NOISE AND ITS ROLE IN INSTRUMENTATION

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ABSTRACT

The role of noise in instrumentation is examined. Generally, the presence of noise limits the sensitivity of a measurement. Various methods of improving the signal-to-noise ratio are described. However, another facet of noise in measurement systems is that it can be used as a measurement tool. This aspect is also examined and some examples of the use of noise in instrumentation are presented.

INTRODUCTION

THE word noise usually connotes an irritating and interfering disturbance. Indeed, in measurement systems it has precisely the same meaning and refers to any unwanted signal other than the quantity being measured. This rather general definition includes disturbances such as the 50 Hz AC mains pick-up. However, since such disturbances can be eliminated (at least in principle) by proper shielding and design of the instruments, we will not consider such man-made disturbances in this paper. We will focus our attention mainly on the spontaneous fluctuations that arise within physical systems.

Noise or fluctuations are ubiquitous in nature. One can trace the origins of fluctuations to the corpuscular nature of matter[1]. Consequently, fluctuations can never be completely eliminated from any physical system.

In instrumentation, the presence of noise is usually regarded as a nuisance as the ultimate sensitivity of any measurement is limited by noise. However, even in measurement systems, noise has other facets. Noise can be used as a measurement tool as is done in the determination of system response by noise excitation[2]. The purpose of this paper is to review the various facets of noise in instrumentation.

The plan of the paper is as follows: In the next three sections, methods of improving the signal-to-noise ratio in measurement systems are presented. Next, examples of the use of noise as a measurement tool and the use of noise in improving the system performance are considered. The last section is devoted to some conclusions.

FILTERS

As mentioned earlier, fluctuations are always present in any physical system and hence we do not ever have an ideal noise-free signal. We can define the 'quality' of a signal by its signal-to-noise ratio (S/N) as

$$(S/N) = \frac{\text{rms value of the signal}}{\text{rms value of the noise}}. \qquad (1)$$

When dealing with very weak signals such as those that arise in pulsed NMR, fluorescence decay studies etc, the presence of noise in the physical processes makes the ratio S/N very poor. In such cases, it is very difficult to make any meaningful measurement unless the ratio S/N is improved.

We cannot really distinguish between signals and noise unless we have some *a priori* information about the signal and/or the noise. The choice of a particular technique of signal-to-noise improvement depends upon the nature of the information. In general, the signal-to-noise improvement ratio (SNIR) defined as

$$\text{SNIR} = (S/N)_{\text{output}}/(S/N)_{\text{input}}, \qquad (2)$$

depends upon the amount of *a priori* information available about the signal.

Often, very little is known about the signal beyond its bandwidth. In this case, the best one can do is to use a filter to pass the information in the signal band and attenuate the noise contribution from frequencies which do not contribute to the signal, as shown in figure 1.

If we assume that the noise has a power spectrum $N(\omega)$, the SNIR obtained by using the filter is

$$\mathrm{SNIR} = \int_{-\omega_\infty}^{\omega_\infty} N(\omega)\,\mathrm{d}\omega \Big/ \int_{-\omega_c}^{\omega_c} N(\omega)\,\mathrm{d}\omega, \quad (3)$$

where ω_∞ is the system cut-off frequency and ω_c is the cut-off frequency of the filter. In the presence of white noise, (i.e. $N(\omega) =$ constant), the SNIR is $(\omega_\infty/\omega_c)^{0.5}$.

From the above, it is evident that SNIR is a maximum when the filter cut-off frequency ω_c is a minimum. One may then be tempted to make ω_c smaller and smaller. However, one cannot reduce the filter bandwidth beyond a point as the signal will then get distorted. If more information is available about the signal and the noise, we can improve upon the rudimentary filter described above. For example, we could design an optimal filter. As the filter bandwidth is made smaller and smaller, the output noise reduces and the signal distortion increases. An optimal filter is one where any reduction in the output noise due to a reduction in the filter bandwidth is exactly offset by the corresponding increase in the signal distortion. Obviously, the design of such a filter requires much more information than

