RECENT DEVELOPMENTS IN TRANSONIC FLOW
Part I – Theoretical and Computational

M. A. RAMASWAMY
Aerodynamics Division, National Aeronautical Laboratory, Bangalore 560034, India.
Presently, visiting professor, Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560012, India.

INTRODUCTION

TRANSONIC flow fields, as the name indicates, have subsonic and supersonic flow regions existing adjacent to each other. However, the case where the free stream and the general flow field are subsonic within which small supersonic regions exist over the body are of much greater interest both from practical as well as phenomenalogical considerations. Our discussions will be confined to this type of flow field only.

As necessary background, I shall first touch upon the inherent difficulties present in this field on account of which the progress had almost come to a stand still during 1950–60. Then the pioneering work done by Pearcey and a few others, which resulted in the possibility of designing shock-free supercritical airfoils would be described. The great benefits in aircraft design that would arise by the use of supercritical airfoils acted as a trigger for greatly renewed efforts in this field. The phenomenal developments that have resulted from these efforts would be the main theme of this paper. However, only the salient points to bring out new concepts, rather than the details would be covered here[1-6].

DIFFICULTIES IN SOLVING TRANSONIC FLOW PROBLEMS

Unlike subsonic or supersonic flow, the simplest of equations to describe transonic flow is non-linear and of mixed type (*i.e.*, elliptic in subsonic and hyperbolic in supersonic regions). Further, the boundary separating the two regions is not known *apriori*. In addition, the embedded supersonic regions, generally terminate in shocks whose position is again not known *apriori*. Also when the shock is strong, there would be strong viscous-inviscid coupling. Therefore inviscid methods with weak viscous coupling may not suffice for many practical problems.

Not only were there these great difficulties, but also, there was no great impetus to tackle these problems for the following reasons: If one considers the flow past an airfoil as the free stream, Mach number (M_∞) is gradually increased; then at a particular M_∞ called the critical Mach number, M_{cr}, the maximum velocity on the airfoil reaches local sonic value. Below M_{cr}, the flow field is purely subsonic. When M_∞ was increased beyond M_{cr}, *i.e.* at supercritical speeds, a small increment in M produced large regions of supersonic flow terminated by strong shocks on all the airfoils known at that time. Because of the shocks, there was a rapid increase of drag beyond the critical Mach number. Many attempts at designing airfoils, such that the supersonic flow decelerated back smoothly to subsonic flow failed. At this stage, the theoretical conclusion of Morawetz[7], arrived at through one dimensional analysis, that downstream disturbances which are always present in the flow would coalesce into a shock and therefore a shock-free flow is impossible, had a retarding effect on transonic flow research. She predicted that even if shock free flow was established, it would be very unstable and the smallest perturbation in geometry or freestream condition would result in strong shocks. Therefore to avoid the drag penalty, aircraft designers tried to avoid the transonic regime (except to fly through it quickly in the case of supersonic aircraft) by either reducing wing thickness, or by increasing the wing sweep. Also the new developments in space research at that time diverted the attention of many workers to that field.

PIONEERING WORK OF DEMONSTRATING SHOCK FREE AIRFOILS

Pearcey[8], by his pioneering experimental investigations, succeeded in developing nearly shock-free airfoils. He further demonstrated that small perturbations from the shock-free design condition, either in geometry or free stream Mach number, resulted in only small changes in the flow field. This was a crucial and important development which gave a tremendous fillip to revival of transonic aerodynamic research throughout the world. A few years later, Spee[9] showed that lateral gradients did permit disturbances to propogate* into the supersonic regions without coalescing and that the earlier mathematical deduction of the impossibility of shock-free flow was a result of gross over simplification of the problem through one dimensional analysis.

Further, Pearcey[8] gave a physical explanation of how the development of supersonic flow depended on the curvature distribution of the airfoil contour in that region and consequently how the contour could be adjusted to support a shock-free flow. His study showed that, the contour of a shock-free airfoil has the general feature that it has a sharper curvature just after the leading edge followed by a flatter upper contour as compared with the contour of a conventional airfoil. A good supercritical airfoil is one which is nearly shock free under design conditions and also does not give rise to strong shocks under slightly off-design conditions.

Since the top surface of supercritical airfoil, as mentioned is flatter than the bottom surface, the bottom surface now experiences slightly higher negative pressures than the conventional airfoil. This results in a slight loss of lift. To compensate for this, supercritical airfoils (particularly for transport aircraft application) are given an aft camber providing increased aft loading.

Figure 1 shows the schlieren photographs, surface pressure distribution and wake pressure data for a typical supercritical airfoil at three Mach numbers[10]. It can be seen (from schlieren pictures and surface pressure data) that at the design Mach number which is slightly higher than 0.811, the shock strength is minimum and the flow is nearly shock free. This is corroborated by the wake pressure data also, where the shaded region is an indication of the loss due to shock on the airfoil surface.

The benefits of using supercritical airfoils in aircraft design, particularly for transport aircraft, as brought out by Goodmanson and Gratzer[11], may be summarised as follows:

(i) For a given thickness of chord ratio, the cruise Mach number can be increased, by nearly 0.1 over the conventional design.
(ii) For a given cruising speed it is possible to have a much thicker wing, compared to the conventional wing, resulting in reduction in structural weight. Consequently more fuel can be carried, increasing the range.
(iii) It appears that direct operating cost can be reduced by nearly 15%.
(iv) Buffet boundary (*i.e.* the boundary in M_∞ and lift coefficient plane beyond which aircraft vibration due to flow separation is unacceptable) can be increased, thereby improving the manoeuvrability of a combat aircraft.

