsilon(4s) \to B^\circ \bar{B}^\circ$. Our analysis will be within the framework of the formalism outlined in the beginning of §2. Analogous to (1) and (2) respectively we now have:

$$|\Psi_0\rangle = [|M^\circ\rangle_L |\bar{M}^\circ\rangle_R - |\bar{M}^\circ\rangle_L |M^\circ\rangle_R]/\sqrt{2}, \quad (16)$$

$$U(t,0)|M_L\rangle = |M_L\rangle \exp(-i\lambda_L t) + |\phi_L(t)\rangle, \quad (17)$$

and a similar equation for $|M_S\rangle$. Taking into account CP non-invariance,

$$|M_L\rangle = N[(1+\varepsilon)|M^\circ\rangle + (1-\varepsilon)|\bar{M}^\circ\rangle],$$
$$|M_S\rangle = N[(1+\varepsilon)|M^\circ\rangle - (1-\varepsilon)|\bar{M}^\circ\rangle],$$

where the normalization factor $N = [2(1+|\varepsilon|^2)]^{-1/2}$ and the parameter ε is a measure of CP violation. CP non-invariance requires $|M_L\rangle$ and $|M_S\rangle$ to be mutually non-orthogonal; $\langle M_L|M_S\rangle = 4N^2$ Re ε.

Unitarity of $U(t,0)$ implies

$$\langle\phi_L(t)|\phi_L(t)\rangle = 1-\exp(-\gamma_L t)$$
$$\langle\phi_S(t)|\phi_S(t)\rangle = 1-\exp(-\gamma_S t)$$

and, $\langle\phi_L(t)|\phi_S(t)\rangle = \langle M_L|M_S\rangle$

$$[1-\exp(i\Delta m-\gamma)t]$$

where, $\Delta m = m_L - m_S$ and

$$\gamma = (\gamma_L + \gamma_S)/2.$$

Referring to M°–$\bar{M}^\circ$ oscillations, the probability $P_{M^\circ\to\bar{M}^\circ}(t,0)$ ($P_{\bar{M}^\circ\to\bar{M}^\circ}(t,0)$) for finding $\bar{M}^\circ$ at time t in a beam which is pure $M^\circ(\bar{M}^\circ)$ at $t=0$ is given by

$$P_{M^\circ\to\bar{M}^\circ}(t,0) = (1/4)|(1-\varepsilon)/(1+\varepsilon)|^2 [\exp(-\gamma_L t)+\exp(-\gamma_S t) - 2\exp(-\gamma t)\cos(\Delta m t)], \quad (19)$$

$$P_{\bar{M}^\circ\to\bar{M}^\circ}(t,0) = (1/4)[\exp(-\gamma_L t) + \exp(-\gamma_S t) + 2\exp(-\gamma t).\cos(\Delta m t)]. \quad (20)$$

For the sake of brevity and in order not to get distracted by irrelevant details, our subsequent discussions will be based on the following rewritten form for the wavefunction $|\Psi(t)\rangle$ evolved from the wavefunction (16) at $t=0$:

$$|\Psi(t)\rangle = C_1|M_L\phi_S\rangle + C_2|M_S\phi_L\rangle + C_3|x\rangle, \quad (21)$$

where C_1, C_2, C_3 are time-dependent constants and the first (second) member of each pair refers to the left (right) hemisphere. $|x\rangle \sim |M_S M_L\rangle - |M_L M_S\rangle$ represents the undecayed piece with $\langle x|x\rangle = 1$. It may be noted that in (21) we have not considered those components of the wavefunction which contain decay products on the left as they are not necessary for the subsequent treatment.

Here we are interested in the total number of $\bar{M}^\circ(\sim|M_L\rangle - |M_S\rangle)$ on the left in the two cases: (A) For no measurement performed on the right; (B) After 'partial collapse' type measurement pertaining to $|\phi_L\rangle$ and $|\phi_S\rangle$ on the right.

Let us first consider the case (A). The density operator ρ^A_{LR} corresponding to $|\Psi(t)\rangle$ is given by

$$\begin{aligned}\rho^A_{LR} = {} & |C_1|^2|M_L\phi_S\rangle\langle M_L\phi_S| \\ & + |C_2|^2|M_S\phi_L\rangle\langle M_S\phi_L| + \\ & + |C_3|^2|x\rangle\langle x| \\ & + \bar{C}_1C_2|M_S\phi_L\rangle\langle M_L\phi_S| + \\ & + \bar{C}_1C_3|x\rangle\langle M_L\phi_S| \\ & + \bar{C}_2C_1|M_L\phi_S\rangle\langle M_S\phi_L| \\ & + \bar{C}_2C_3|x\rangle\langle M_S\phi_L| + \\ & + \bar{C}_3C_1|M_L\phi_S\rangle\langle x| \\ & + \bar{C}_3C_2|M_S\phi_L\rangle\langle x|. \end{aligned} \quad (22)$$