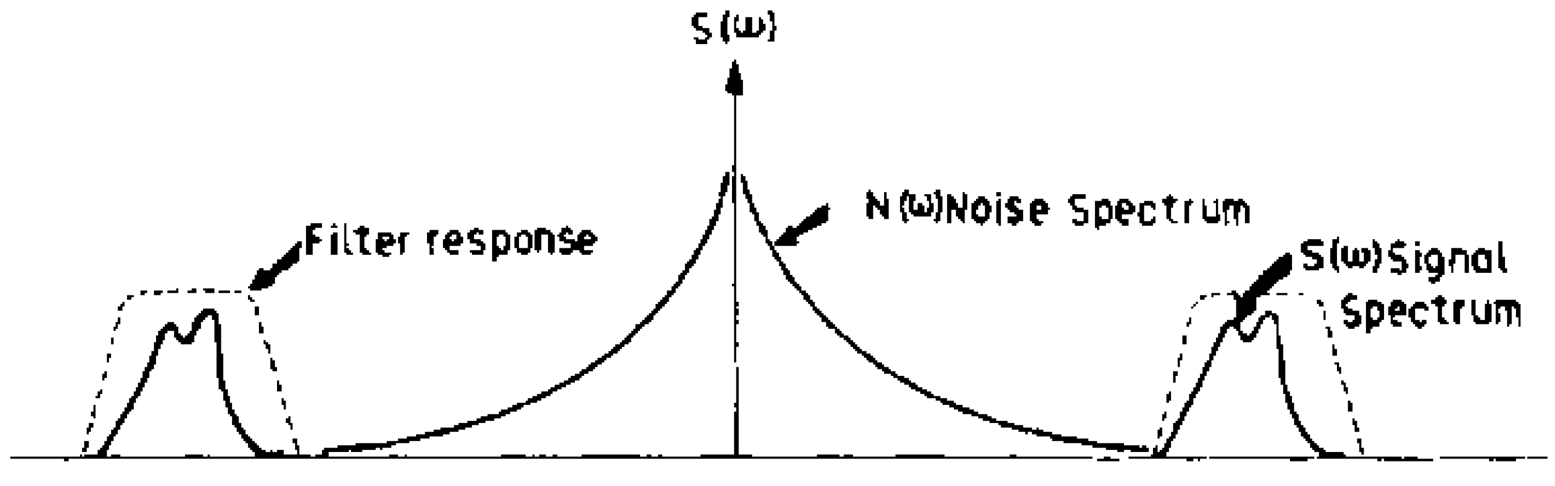

Figure 1. The use of a filter to limit the effects of noise.

merely the signal bandwidth[3]. For example, if the signal waveshape is known (as is the case in radar systems), we can use a matched filter. The matched filter is the optimum filter when a signal of known shape is corrupted by additive noise. It yields a higher signal-to-noise ratio for the shape to which it is matched than for any other shape with the same energy.

Suppose that the signal is $f(t)$, then it can be shown that the impulse response of the matched filter in the presence of white noise is

$$h_o(t) = G\,f(t_1 - t), \quad (4)$$

where G is the filter gain constant and t_1 is a time shift to assure physical realizability. The output of the filter for the signal $f(t)$ is then

$$u(t) = \int_{-\infty}^{t} h_o(t-\tau)\,f(\tau)\,\mathrm{d}\tau. \quad (5)$$

One can see that the matched filter essentially acts as a correlator.

While filters do give some amount of signal-to-noise improvement, they are quite inadequate in cases where the signals are entirely buried in noise. The use of optimal filters too does not really help as these improve the S/N by 1 or 2 dB compared to well designed non-optimal filters.

SIGNAL AVERAGING

Signal averaging is a very powerful method of extracting signals entirely buried in noise[4,5]. The use of signal averaging requires that the signal be repetitive and a reference trigger to mark the occurrence of the signal be available. These conditions are satisfied in a variety of experimental situations such as pulsed NMR studies, ultrasonic flaw detection etc. The basis of signal averaging lies in the assumption that the noise, being uncorrelated to the signal, will average out in an incoherent fashion while the signal builds up coherently when averaged over several repetitions.