Because of such great benefits that a supercritical airfoil offered, once the feasability of designing such airfoils was demonstrated by Pearcey, a new surge of activity in transonic flow research began to take place and resulted in phenomenal developments which I shall briefly discuss in the remaining part of the paper.

RECENT DEVELOPMENTS

(i) *Method based on hodograph equations*: Nieuwland[12], Boerstoel and Huizing[13], and Bauer, Garabedian and Korn[14, 15] succeeded in designing shock-free airfoils using the hodograph equations where the velocity components or their equivalents are used as independent variables as this resulted in a linear equation. However, the methods employed by all of them are rather complicated and are not convinient to be used as tools by the designer. It is however

*The wave front of a disturbance can propogate into the supersonic flow if the direction of propogation is such that the component of the flow velocity in that direction is less than sonic velocity.

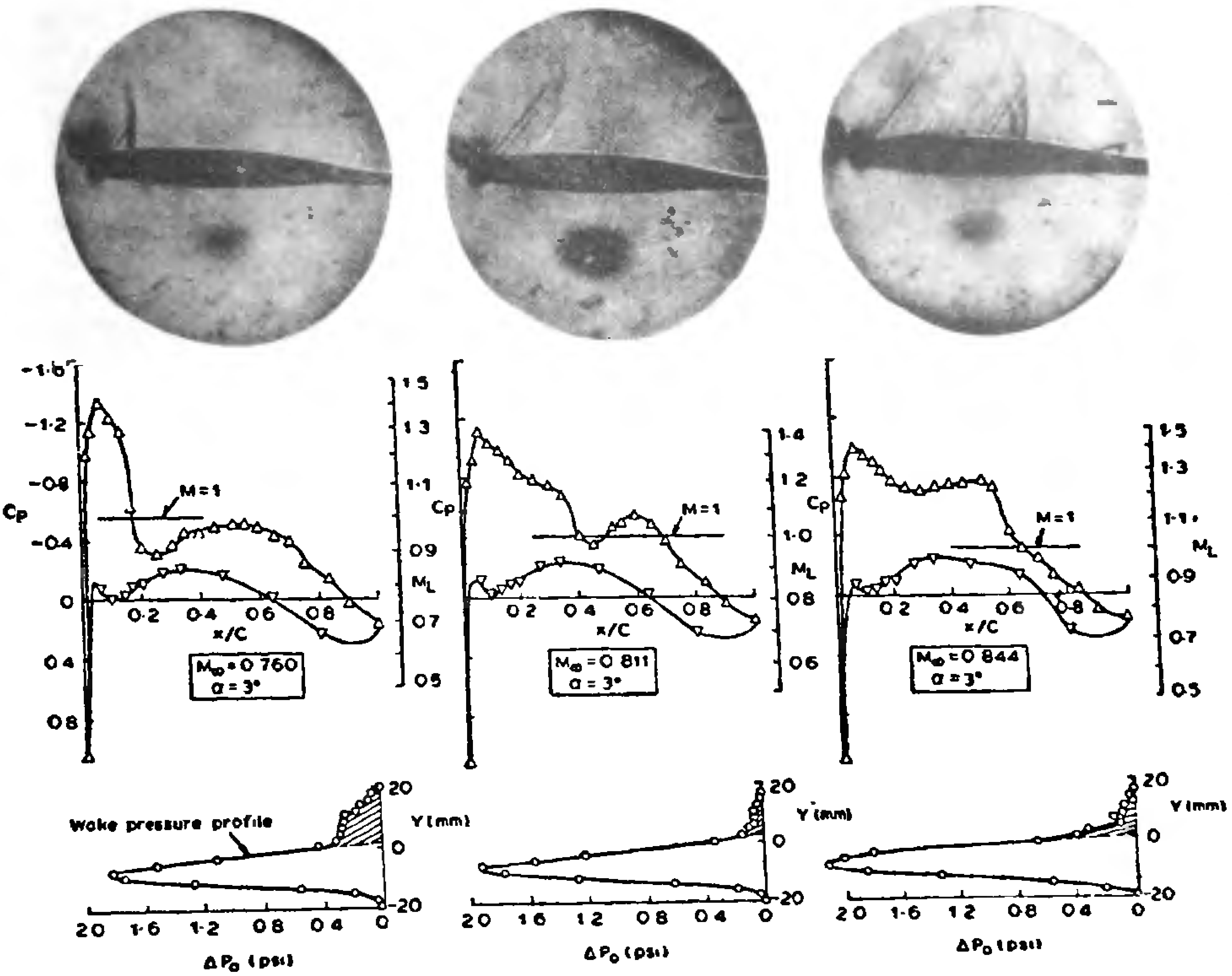

Figure 1. Flow development on NAL-3 Aerofoil

remarkable that in references 13, 14 and 15, airfoils having extended supersonic regions and aft loading similar to the practical airfoils developed empirically by Whitcomb[16] were designed.

