PLANE AND CYLINDRICAL STRONG SHOCKS IN MAGNETOGASDYNAMICS

RAM PRAKASH and SANTOSH KUMAR*
Physics Department, Govt. Degree College, Sagar 470 006, India.
**Physics Department, Maiduguri University, Nigeria.*

ABSTRACT

The method of Chisnell[1], Chester[2] and Whitham[3] has been used to study the propagation of strong diverging plane and cylindrical shock waves through an ideal gas in the presence of magnetic field. Magnetic field is assumed to have all the three components. Analytical expressions for shock velocity and shock strength have been obtained for an initial distribution $\rho_0 = \rho' r^{-w}$, where ρ' is the density at the axis of symmetry and w is a constant. The expressions for the pressure, the density and the particle velocity immediately behind the shock have also been derived.

INTRODUCTION

Propagation of plane and cylindrical hydromagnetic shock waves has received attention of many workers. Chaturani[4] has obtained similarity solutions describing the propagation of diverging strong plane and cylindrical shock waves through an ideal gas in presence of a magnetic field having only constant axial and variable azimuthal components. Kumar *et al*[5] used CCW method and obtained analytical relations for flow variables for weak hydromagnetic shock waves in the presence of an axial magnetic field. Recently, Kumar[6] investigated analytically Chaturani's problem simultaneously for both weak and strong cases. In the present paper the propagation of strong diverging plane and cylindrical shock waves through an ideal gas in presence of a magnetic field having all the three components $(\sqrt{r}\, H_{z0}\sqrt{\eta},\ \sqrt{r}\, H_{z0},\ H_{z0})$ has been investigated. Assuming an initial density distribution $\rho_0 = \rho' r^{-w}$, the analytical relations for shock velocity and shock strength have been obtained. Finally, the expressions for the pressure, the density and the particle velocity immediately behind the shock have also been derived.

It is essential to emphasize here that for flows with cylindrical symmetry it is assumed that u is radial and that all flow quantities are functions of the radial distance r and the time t. However, the transverse and axial components H_θ, H_z of H need not vanish. Indeed they are of primary importance. Further, for $H_r \neq 0$, the only feasible case turns out to be $H_\theta = 0 = H_z$. But then the flow problem is independent of the magnetic field. Therefore, taking for instance $H_{r0}^2 = \eta r H_{z0}^2$, $H_{\theta 0}^2 = r H_{z0}^2$ and H_{z0} as constant, the present problem becomes more general (η is a constant).

BASIC EQUATIONS, BOUNDARY CONDITIONS, AND ANALYTICAL EXPRESSIONS FOR SHOCK VELOCITY AND SHOCK STRENGTH

The equations of motion for the gas enclosed by the front are:

$$\begin{aligned}
&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{2\rho}\frac{\partial H^2}{\partial r}+\frac{\mu}{\rho r}(H_\theta^2+H_r^2)=0\\
&\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\left(\frac{\partial u}{\partial r}+\alpha\frac{u}{r}\right)=0\\
&\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}+\gamma p\left(\frac{\partial u}{\partial r}+\alpha\frac{u}{r}\right)=0,\\
&\frac{\partial H_\theta}{\partial t}+u\frac{\partial H_\theta}{\partial r}+H_\theta\frac{\partial u}{\partial r}=0,\\
&\frac{\partial H_r}{\partial t}+u\frac{\partial H_r}{\partial r}+H_r\frac{\partial u}{\partial r}=0,\\
&\frac{\partial H_z}{\partial t}+u\frac{\partial H_z}{\partial r}+H_z\left(\frac{\partial u}{\partial r}+\alpha\frac{u}{r}\right)=0,
\end{aligned}\tag{1}$$

where u, p, ρ, μ, H_r, H_θ, and H_z are, respectively, the velocity, the pressure, the density, constant magnetic permeability, radial, azimuthal and axial components of the magnetic field; $\alpha = 0$ and 1 apply, respectively, to the plane and cylindrical symmetry of the problem.