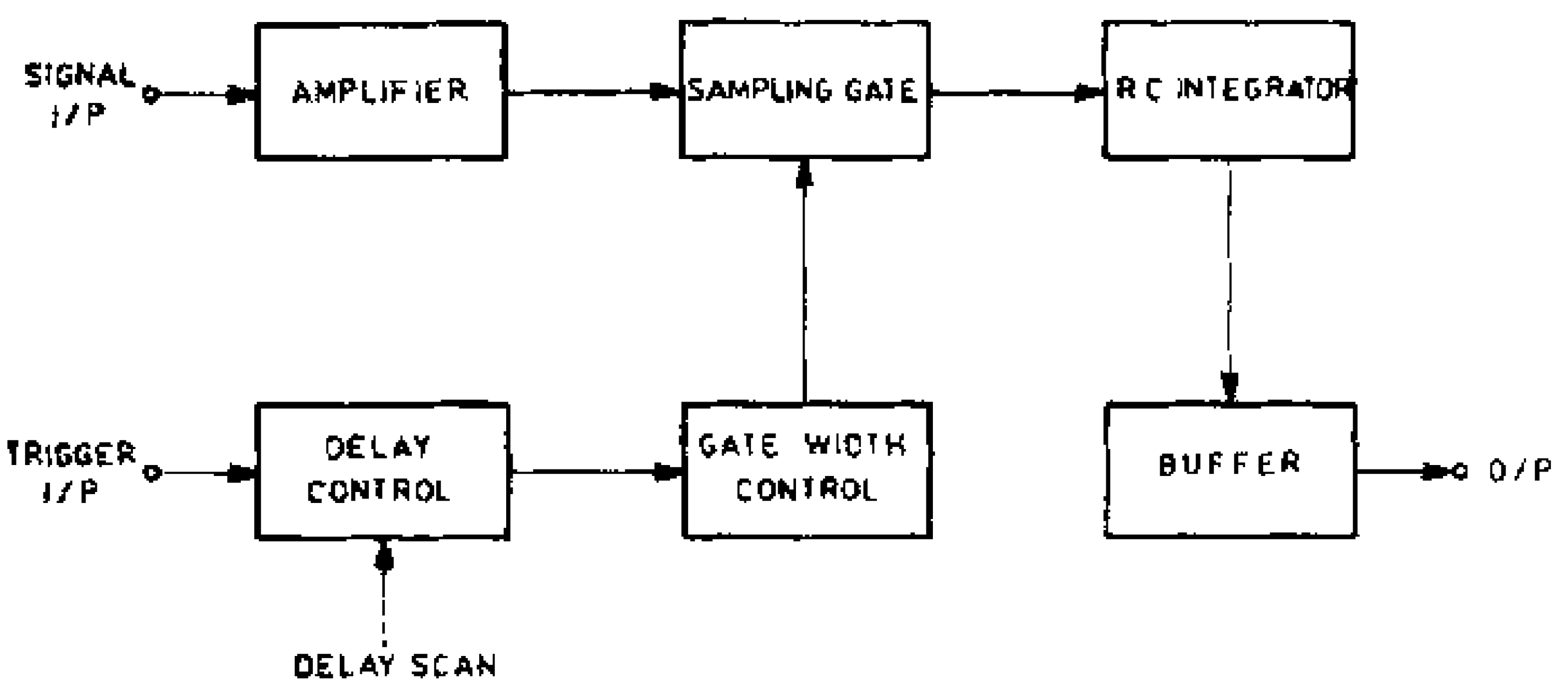

Figure 2a. Block diagram of a box-car averager.