(ii) *Methods using steady potential flow equations*: A major break through in the computation of steady transonic flow fields was achieved by Murman and Cole[17] when they introduced a type dependent difference scheme (*i.e.*, central difference scheme at subsonic points and backward difference scheme at supersonic points) to solve the steady flow past an airfoil using Transonic small perturbation (TSP) equation. The physical idea in using backward difference scheme at supersonic points is that, at supersonic points, there can be no upstream influence and hence the scheme should use only the upstream points and no downstream point. Further the backward difference scheme introduces an artificial numerical viscosity, as a consequence of which the shocks can be captured automatically. Though this idea may appear obvious now, it is a very significant contribution of Murman and Cole to transonic flow computations. Though TSP equations were used initially by Murman and Cole to simplify grid generation and application of boundary conditions, this basic idea of type dependent difference scheme applies in general to the solution of steady flow problems using potential flow equations.

There have been significant developments and refinements in solving transonic flow problems using full potential flow equations in terms of grid generation for complex configurations, numerical schemes for faster convergence etc. However, these aspects will not be covered here due to limited space and also as they do not involve any new concepts from the fluid mechanics point of view. (For details see review paper by Nixon and Kerlick in Ref. 1). Here the results of flow computation on the ONERA M6 wing at $M = 0.84$ and an incidence of 3° by Jameson and Caughey[18] is presented as a typical example in figure 2.

Though the increase in entropy across weak shocks are small enough to be neglected, the imposition of isentropy across the shock in the potential flow equations give rise to the physical inconsistency that all the three conditions of conservation of mass, momentum and energy (implied for smooth flows) cannot be completely satisfied across the shock, as there are more equations than unknowns. It is generally accepted that under these circumstances, the best recourse is to drop the momentum equation and form the "Mass conserved potential flow equations", where mass, energy and entropy are conserved.

(iii) *Design of lifting symmetric supercritical airfoils using fictitious gas method*: The fictitious gas method is an ingenious method developed by Sobieczky[19] to modify the contour of a base line airfoil such that the shock that the base line airfoil would be generating at a desired M_∞ and incidence α is eliminated by the modified contour. The outline of the method is as follows:

The transonic flow field past the base line airfoil is computed at the desired M_∞ and α using an available potential flow code with the modification that, when the local flow is supersonic, the relationship between the density ρ and the total velocity q is taken to be

$$\frac{\rho}{\rho^*} = \left(\frac{q^*}{q}\right)^p, \; p < 1.0 \text{ for } q > q^*$$

Where ρ^* and q^* are respectively the density and velocity corresponding to sonic conditions. This relationship could correspond to a gas law for a fictitious gas. With this gas law the flow is elliptic even in the supersonic region (for *e.g.* if $P = 0$, $\rho = \text{Constant} = \rho^*$). Therefore the flow over the airfoil, thus obtained would be absolutely shock free and a smooth sonic line connecting two points on the airfoil is generated. It has been shown[19], that the flow field every where except in the supersonic region would very closely correspond to the real gas flow. Only the flow field within the supersonic region is fictitious. In order to generate a real flow within this region, the flow directions and the sonic velocity on the sonic line