The reduced density operator ρ^A_L for the undecayed system on the left is obtained by taking the trace of ρ^A_{LR} over a complete set of ortho-normal states of the system on the right. Then using

$$\langle\phi_S|\phi_L\rangle = \langle\phi_L|\phi_S\rangle$$
$$= \alpha(t) \text{ (say)},$$
$$\langle\phi_S|M_{L,S}\rangle = \langle\phi_L|M_{L,S}\rangle = 0,$$

and

$$\langle\phi_{L,S}|\phi_{L,S}\rangle = 1-\exp(-\gamma_{L,S}t) = F_{L,S}(t),$$

we get

$$\begin{aligned}\rho^A_L = {} & |C'_1|^2|M_L\rangle\langle M_L| + |C'_2|^2|M_S\rangle \\ & \langle M_S| + \bar{C}_1C_2\alpha|M_S\rangle\langle M_L| + \\ & + \bar{C}_2C_1\bar{\alpha}|M_L\rangle\langle M_S| \\ & + |C_3|^2\rho_L(x), \end{aligned} \quad (23)$$

where $\rho_L(x) = \sum_{R_i} \langle R_i|x\rangle\langle x|R_i\rangle$

with the kets $|R_i\rangle$ forming a complete orthonormal basis for the system on the right and $|C'_{1,2}|^2 = |C_{1,2}|^2 F_{S,L}(t)$. Note that $\langle x|x\rangle = 1$

implies $\sum_{L_i} \langle L_i|\rho_L(x)|L_i\rangle = 1$, where the kets $|L_i\rangle$ form a complete orthonormal basis for the system on the left.

Turning now to the case (B), we consider measurements on the right pertaining to the physically observable attributes of the non-orthogonal states $|\phi_L\rangle$ and $|\phi_S\rangle$ resulting in 'partial collapse' to a mixed state comprising of $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle$ with the respective probabilities $p_1, p_2, p_3, p_4 (= |C_3|^2)$, where $|\psi_1\rangle = C_1|M_L\,\phi_S\rangle + C_2|M_S\,\phi_L\rangle$, $|\psi_2\rangle = |M_L\,\phi_S\rangle$, $|\psi_3\rangle = |M_S\,\phi_L\rangle$ and $|\psi_4\rangle = |x\rangle$. Note that in the limit of non-orthogonality (i.e. $\alpha = 0$) there is 'total collapse' in which case $p_1 = 0, p_2 = |C_1|^2$, and $p_3 = |C_2|^2$.

After the 'partial collapse' type measurement, the density operator ρ^B_{LR} is given by

$$\rho^B_{LR} = p_1|\psi_1\rangle\langle\psi_1| + p_2|\psi_2\rangle\langle\psi_2| + p_3|\psi_3\rangle\langle\psi_3| + |C_3|^2\,|x\rangle\langle x|, \tag{24}$$

whence the reduced density operator ρ^B_L corresponding to the undecayed system on the left is obtained to be

$$\rho^B_L = (p_1|C'_1|^2 + p'_2)|M_L\rangle\langle M_L| + (p_1|C'_2|^2 + p'_3)|M_S\rangle\langle M_S| + p_1\bar{C}_1\,C_2\,\alpha|M_S\rangle\langle M_L| + p_1\bar{C}_2\,C_1\,\bar{\alpha}|M_L\rangle\langle M_S| + |C_3|^2\,\rho_L(x), \tag{25}$$

where $p'_{2,3} = p_{2,3}$, $F_{S,L}(t)$. If one invokes probability conservation in the 'partial collapse' type measurement then $\mathrm{Tr}(\rho^A_{LR}) = \mathrm{Tr}(\rho^B_{LR})$, whence we get

$$p'_2 + p'_3 = (1 - p_1)\,(|C'_1|^2 + |C'_2|^2 + \bar{C}_1\,C_2\,\alpha\beta + \bar{C}_2\,C_1\,\bar{\alpha}\beta), \tag{26}$$

where we have used $\langle M_L|M_L\rangle = \langle M_S|M_S\rangle = 1$, and $\langle M_L|M_S\rangle = \langle M_S|M_L\rangle = \beta$. Note that α and β are related and they both vanish in the limit of CP conservation.