The magnetohydrodynamic shock conditions can be written in terms of a single parameter $\xi = \rho_1/\rho_0$ as

$$\rho_1 = \rho_0\xi,\ H_1 = H_0\xi,\ u_1 = U\frac{\xi-1}{\xi},$$

$$U^2 = \frac{2\xi}{(\gamma+1)-(\gamma-1)\xi} \times \left[a_0^2 + \frac{b_0^2}{2}\left\{(2-\gamma)\xi+\gamma\right\}\right], \tag{2}$$

$$p_1 = p_0 + \frac{2\rho_0(\xi-1)}{(\gamma+1)-(\gamma-1)\xi} \times \left\{a_0^2 + \frac{\gamma-1}{4} b_0^2(\xi-1)^2\right\},$$

where 0 and 1, respectively, stand for the states immediately ahead and behind the shock front, U is the shock velocity, a_0 is the sound speed $(\gamma p_0/\rho_0)^{1/2}$ and b_0 is the Alfven speed $(\mu H_0^2/\rho_0)^{1/2}$.

The equilibrium state of the gas is assumed to be specified by the condition $(\partial/\partial t) = 0$ and H_{z0} = constant. Consequently, the equilibrium condition prevailing in front of the shock can be written as

$$\frac{d\rho_0}{dr} + \frac{\mu}{2}\frac{dH_0^2}{dr} + \mu\left(\frac{H_{\theta 0}^2}{r} + \frac{H_{r0}^2}{r}\right) = 0 \tag{3}$$

Assuming $\rho_0 = \rho' r^{-w}$, $H_{\theta 0}^2 = rH_{z0}^2$ and $H_{r0}^2 = \eta r H_{z0}^2$

(where η is a constant), integration of (3) yields

$$p_0 = K - \frac{3}{2}\mu H_{z0}^2 (1+\eta) r$$

$$a_0^2 = \gamma r^w \frac{K}{\rho'}\left\{1 - \frac{3\mu H_{z0}^2}{2\ K}(1+\eta) r\right\}, \tag{4}$$

where K is constant of integration.

Strong shocks: In the case of strong shocks, p_1/p_0 is large. For the magnetic case this is achievable in two ways:

Case I. The purely non-magnetic way when $\xi-(\gamma-1)/(\gamma+1)$ is small, or

Case II. When $b_0^2 \gg a_0^2$ or when $\mu H_0^2 \gg p_0$ i.e. the magnetic pressure is much greater than the gas pressure in the equilibrium state. In terms of ξ the boundary conditions now become

$$\rho_1 = \rho_0\xi,\ H_1 = H_0\xi,\ u_1 = \frac{\xi-1}{\xi}U,$$

$$\frac{p_1}{p_0} = \chi(\xi)\frac{U^2}{a_0^2} + A' \tag{5}$$

where,

$$\chi(\xi) = \frac{\gamma(\gamma-1)(\xi-1)^3}{2\xi\{(2-\gamma)\xi+\gamma\}}$$

and,

$$A' = \frac{(\gamma+1)\xi-(\gamma-1)}{(\gamma+1)-(\gamma-1)\xi}.$$

For diverging shocks, the characteristic form of system of equation (1) (i.e. the form in which each equation contains derivatives in only one direction in the (r, t) plane) is

$$dp + \mu(H_\theta\, dH_\theta + H_z\, dH_z + H_r\, dH_r) + \rho c\, du + \mu H_\theta^2 \frac{dr}{r} + \mu H_r^2 \frac{dr}{r} + \alpha c^2 \frac{u}{u+c}\frac{dr}{r} = 0 \tag{6}$$

where $c^2 = a^2 + b^2 = (\gamma p/\rho) + (\mu H^2/\rho)$.

Substituting the shock conditions (5) into (6) and respective values of various quantities, we get

$$\frac{dU^2}{dr} + U^2\left(\frac{L}{r} - M\beta^2 - Nr\beta^4\right) - p\beta^2 r^w = 0, \tag{7}$$

where $p = \frac{1.5}{0_1}(A'-\xi^2)a'^2$; $M = \frac{1.5}{KP}\chi(\xi)p'(1+\eta)$;

$\beta^2 = \frac{\mu H_{z0}^2}{\gamma p'}$, $N = \frac{2.25}{Kp}\chi(\xi)p'^2(1+\eta)^2$;

$$L = \frac{\chi(\xi)}{0_1}\left\{\frac{\alpha(\xi-1)}{(\xi-1)+\sqrt{\xi\chi(\xi)}} - w\right\}$$

and $0_1 = \frac{\chi(\xi)}{\gamma} + \frac{(\xi-1)}{2}\left(\frac{\chi(\xi)}{\xi}\right)^{1/2}$