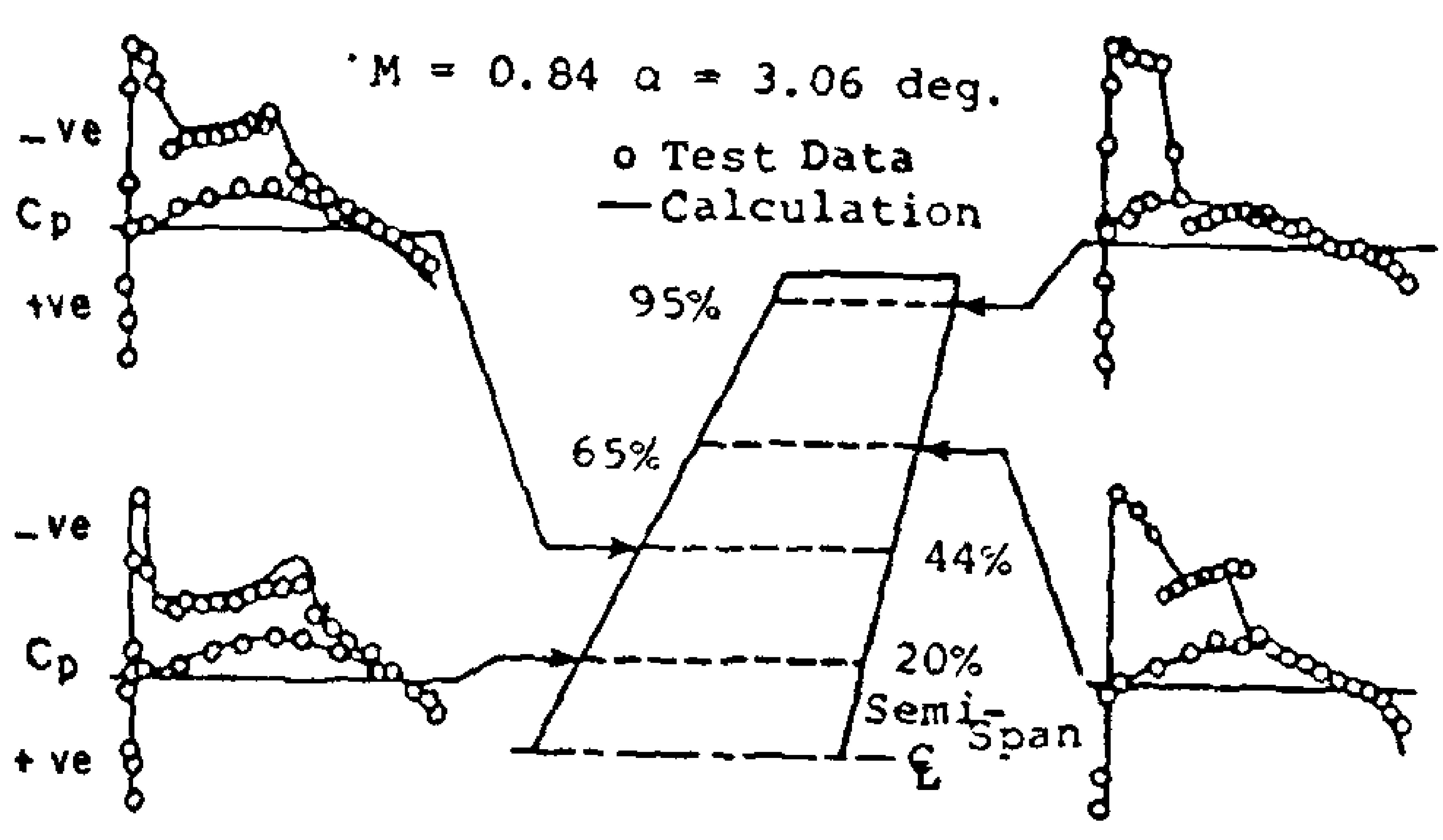

Figure 2. Comparison of FLO-22 results with experiment for ONERA wing M-6; test conducted at $R_c = 18 \times 10^6$

are taken as initial conditions and the flow in this region is solved for a real gas using the characteristic method which lends itself admirably for supersonic flow computations. The solution to this real flow field would generate a new contour on the airfoil between the sonic points so as to be compatiable with the real gas flow and also it would generate the corresponding pressure distribution on this modified surface. Thus the contour over a portion of the basic airfoil is modified to make it shock free at the desired M_∞ and α.

Using this method, Nandanan and Ramaswamy[20,21] have designed lifting symmetric supercritical airfoils. This can be considered a new concept in the design of supercritical airfoils, since all other lifting supercritical airfoils that have been designed, to the best knowledge of the author, have camber (reflex camber in many cases). An airfoil with camber is not beneficial if it has to operate at supersonic speeds, as may be the case for combat aircrafts, since camber gives raise to unwarranted wave drag. Also camber tends to produce large pitching movements, an undesirable feature for helicopter rotor blades. These disadvantages are overcome by lifting symmetric supercritical airfoils.

Figure 3 shows the characteristics of a lifting symmetric supercritical airfoil derived from NACA-0012. Its improved performance over the basic airfoil is clearly seen.

Figure 4 shows the pressure distributions along the chord at two span-wise stations for a wing with the original RAE 102 airfoil as well as with a new lifting symmetric supercritical airfoil derived from this original airfoil. From this figure, it may be inferred that the use of symmetric lifting supercritical airfoils to finite wings is also beneficial[22].