Now, using (26), we obtain from (23) and (25)

$$\rho^B_L - \rho^A_L = [(p_1 - 1)|C'_1|^2 + p'_2]\,(|M_L\rangle\langle M_L| - |M_S\rangle\langle M_S|) - 2\beta(p_1 - 1)\,(\mathrm{Re}\,\bar{C}_1\,C_2\,\alpha)\,|M_S\rangle\langle M_S| + (p_1 - 1)\,(\bar{C}_1\,C_2\alpha\,|M_S\rangle\langle M_L| + \bar{C}_2\,C_1\,\bar{\alpha}\,|M_L\rangle\langle M_S|). \tag{27}$$

It is transparent from (27) that $\rho^B_L \neq \rho^A_L$ which is a signature of non-locality at the statistical level i.e. the statistical properties of the undecayed system on the left would change due to the 'partial collapse' type measurement on the right. In the limit of orthogonality (No CP violation), $\alpha = \beta = 0$ and $p_1 = 0, p_2 = |C_1|^2$ whence $\rho^B_L = \rho^A_L$.

Now, to be more specific, we compute $\Delta\bar{M}^\circ$, the difference in the number of $\bar{M}^\circ$ observed on the left for (A) and (B). Using the relevant formulae given in Ref. 19 we obtain from (27):

$$\begin{aligned}\Delta\bar{M}^\circ &= \langle\bar{M}^\circ|\,(\rho^B_L - \rho^A_L)\,|\bar{M}^\circ\rangle \\ &= (1 - \beta^2)\,(1 - p_1)\,\mathrm{Re}(\bar{C}_1\,C_2\alpha) \\ &= \{(p_1 - 1)/2\}\,\beta\exp(-\gamma t)\,(\cos\Delta\,mt - \exp(-\gamma t)).\end{aligned} \tag{28}$$

In the second line we have substituted the actual expressions for C_1, C_2 and α from reference 19. Notice that for CP invariance, $\beta = 0$ and hence $\Delta\bar{M}^\circ = 0$. The non-local effect, therefore, crucially hinges on the non-orthogonality between the states $|M_L\rangle$ and $|M_S\rangle$. This is the crux of the essential result also obtained in Ref. 19, albeit using a different approach which was less rigorous than the treatment outlined here. A comment on the probabilities $p_i (i = 1, 2, \ldots)$ is in order. As stated above, these probabilities can be calculated unambiguously in the limit of CP conservation. In the presence of CP violation, their precise values are, however, not calculable due to non-orthogonality of the basis states and hence we treat them as phenomenological parameters. It is, however, interesting to note that the non-local effect at the statistical level (eq. (28)) does not depend on these details and is non-vanishing unless $p_1 = 1$, a value which is ruled out for obvious reasons (see eq. 26).

The genesis of this intriguing non-locality at the statistical level lies in the possibility of distinguishing the non-orthogonal states $|\phi_L\rangle$, $|\phi_S\rangle$ through their physical attributes e.g. the probability distributions of the total invariant mass of the decay products corresponding to these states (since these distributions overlap, coherence of the original wavefunction is only 'partially' destroyed). Of course, if one chooses to confine attention only to the invariant mass of the individual decay-product components (mutually orthogonal) of $|\phi_L\rangle$ and $|\phi_S\rangle$, then this non-locality at the statistical level would not manifest. This peculiarity is a reflection of the sensitive dependence of the nature of the wavefunction collapse on the type of measurement one decides to perform or the level of information one chooses to obtain through the measurements involved. Inherent ambiguity in this process has recently been sharply focussed by Greenberger and YaSin[21] in terms of what they call a 'haunted' version of the EPR experiment. It should be interesting to probe the relationship between their example and that discussed in this section.

These investigations atleast serve to indicate that CP non-invariance introduces certain new features in the EPR-type example, hitherto left unexplored, whose intricacies call for further clarification and deeper studies. Of course, what has been conceived in this section is a purely gedanken experiment; the difficulties in actually realising such a case in practice are a real, but separate issue.

EPILOGUE

In the light of the discussions generated by this work it seems necessary to incorporate the following clarifying remarks, particularly concerning the notion of 'partial collapse' type measurement introduced in the treatment §4. The process envisaged as 'partial collapse' essentially implies a special type of transition from the pure state given by (21) into a mixed state comprising of $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle$ [(notation used in (24)]. This presumes the possibility that atleast for some of the events, the non-orthogonal states $|\phi_L\rangle$ and $|\phi_S\rangle$ can be distinguished through certain physical attributes, so that for the selected sub-ensembles designated by the states $|\psi_3\rangle$ and $|\psi_2\rangle$ the projection operators corresponding to the states $|\phi_L\rangle$ and $|\phi_S\rangle$ have definite values $(= +1)$ respectively. This selection process is inherently different from the standard quantum measurements and hence falls outside the ambit of the usual formalism invoked to describe measurements in quantum mechanics. Nevertheless, it can be self-consistently treated by taking into account the probability conservation (as shown in §4).