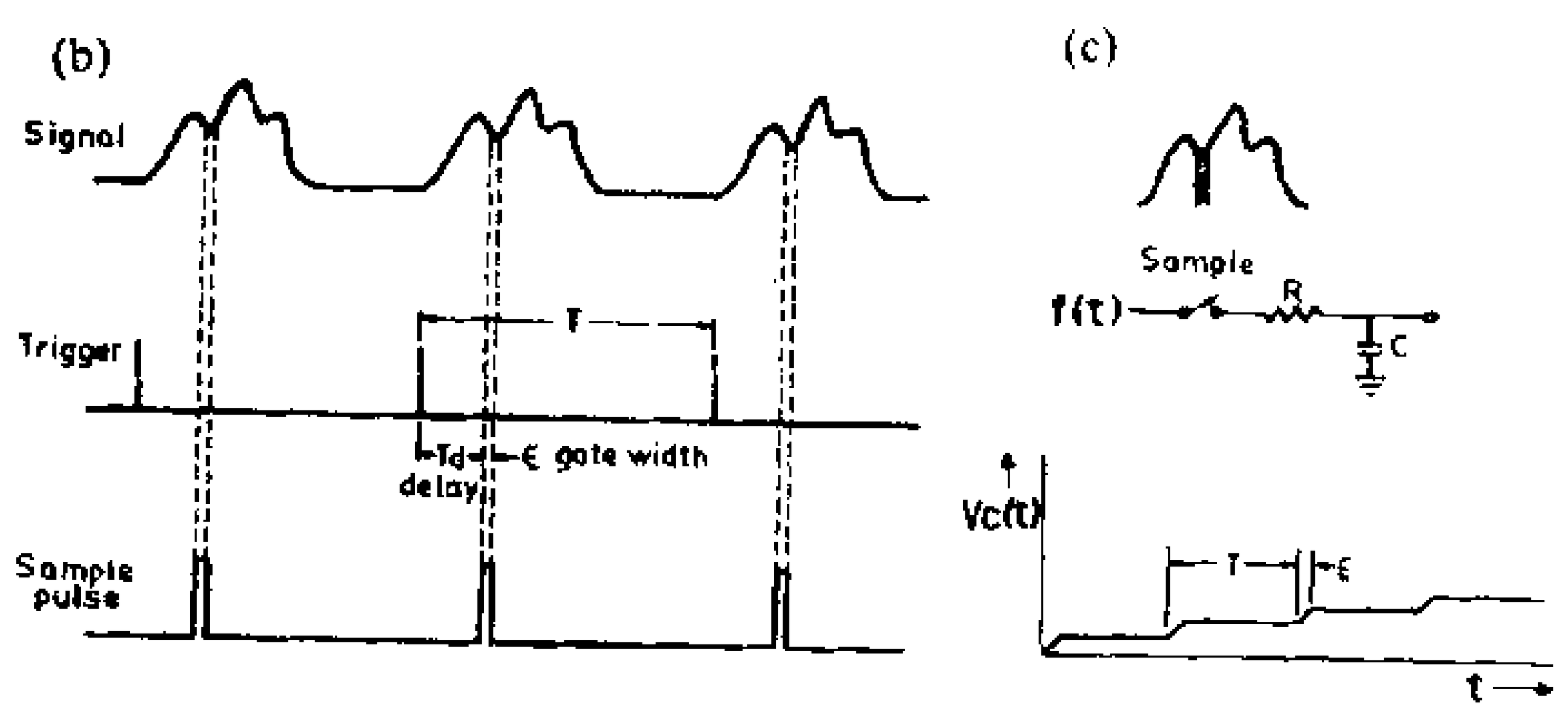

Figure 2b-c. b. Details of the sampling process; **c.** Capacitor voltage during the sampling process.

The block diagram of a box-car averager is shown in figure 2(a). The noisy signal is amplified and fed to an analog gate. The analog gate is opened for a time ε by a gating pulse which is derived by delaying the trigger by time T_d. The output of the analog gate is fed to a simple RC integrator. The integrated value of the sampled signals forms the output.

The timing diagram is shown in figure 2(b) while the capacitor voltage during the sampling process is shown in figure 2(c). The output builds up to its steady state value as a result of integration over several samples. It can be shown that the output voltage after n samplings is given[6] by

$$v_o = \lambda \sum_{k=1}^{n} \exp(-\lambda(k\varepsilon + (n-k)T)) \times \int_{(n-k)T}^{(n-k)T+\varepsilon} dt' \exp(\lambda t')\ v(t')$$

$$\lambda = 1/RC. \tag{6}$$

We see that the samples are exponentially weighted and hence this is also called an exponential averager. The SNIR in the presence of white noise is given[4-6] by $(2RC/\varepsilon)^{1/2}$.

The box-car averager is relatively inexpensive as it uses a capacitor as a memory element to perform the averaging. The disadvantage is that the SNIR is limited. One cannot increase RC/ε indefinitely as this would increase the number of samples required for the output to build up to its steady state value. Since the integrating capacitor cannot hold charge indefinitely, RC/ε is limited. On the other hand, we could use linear averagers which digitize the samples and compute the arithmetic mean by processing the digital information. Here the SNIR is proportional to $\sqrt{n}$ where n is the number of samples averaged. The SNIR is (theoretically) unlimited.

In practice, the maximum SNIR obtainable[5] from a box-car averager is about 40–45 dB. If more information about the signal is available, then one can improve the S/N even further. For instance, the lock-in-amplifier[7] can give an SNIR of as much as 60 dB. The lock-in-amplifier requires a periodic signal of known wave-shape together with a reference. The block diagram of the instrument is shown in figure 3. The noisy signal is amplified and multiplied by the phase-shifted reference. If the signal is $X\cos(\omega t)$ and the phase-shifted reference is $R\cos(\omega t + \theta)$, then the multiplier output $O(t)$ is

$$O(t) = XR\cos(\omega t)\ \cos(\omega t + \theta) = XR/2\ \cos(\theta) + XR/2\ \cos(2\omega t + \theta). \tag{7}$$

The high frequency term is filtered by the low pass filter and the phase θ is manually adjusted to maximize the output.