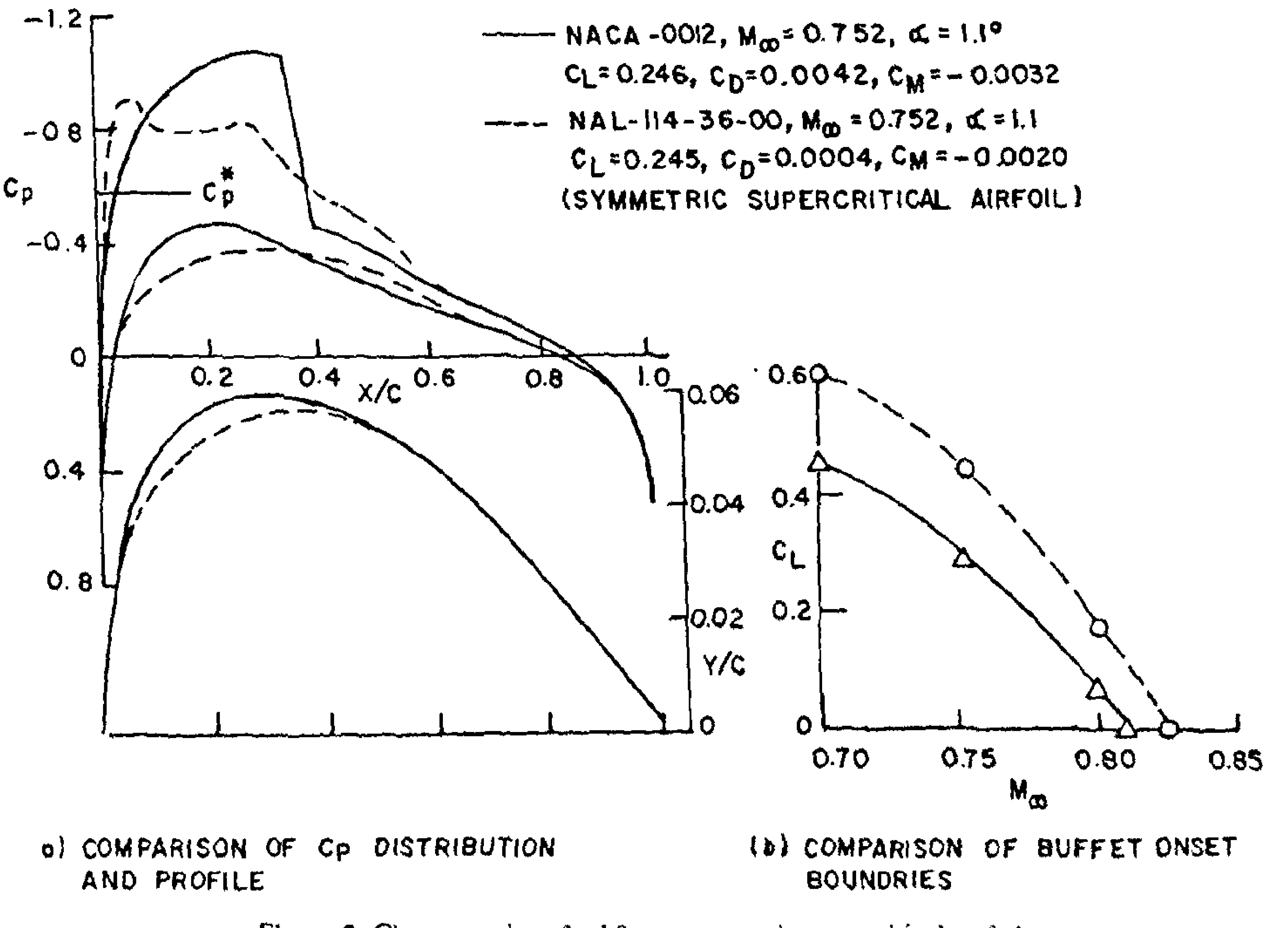

Figure 3. Characterstics of a lifting symmetric supercritical airfoil

(iv) *Solution to Euler Equations:* To avoid the problem encountered in potential flow method, namely non conservation of momentum across the shock, the obvious course is to deal with Euler's equations of motion where entropy conservation is not imposed. However, this implies now that one has to consider the components of the velocity and pressure as variables as against one single variable, the potential. This naturally demands a computer with higher memory and speed. Therefore numerical codes based on Euler equations have not yet become a regular tool in the aeronautical industry, but the trends are towards that direction, since the Euler equations appear to be capable of providing separated flow solutions also. This may appear rather intriguing since Euler equations are purely inviscid equations. The explanation offered is that the Euler equations allow rotational flows and the separated solution, one gets is considered to be the solution to the Navier stokes equation, with Reynolds number going to infinity.

Though one is interested in the solution to the steady Euler equations, it has been found to be more convenient to treat the unsteady Euler equations, since the equations are then hyperbolic in time and one can march in time. The asymptotic solution at large times, converged to the desired accuracy, is taken to be the steady state solution. References 23 to 26 give details of the various techniques that have been employed by various workers to numerically solve the Euler equations.

Results of flow computation over a Delta wing at the incidence at transonic speeds obtained by Desai and Sinha[27], shown in figure 5 illustrates