On integration (7) yields

$$U^2 = r^{-L}\exp\left\{\beta^2\left(Mr + \frac{Nr^2}{2}\right)\right\}[\beta^2\{Rr^{L+1+w} - S\beta^2 r^{L+2+w} - T\beta^4 r^{L+3+w}\} + K_1], \tag{8}$$

where $R = P/(L+1+w)$; $S = PM/(L+2+w)$; $T = PN/(L+3+w)$

and K_1 is a constant of integration. And

$$(L'a_0)^2 = (p'/K\gamma)^{1/2} r^{-(L+n)/2}$$

$$\times \exp\left\{\frac{\beta^2}{4}(2M + Nr^2)\right\} . [\beta^2\{Rr^{L+1+w}$$

$$-S\beta^2 r^{L+2+n} - T\beta^4 r^{L+3+n}\} + K_1] . \qquad (9)$$

DISCUSSION

Equation (8) representing the propagation of strong diverging plane and cylindrical shock waves contains two types of terms involving the propagation distance r one with positive power and the other with negative power of r. Therefore, the shock velocity initially decrease as the shock advances and attains a minimum value for certain propagation distance 'Umin. This agrees with earlier results[6,7]. Chaturani[4] has, however, not observed the increasing trend of shock velocity with propagation distance. Obviously this is due to the limitation of the power series form of representation of the flow variables i.e. the solutions are inadequate for large values of r. The occurrence of exponential term in the present expression permits the parameters governing the propagation to attain theoretically infinite values.

Finally, the expressions for the pressure, the density and the particle velocity immediately behind the shock can be written as

$$P = \frac{\chi(\xi)}{\gamma}\rho' r^{-(L+n)} \exp\left\{\frac{\beta^2}{2}(2Mr + Nr^2)\right\}$$

$$\times[\beta^2\{Rr^{L+1+n} - S\beta^2 r^{L+2+n}$$

$$- T\beta^4 r^{L+3+n}\} + K_1]^{1/2}$$

$$+ A'K - 1.5A'\beta^2\gamma p'(1+\eta)r,$$

$$p = \rho'\xi r^{-n} \qquad (10)$$

$$\text{and}, u = \frac{\xi - 1}{\xi} r^{-L/2} \exp\left\{\frac{\beta^2}{4}(2Mr + Nr^2)\right\}$$

$$\times[\beta^2\{Rr^{L+1+w} - S\beta^2 r^{L+2+n}$$

$$- T\beta^4 r^{L+3+n}] + K_1]^{1/2}$$

30 April 1986

1. Chisnel, R. F., *Proc. R. Soc. London*, 1965, **A232,** 350.
2. Chester, W., *Philos. Mag.*, 1954, **45,** 1293.
3. Whitham, G. B., *J. Fluid Mech.*, 1958, **4,** 337.
4. Chaturani, P., *Shock wave propagation*, Ph. D. thesis, 1968, I.I.T. Bombay, India.
5. Kumar, S., Kulshrestha, A. K. and Chaturani, P., *Nuovo Cimento*, 1982, **B70,** 39.
6. Kumar, S., *Astrophys. Space Sci.*, 1984, **106,** 53.
7. Sachdev, P. L., *J. Fluid Mech.*, 1971, **50,** 669.

NEWS

ORGANIC COMPOUNDS ACCUMULATING SO- LAR ENERGY

Organic compounds to accumulate solar energy and supply it at the will of the experimenter have been developed by the staff members of the Institute of Chemistry of the Bashkirian branch of the USSR Academy of Sciences. The products of petrochemical synthesis make the basis of these substances. Under the influence of solar light, chemical transformations take place in them resulting in a new product which can keep the accumulated heat. Prof. Genrikh Tolstikov, one of the authors of this project, said that one kilogram of such product can accumulate 300 kilo-calories sufficient to heat a few dozen litres of water to the boiling point. In order to release thermal energy, it is necessary to affect the compound with a special catalyst. If needed, the substance-accumulator can be charged from solar rays practically for an unlimited number of times.

It is hoped that in future the new compounds will replace portable power stations at the places which are far from energy sources. (*Soviet features*, Vol. XXVI, No. 4, p. 5, 1987; Information Department, USSR Embassy in India, P.B. No. 241, Barakhamba Road, New Delhi 110 001).