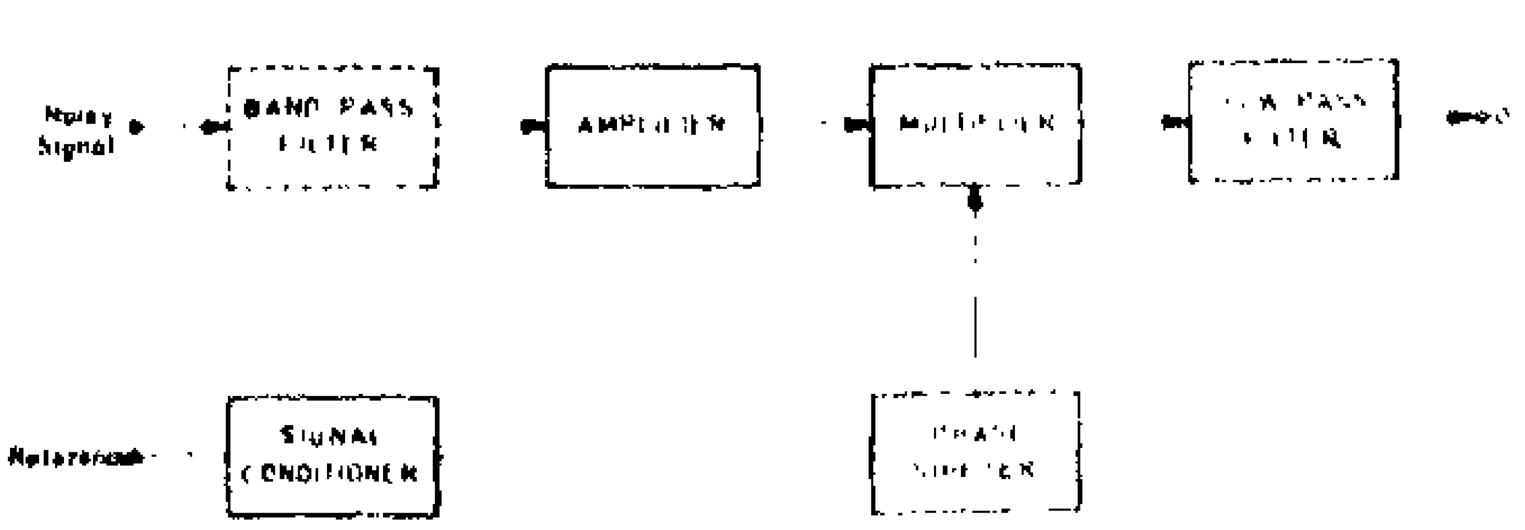

Figure 3. Block diagram of a lock-in-amplifier.

One can view the lock-in-amplifier as a bandpass filter with a very narrow pass band or as a special case of the box-car averager[8] with $\varepsilon = \pi/\omega$. In the case of a box-car averager, the passband cannot be made very narrow as the signal wave-shape (and hence its spectrum) is unknown. Since the wave-shape is known in the case of a lock-in-amplifier, it suffices to determine the amplitude of a single frequency (ω) and therefore the passband (and consequently the noise bandwidth) can be made extremely narrow.

Very often one encounters situations where the signal cannot be made periodic or even repetitive. One cannot use the method of averaging on such single shot phenomenon. However, one can still extract partial information by resorting to the autocorrelation measurements. If the noisy signal $f(t)$ is made up of the true signal $s(t)$ and the noise $n(t)$, i.e.

$$f(t) = s(t) + n(t), \tag{8}$$

then the autocorrelation function of $f(t)$ is given by

$$\begin{aligned} R_{ff}(\tau) &= \langle [s(t) + n(t)]\,[s(t+\tau) + n(t+\tau)] \rangle \\ &= R_{nn}(\tau) + R_{ss}(\tau), \end{aligned} \tag{9}$$

where $R_{nn}(\tau)$ is the autocorrelation of the noise and $R_{ss}(\tau)$ is the signal autocorrelation function. The autocorrelation function of the noise is expected to die down rapidly so that $R_{ff}(\tau)$ represents the signal autocorrelation, after a sufficient time lag. The autocorrelation function provides a S/N improvement of the order of $\sqrt{N}$ where N is the number of points over which the autocorrelation function is computed[9,10]. Hence this method is useful in cases where the signal parameters of interest can be inferred from the autocorrelation function. This method also provides a means of reducing the measurement time[10] as some improvement in the S/N is inherent even with a single occurrence of the signal.

S/N IMPROVEMENT IN PULSE MEASUREMENTS

In many situations, we encounter signals in the form of pulses rather than as continuous voltages or currents. Usually in such situations, the quantity of interest is either the pulse rate or the total number of pulses. An example of this is the photon counting system used in light scattering experiments. The scattered light intensity is usually so low that a photomultiplier tube is used to provide a high gain. When light falls on the photocathode of the photomultiplier, single photoelectrons are emitted. These photoelectrons are then multiplied by a cascaded secondary emission process to produce pulses of charge at the anode. At high light intensities the pulses overlap and we get a steady current but at low intensities the pulses do not overlap and the intensity is directly proportional to the pulse rate[11].