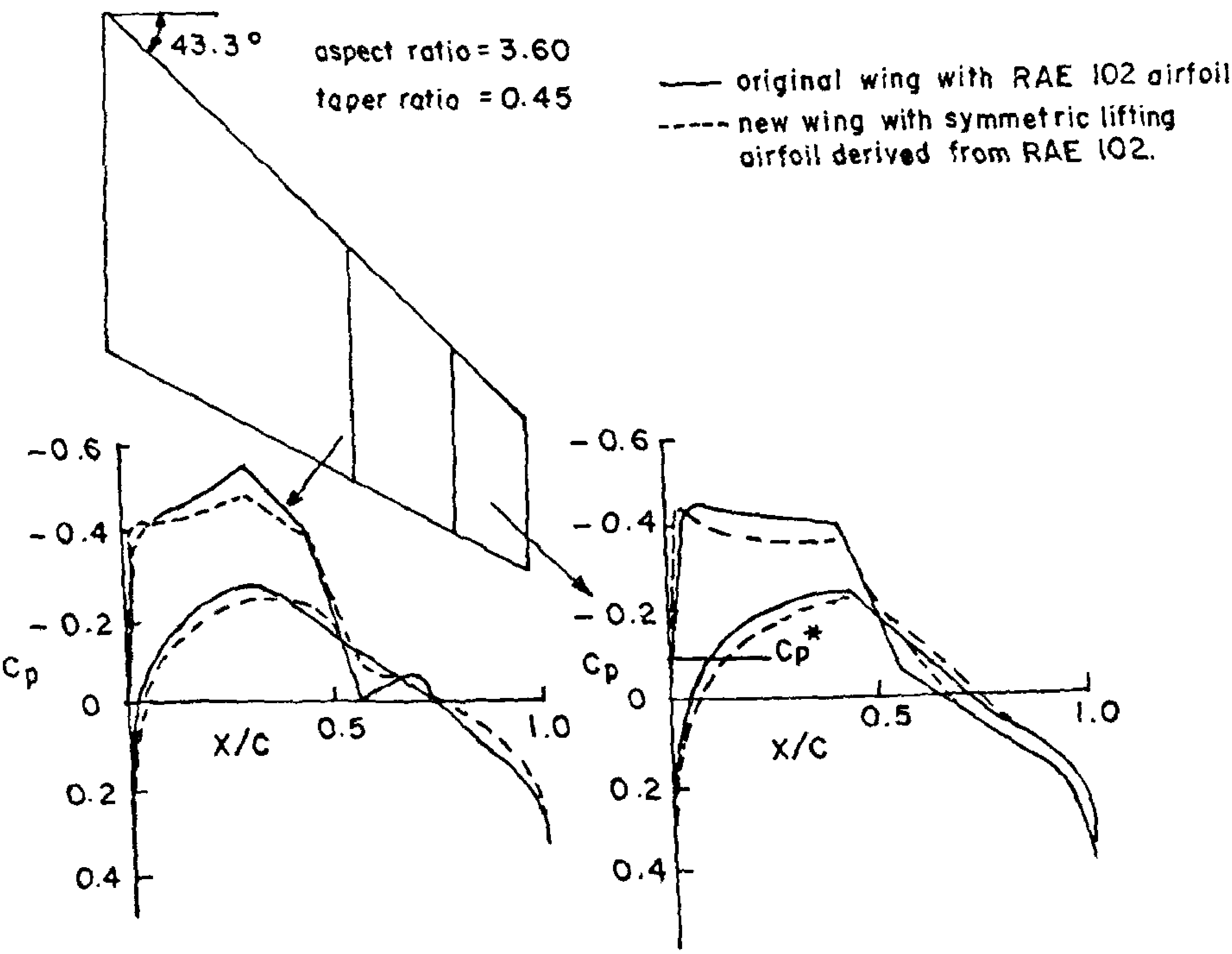

Figure 4. Comparison of surface pressure distributions ($M = 0.95$, $\alpha = 1.45°$, $C_L = 0.1$)

the capability of Euler equations to predict leading edge separation, and the roll up vortex. This is remarkable, considering that inviscid equations are used and no kutta condition to force the separation at the leading edge is applied.

Though the jump solution for the shock obtained by the Euler equations, away from the airfoil surface, is correct and satisfactory, there is an inherent inconsistency at the foot of the shock on a convex surface[28]. From flow tangency requirements, the shock has to be normal to the surface. Ahead of the shock, the Mach number and velocity decrease as one moves away from the convex surface. However solution to the normal shock provides a condition down stream where Mach number and velocity increase away from the convex surface which is physically inconsistent with the requirement of equillibrium of pressure and centrifugal forces. This inconsistency is avoided if proper shock wave boundary layer interactions are taken into account, which is possible if one solves the Navier stokes equation.

(v) *Solution to Navier Stokes Equations:* The shock wave boundary layer interaction effects in Transonic flow can often be quite strong to cause large scale flow separations and in some cases to even cause periodic oscillations of the flow with