To conclude, the curious result obtained in §4 gives rise to the following key conceptual questions whose enumeration may help to orient the outlook for future studies:

(a) Is the peculiarity of the $K°-\bar{K}°$ type example an artifact of the theoretical formalism (Effective Hamiltonian Approach) entailing the approximations required to describe the decaying systems and CP non-invariance, or an indicative of a genuine non-local effect predicted by quantum mechanics at the statistical level?

(b) Since CP nonconservation implies time-reversal asymmetry, does this example imply that time-irreversibility, in general, introduces a qualitatively new element in the quantum mechanical treatment of the EPR-type examples, leading to non-local statistical effects?

(c) To what extent can the notion of 'partial collapse' involving non-orthogonal states be rigorously analysed by extending the standard framework of the quantum mechanical description of measurement processes? In this context it is pertinent to note the very recent investigation by Ivanovic[22] as regards how to differentiate between the non-orthogonal states.

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21 December 1987

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Note added in proof

M. J. W. Hall (*Phys. Lett.*, 1987, **A125,** 89) has argued that the operation-effect formalism provides a way to analyse the conceptual tenability of 'partial collapse' type measurement discussed in §4. However, this argument hinges on the extrapolation of the First Representation Theorem to the case of 'partial collapse' involving non-orthogonal states where justification for this theorem requires to be more critically examined before drawing any firm conclusion. On the other hand, M. Namiki has pointed out (private discussion) that the many-Hilbert space formalism of the measurement theory (see S. Machida and M. Namiki, In: *Proc. 1st. Int. Symposium on Foundations of Quantum Mechanics*, (ed) S. Kamefuchi *et al.*, Physical Society of Japan, Tokyo, 1984) provides a viable framework to analyse the idea of 'partial collapse' type measurement considered in the DHR example of §4.

In a very recent investigation, G. C. Ghirardi *et al.* (to appear in *Europhys. Lett.*) have argued that the type of wavefunction collapse used in the treatment of §4 can be consistently described within the generalized description of measurement processes and that the origin of the curious non-local effect at the statistical level lies in the peculiar non-local character of such a measurement procedure. They contend that this does not imply any action at a distance. Generality of this argument to cover all possible models of measurement in this case is being examined. One may also note here the recent paper by D. Dieks (*Phys. Lett.*, 1988, **A126,** 303) which studies in depth the question of partial discrimination between non-orthogonal states. The present status of the DHR example, along with its controversial aspects, is summarized in a forthcoming review paper by D. Home and F. Selleri (to appear in *Phys. Rep.*,).

ANNOUNCEMENTS

STRUCTURE AND DYNAMICS OF THE INDIAN LITHOSPHERE

A three-day international symposium on the structure and dynamics of the Indian continental and oceanic lithosphere, sponsored by ICL and IASPEI, will be held at the National Geophysical Research Institute, Hyderabad, India, during February 1–3, 1989. We request research contributions in the following areas: (i) Structure and tectonics (geophysical, geological and geochemical); (ii) Intraplate stress regimes; (iii) Kinematics of Indian plate and Indian continent; (iv) Lithosphere—asthenosphere interactions; and (v) Mantle convection and thermal history.

Extended abstracts (not exceeding 500 words) with figures (in A-4 size suitable for reproduction) should arrive before 30 November 1988.

For further information contact: Dr R. N. Singh, Symposium Secretary, National Geophysical Research Institute, Uppal Road, Hyderabad 500 007, India.

IAGA MEETING, EXETER, U.K. (JULY 24 TO AUGUST 4, 1989)

Topics for discussion

Geophysical anomalies of Gondwana ($\frac{1}{2}$ day); Magnetic petrology in magnetic anomaly interpretation (1 day); Magnetic anomalies due to sulphides ($\frac{1}{2}$ day); Interpretation of long-wavelength anomalies ($\frac{1}{2}$ day); Tectonic implications of magnetic anomalies in Europe ($\frac{1}{2}$ day); Poster session: Magnetic maps and their correlation with other geophysical and geological observations ($\frac{1}{2}$ day).

Those intending to participate in this meeting and contribute papers may communicate with the Chairman, Dr W. J. Hinze, Department of Earth and Atmospheric Sciences, Purdue University, West Lafayette, Indiana 47907, USA, with a copy to Dr V. K. Gaur, Director, National Geophysical Research Institute, Hyderabad 500 007.