The noise in the photomultiplier affects the sensitivity of the measurement. Since the expected range of pulse amplitudes (and the pulse shape) is known *a priori*, one can enhance the S/N by using amplitude discriminators or single channel analysers as shown in figure 4. It must be noted that this method does

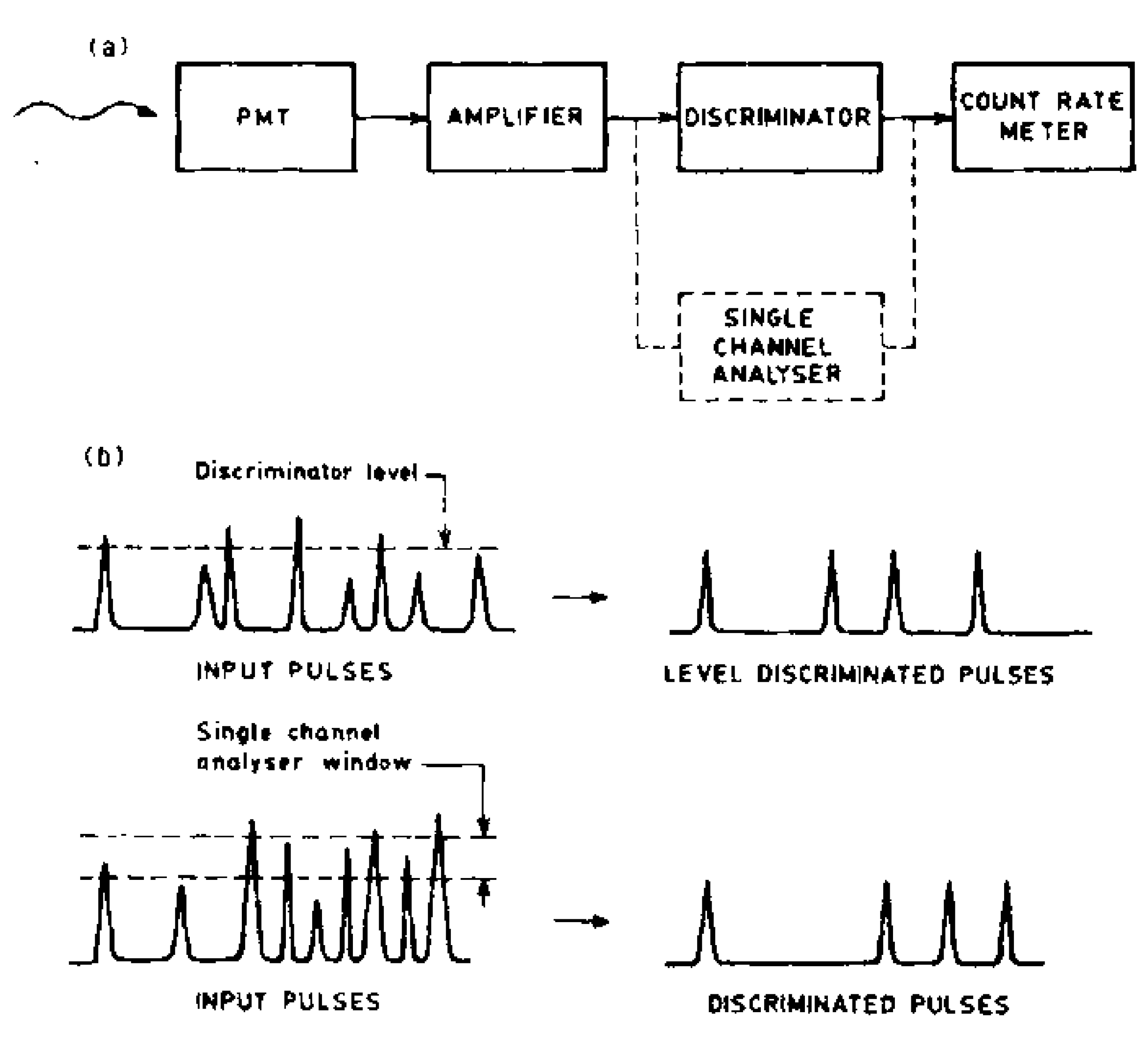

Figure 4a-b. a. A photon counting systems; **b.** Details of amplitude discrimination to eliminate spurious pulses.