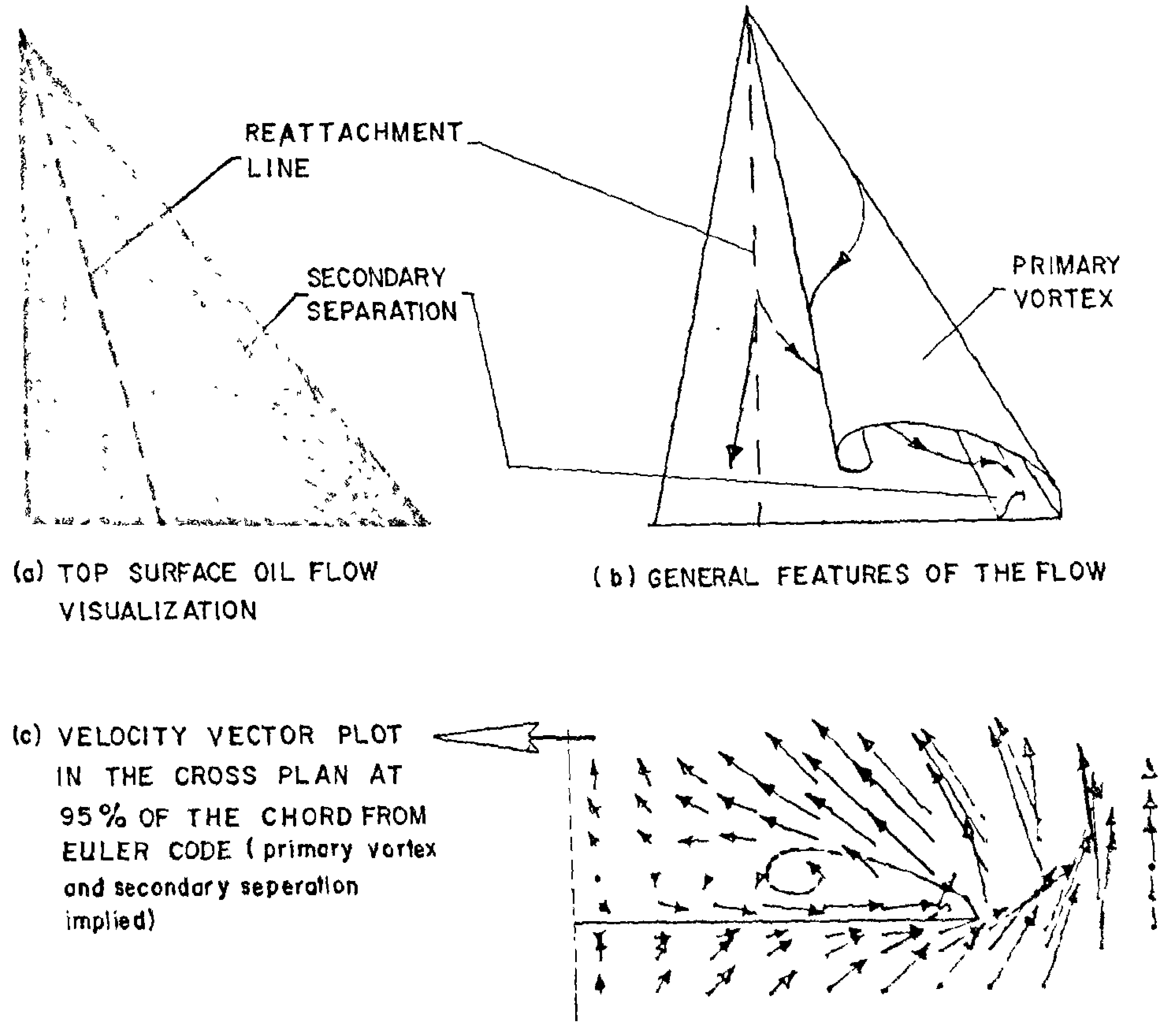

Figure 5. Flow over delta wing with sharp leading edge (50° leading edge sweep; incidence = 10°; $M = 0.7$)

relatively large amplitudes (even though the incoming free stream flow is steady). If one wants to compute these types of flows, one has got to use the Navier-stokes (NS) equations. Here, by NS equations, it is implied that one is considering the mass weighted, time averaged NS equations. The present state of art in the computation of transonic flows using NS equations is given by Mehta and Lomax[29]. Here only the highlights are presented.

The main difficulty in the use of NS equations, apart from the uncertainties in modelling the turbulence, is the fine grids that are necessary to resolve the flow variations within the thin viscous layer of the high Reynold's number flows. Because of this, the memory of the computer needed is large. The speed also has to be very high to get converged solutions in reasonable time of computation. This arises because from the stability and accuracy considerations; the time step depends directly on the space step. Because of the small step, needed from the resolution point of view, the time step also has to be small and hence large computational time if the computational speed is not high. It must be further mentioned that unlike the case of seeking a steady state solution from an unsteady time dependent equation where the Pseudo time step has no bearing with the physical time, in case of NS equations, the time step must bear a direct relation to physical time, since fluctuating phenomena are being studied. The time step has to be small compared to the time scale of the unsteadiness of the flow that is of interest but much larger than the high frequency small scale turbulent fluctuations which have been averaged out in these equations. In view of these high requirements, even the best computer available in the world today cannot tackle the problem of finding the solution to the flow past a complicated body like an aircraft using the NS equations.

Nevertheless, the detailed flow phenomena, on simple configurations, that the NS equations has been able to predict numerically, inspite of uncertainties in turbulence modelling, is rather remarkable. The computation of the flow past a 18 % bi-convex airfoil by Levy[30] can be cited as one of the examples. The computations showed a region of self exicted oscillating flow in the Mach number range between 0.72 and 0.79 for a Reynold's number of 11×10^6, in agreement with the experimental results. Further the computed fluctuating pressure data under oscillating flow conditions agreed reasonably well with the experimental data. (See ref 30 for more details). Another remarkable example is the computation of the complicated three dimensional boat tail after body flow field at an angle of attack, by Deiwert[31], which showed good agreement with experimental surface pressure data and surface flow phenomena as observed from oil flow visualisation technique. Figure 6 shows the complicated flow pattern with separation and reattachment lines that the NS equations have been able to predict.

Though it may take a long time before one can compute the flow past, a complicated configuration like an aircraft using NS equations, it is believed that this goal would eventually be reached and with that hope work continues to be pursued. In fact, this goal is one of the factors pacing the development of super computers.

CONCLUSIONS

An attempt has been made to give a glimpse of the phenomenal developments in the theoretical

