SIGNIFICANCE OF ATOMIC AND GRAVITATIONAL CLOCKS IN SCALE-COVARIANT THEORY

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ABSTRACT

A theoretical method for the determination of the unknown scale function within the scale-covariant theory with the help of atomic and gravitational clocks has been proposed.

BOHR and Rosenfeld[1] argued that a physical theory provides its own unit of measurement. Later Eddington[2] and Dirac[3] supported this argument to explain dimensionless large numbers constructed with the help of the constants of nature. Recently, Dirac[4], Canuto *et al*[5] and Rosen[6] proposed scale-covariant theory on this basis.

According to Dirac[4], there are two types of units in nature: atomic unit and gravitational unit. The atomic unit is constructed from the constants of atomic physics and the gravitational unit is constructed from the constants of gravitational physics. The constants of atomic physics are the Planck constant, the speed of light c, the electron charge e, and the masses of microscopic particles and the constants of gravitational physics are the constant of gravitation G, the speed of light c, and the masses of macroscopic objects[5].

Dirac[4] proposed that the metrics ds_A and ds_G of the space-time measured in atomic and gravitational units, respectively, are related by

$$ds_G = \beta \, ds_A, \tag{1}$$

where β is a dimensionless, positive and real function of space-time. It is known as a scale function. The metric ds_G behaves like a Riemanian space-time for gravitational physics, whereas, for atomic physics, it behaves like an integrable Weyl space-time. The metric ds_A behaves like a Riemanian space-time for atomic physics, whereas, for gravitational physics, it behaves like an integrable Weyl space-time. The formulation of physical laws in space-time with consideration of the structures of space-time corresponding to atomic and gravitational physics gives rise to the scale-covariant theory.

A serious drawback of the scale-covariant theory at least for the study of the gravitational physics in atomic unit is the unknown scale function β. It is due to the fact that the formulation of a physical law in an integrable Weyl space-time is covariant under the Weyl gauge transformation of the form

$$ds' = \beta \, ds \tag{2}$$

and hence, we may not have a field equation for β, just as we do not have field equation for coordinate systems. This is the reason that Dirac[4], Canuto *et al*[5] and Rosen[6] have proposed some other means outside the scale-covariant theory to determine the function β. Different consideration such as variable G, large number hypothesis etc give rise to different types of variation of the function β. Therefore, it becomes natural to find out a method to single out a variation of β out of all possible variation of β obtained from different considerations. An experiment has been performed with the viking satellite in this direction and the experimental result is still under examination.

In the present paper we propose a theoretical method to determine the scale function within the scale-covariant theory and independent of any other related concepts.

In the scale-covariant theory the mass M_G of a macroscopic object in gravitational unit is a constant and the mass M_A of the macroscopic object in atomic unit is variable according to the relation

$$M_G \beta = M_A. \tag{3}$$

A similar relation is true for the atomic mass m_A and gravitational mass m_G of a microscopic particle, that is,

$$m_G \beta = m_A \tag{4}$$

but, the mass m_A is constant and the mass m_G is variable.

As pointed out earlier the formulation of a physical law in an integrable Weyl space-time is covariant under the coordinate transformation as well as under a transformation of the form (2). Therefore, the equations governing the physical law provide the measurement of Weyl gauge invariant quantities only in the Weyl space-time. A quantity T (scalar, vector, tensor or spinor) is said to be Weyl gauge invariant if it transforms as

$$T' = T \tag{5}$$

under the transformation (2). Since, in a Riemannian space-time, the formulation of a physical law is covariant under the coordinate transformation only, the equations governing a physical law provide the measurement of those quantities which are invariant under the coordinate transformations in the Riemannian space-time. Therefore, the electron mass m_{eA} in atomic unit and the metric ds_G of the space-time in gravitational unit are measurable quantities. The metric ds_G may be measured according to the method of Marzke and Wheeler[7] or according to the method of Kundt and Hoffmann[8].

Let us construct atomic clocks via the atomic transitions of hydrogen atom in gravitational units. For this we need the formulation of the Dirac field in gravitational units. Following Lord[9], the Lagrangian density for the Dirac field in gravitational units (in integrable Weyl space-time) is given by

$$2L = (-g)^{1/2}[-i\bar{\psi}_G\gamma^\mu_G\psi_{G\|\mu} + i\bar{\psi}_{G\|\mu}\gamma^\mu_G\psi_G + 2m_{eG}\bar{\psi}_G\psi_G + 2e\bar{\psi}_G\gamma^\mu_G A_\mu\psi_G] \tag{6}$$

where for simplicity we have taken the speed of light and the planck constant as unity. $-g$ is the determinant of the metric tensor corresponding to the metric ds_G. ψ_G and $\bar{\psi}_G$ are the four-spinor and its conjugate, respectively, γ^μ_G the Dirac matrices, m_{eG} the electron mass, the symbol '$\|$' denotes the Weyl covariant derivative and A_μ the electromagnetic potential. The quantities $\bar{\psi}_G$, ψ_G, γ^μ_G, m_{eG} are related to their corresponding quantities in atomic unit as follows:

$$\bar{\psi}_G = \beta^{-3/2}\bar{\psi}_A,\ \psi_G = \beta^{-3/2}\psi_A,$$
$$\gamma^\mu_G = \beta^{-1}\gamma^\mu_A,\ m_{eG} = \beta^{-1}\mathrm{m}_{eA} \tag{7}$$

and the quantities e and A_μ are the same as the corresponding quantities in atomic unit because of their Weyl gauge invariant nature in gravitational unit. The Dirac field equation in gravitational unit is then derived from the Lagrangian density (6) under the condition

$$\delta L/\delta\bar{\psi}_G = 0 \tag{8}$$

as

$$i\gamma^\mu_G\psi_{\|\mu} - e\gamma^\mu_G A_\mu - m_{eG}\psi_G = 0. \tag{9}$$

It may not be possible to obtain general expression for the energy-separations in an arbitrary space-time. As a suitable example we consider spherically symmetric space-time, whose metric is given by

$$ds^2_G = T\,dt^2 - H(dx^2 + dy^2 + dz^2),$$
$$x^\mu = (x, y, z, t), \tag{10}$$

where T and H are space-time functions.

We consider a hydrogen atom static for a moment in the space-time (10) and assume that the region in which the electron moves around the nucleus during the transition period, is very small and therefore, for the determination of the energy-separation T, H and m_{eG} can be treated as constants.

There are three types of transitions of hydrogen: (i) between principal levels, (ii) between fine-structure levels within a principal level, and (iii) between hyperfine levels. Following Will[10], the energy separations ΔE_p, ΔE_F and ΔE_H corresponding to the three types of transitions, respectively, are given by

$$\Delta E_p = -\frac{1}{2}m_{eG}e^4T^{1/2}\Delta\left(\frac{1}{n^2}\right), \tag{11}$$

$$\Delta E_F = -\frac{1}{2}\frac{m_{eG}e^8T^{1/2}}{n^3}\Delta\left(\frac{1}{j+\frac{1}{2}}\right), \tag{12}$$

and

$$\Delta E_H = \frac{4\pi}{3}\frac{m^2_{eG}e^8}{M_{pG}}T^{1/2}gp\langle 0|\sigma_e\cdot\sigma_p|0\rangle, \tag{13}$$

where n is the principal quantum number, j the angular momentum quantum number, g_p the gyromagnetic ratio, σ_e, σ_p the spins of the electron and proton, respectively, M_{pG} the proton mass, in gravitational unit. The quantities n, g_p, σ_e and σ_p are the same as their corresponding values in atomic unit, because of their Weyl gauge invariant nature in gravitational unit and M_{pG} is related with the corresponding quantity M_{pA} in atomic unit according to the relation (4) as

$$M_{pG}\beta = M_{pA}. \tag{14}$$

From (11), (12) and (13) it may be seen that the atomic clocks constructed *via* any of the types of atomic transitions measure the Weyl gauge invariant quantity $m_{eG}T^{1/2}$ in gravitational unit. Furthermore, the hydrogen atom has been assumed static during the transition period, therefore, in general, an atomic clock would measure $m_{eG}ds_G$.

Thus, we have three measurable quantities m_{eA}, ds_G and $m_{eG}ds_G$ in the scale-covariant theory, from which we may construct the quantity $m_{eA}ds_G/m_{eG}ds_G$, and in view of (4), we have

$$\beta = m_{eA}ds_G/m_{eG}ds_G, \tag{15}$$

This shows that we may determine the scale function β within the scale-covariant theory and without any other consideration from outside the theory.

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NEWS

THE SUPERNOVA MAN

. . . "Searching from his backyard, Robert Evans discovers supernovae [the distant nuclear explosions of collapsed stars] by coupling a small telescope with a keen eye and a mental map of hundreds of galaxies. . . . On clear nights, Evans aims his home-built telescope skyward and scans nearby galaxies for a discrepant dot of light. Several times a year he finds one. 'He's absolutely unbelievable,' says Gerard de Vaucouleurs [U. Texas]. '[His is] a rate of detection that has never been achieved by anyone else, and it shows that there are a lot of supernovae that we are missing'. . . . Over the years, Evans has gained familiarity with more than 700 galaxies, each mentally catalogued with its distinctive surrounding stars. No other person can claim such a vast mental storehouse of familiar galaxies, yet Evans modestly likens his talent to remembering a person's face. . . . Using Milky Way stars as guideposts, he hops from one fuzzy patch to the next at the rate—on good nights—of one per minute. With such speed, Evans can sift through as many as 500 galaxies in a session, dramatically boosting his chances of spotting a supernova. . . Evans has made 10 discoveries— eight in the last two years—and four times he spotted the supernova before it had reached its maximum brightness, testimony to his diligence."

[(John R. Gustafson in *Science 85* 6(3):78, 80, 82, Apr 85) (Reproduced with permission from Press Digest, *Current Contents*®, No. 20, May 20, 1985, p. 15. Published by the Institute for Scientific Information®, Philadelphia, PA, USA.)]