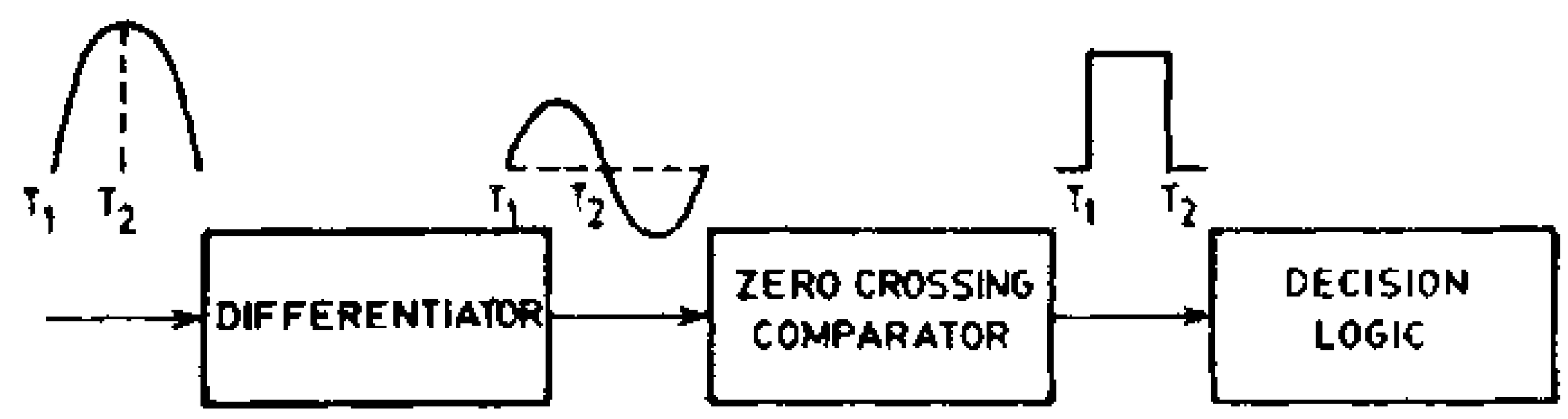

Figure 5. A pulse shape discriminator.

not by itself reduce the variance in the pulse rate but merely eliminates spurious pulses.

A similar arrangement is used in nuclear reactor instrumentation for measuring the reactor power from the neutron counts. In a reactor, the neutron detectors see both neutrons as well as the γ radiation. The detector output for γ radiation is expected to be lower in amplitude than the output pulses due to neutrons. Again an arrangement such as the one shown in figure 4 helps to eliminate spurious pulses.

A slightly different method is used at times in spectroscopy. The pulse shapes (in terms of the rise and fall times) are expected to be different for noise pulses. The discrimination in this case is provided by the pulse shape rather than amplitude. This is advantageous because the amplitudes of the pulses often vary over a wide range so that amplitude discrimination becomes difficult. The pulse shape discriminator usually consists of an RC differentiator followed by a zero crossing comparator[12] as shown in figure 5. The pulse is accepted if the time difference $T_2 - T_1$ is within specified limits.

NOISE AS A MEASUREMENT TOOL

So far we have considered methods of improving the signal-to-noise ratio of signals. However, the presence of noise in systems need not always be detrimental. In fact noise itself can sometimes be used to measure some system parameter—an example being noise thermometry[13]. This uses the well-known relationship for the mean square noise voltage in a resistor i.e.

$$V^2 = 4kTR\,\Delta f, \tag{10}$$

where k is the Boltzmann's constant, T the absolute temperature, R the resistance and Δf the system bandwidth. The available power[14]

$$P_{av} = kT\Delta f, \tag{11}$$

is independent of the resistance value and depends only on k and Δf. This is perhaps the only truly linear temperature sensor available.

Another example where the noise inherent in the system is used to advantage is in the nuclear reactor power measurements by the so-called Campbell technique. The technique uses the fact that the pulses from a neutron detector have a Poisson distribution and so the variance is a direct measure of the mean value[15]. Thus the noise power from the neutron detector is monitored rather than the mean current. This method has the advantage of being less sensitive to γ radiation. The discrimination occurs because the neutron pulses have a larger amplitude than the pulses due to γ radiation. Since the mean-square current is measured in the Campbell techniques, the contribution of the γ pulses becomes relatively small.

Noise excitation can be used to determine rapidly the transfer function of complex systems. If the system has a transfer function $H(j\omega)$, and is excited by a white noise $x(t)$, then the cross correlation $R_{yx}(\tau)$ between the input and output is given by

$$R_{yx}(\tau) = \int_{-\infty}^{\infty} R_{xx}(\tau-\lambda)\, h(\lambda)\, d\lambda = \sigma^2\, h(\tau), \tag{12}$$

where $h(t)$ is the impulse response of the system and $R_{xx}(\tau)$ is the autocorrelation of the input. The cross-spectrum $\phi_{yx}(j\omega)$ is given by

$$\phi_{yx}(j\omega) = S_{xx}(\omega)\, H(j\omega) \propto H(j\omega). \tag{13}$$

Noise can actually be used to improve the resolution of a measurement. Consider the 2-bit flash ADC shown in figure 6. The input signal is compared against 0.75 V_{ref}, 0.5 V_{ref} and 0.25 V_{ref}. The outputs of the comparator