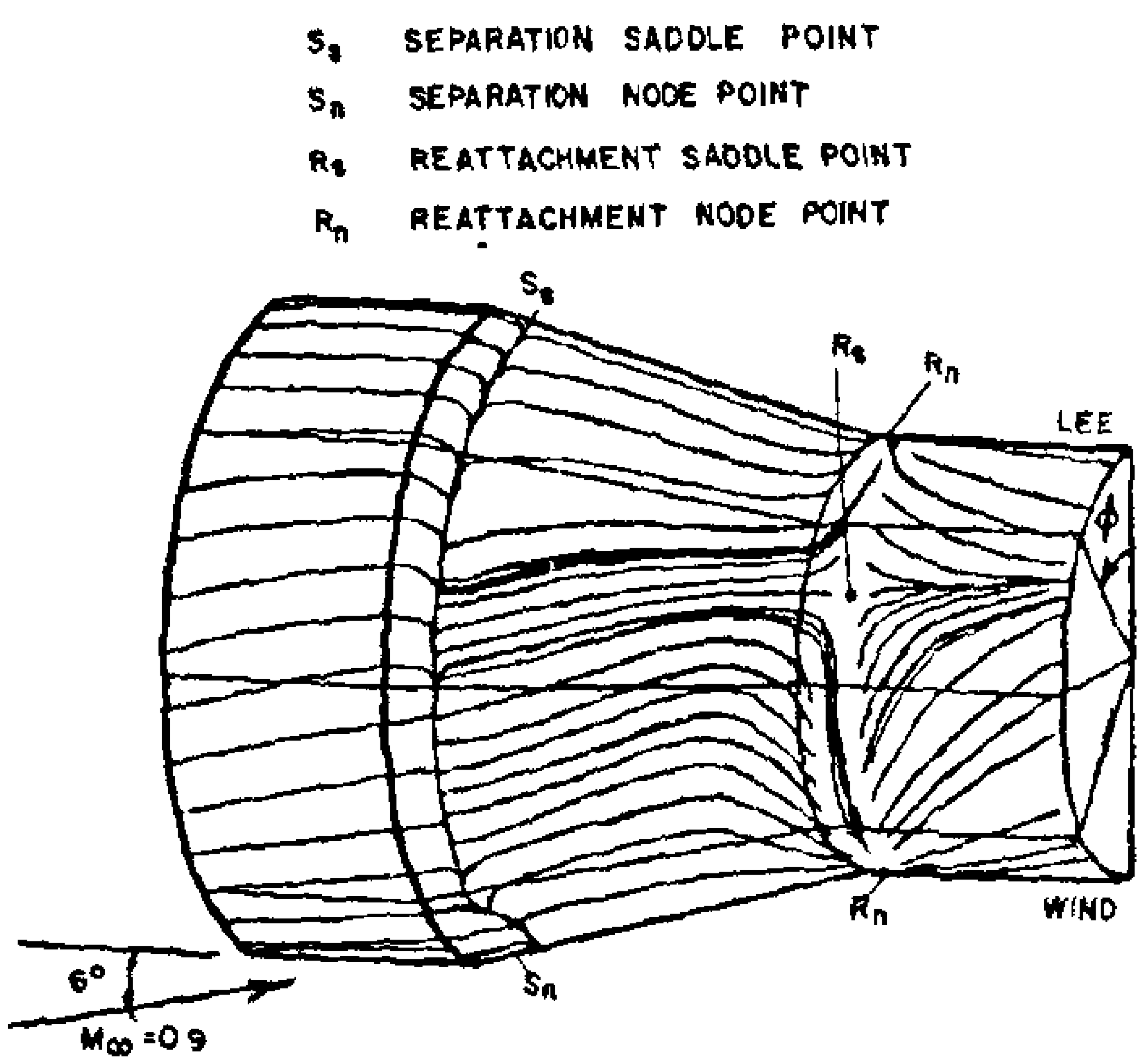

Figure 6. Computed surface flow pattern over after body by N–S equations; $Re_d = 2.9 \times 10^6$

and computational aspects of Transonic flow research during the past two decades. I have chosen to deal with the fluid mechanics aspects rather than numerical analysis aspects not because they are any less significant, but only because of space limitation. Even this aspect could not be covered in breadth or in depth because of the vastness of the developments. However, I hope, I have been able to convey the fact that transonic flow research in recent times has been a pace setter in computational fluid mechanics and also in the development of super computers. Developments in the experimental aspects of transonic flow research would be covered in part II.

24 September 1985

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NEWS

DESIGNING SINGLE-MINDED CATALYSTS

..."Today a new way of making catalysts is emerging, thanks to better understanding of the reactions involved, new instruments that allow tantalizing peeks into the processes that catalysts initiate, and the increased power of computers. The process is called molecular engineering, and it's helping researchers reach their long-sought goal of making catalysts that work as deftly as enzymes do. . . . One of the most ambitious efforts in catalyst design is aimed at exploiting natural gas as a future source of gasoline, other fuels, and chemicals. Many of the petroleum industry's long-range strategists expect oil prices to start rising again eventually. They believe that any company that successfully devises a catalytic means of converting methane (the principal ingredient of natural gas) into methanol, or methyl alcohol, in a single step at the well site, will hit a gusher of revenue. Exotic catalysts already can take the next step, turning methanol into high-octane gasoline. The first commercial plant using them for that purpose will start production in New Zealand this fall."

[(Gene Bylinsky in *Fortune* 27 May 85, p. 82–4, 88 [pd 3001j]*). Reproduced with permission from Press Digest, *Current Contents*®, No. 30, July 29, 1985. p. 11. (Published by the Institute for Scientific Information®, Philadelphia, PA, USA.)]

SAVING THE DYING GANGA

..."Thanks to the free flow of bio-non-degradable industrial effluents into [the Ganga river in India], the increasing pollution of its tributaries and the construction of dams on them, the river has lost its regenerative capacities. It will be impossible to resuscitate it unless a series of parallel measures are taken to relocate industries or to make total effluent treatment mandatory, to provide sanitation facilities in towns and villages, to regulate the use of pesticides and toxic agro-chemicals and simultaneously, to cleanse the tributaries. Difficult as it is, the task is urgent. And not only for the Ganga. Most Indian rivers need to be cleaned up. The danger is that, like cleansing the Thames in Britain, depolluting the Ganga will become a spectacular project whose success has no bearing upon other, equally relevant, aspects of pollution of the environment. Fortunately, effluent treatment is not an unattractive economic proposition. Several million tons of soil nutrients and large quantities of bio-gas and water are its by-products."

[(In *Times of India* 1 May 85, p. 8 [pd 3007j]). Reproduced with permission from Press Digest, *Current Contents*®, No. 30, July 29, 1985, p. 12. (Published by the Institute for Scientific Information®, Philadelphia, PA, USA.)]