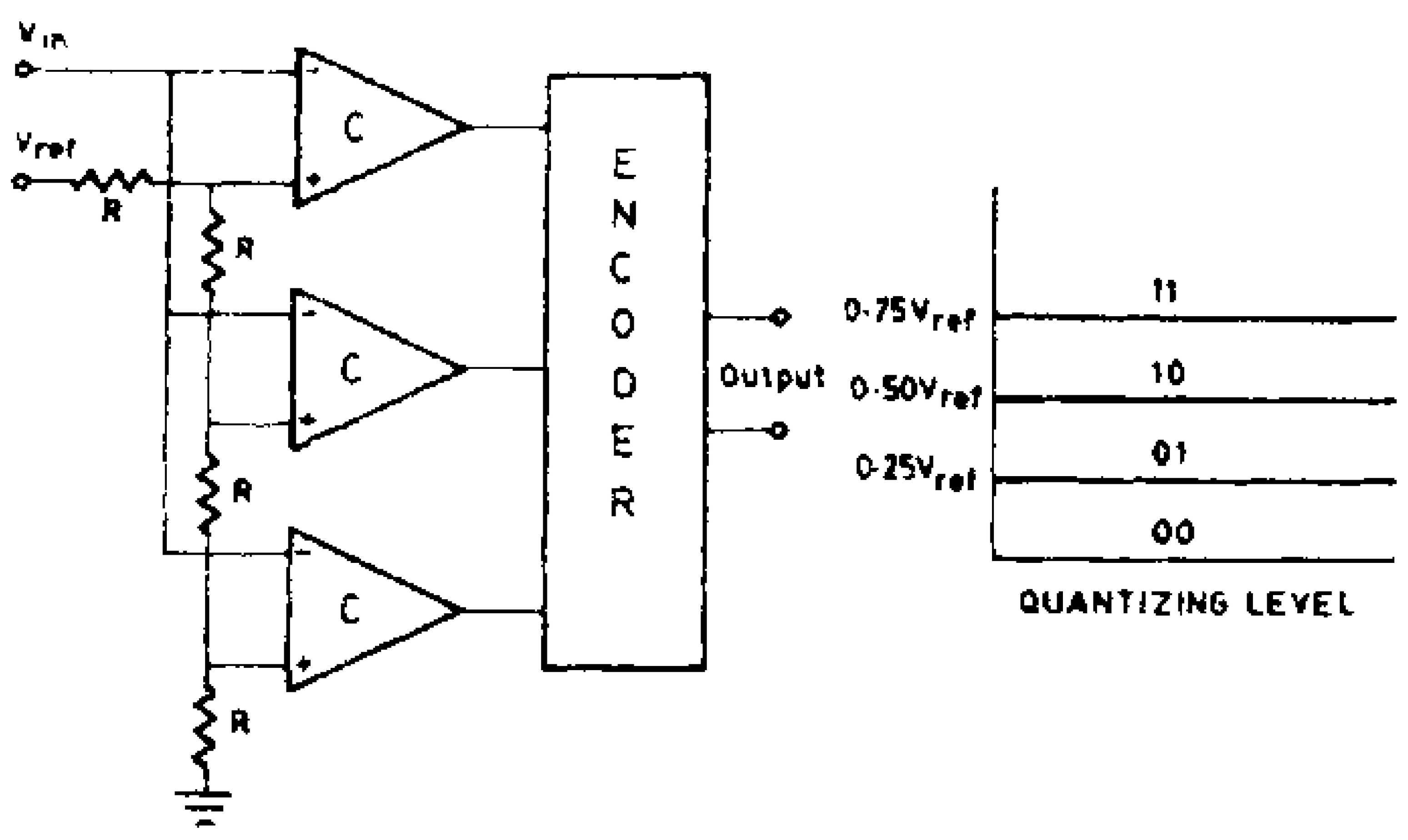

Figure 6. A 2-bit flash ADC.

are encoded to produce digital values quantizing the input signal. The resolution is $V_{ref}/4$ and the ADC gives the same digital output (01) for $0.25\ V_{ref} < V_{in} < 0.50\ V_{ref}$. If we add a noise to the input signal of sufficient amplitude, the digital output will fluctuate between 01 and 10. By averaging over these values, we can enhance the resolution of the ADC. For example if the input is $0.375\ V_{ref}$, the mean will be 01.1 (binary) as 01 and 10 will occur with equal probability.

CONCLUSIONS

Fluctuation or noise has two facets in measurements systems. By and large, it detracts from the sensitivity of the measurement and therefore one has to resort to methods of improving the S/N. The efficacy of the various methods of improving the S/N depends upon the amount of *a priori* information available about the signal and the choice of a particular technique depends upon the nature of the signal and the noise.

However, noise need not always be a nuisance. There are several experimental situations where the presence of noise can be used to advantage. Indeed one may even deliberately add noise to improve the resolution of the measurement.

Noise contains information about the detailed microscopic mechanisms that govern the macroscopic response of the physical system. It is therefore not surprising that noise can be used to advantage in measurements.

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