GRAVITATIONAL ATTRACTION DUE TO A TWO-DIMENSIONAL ASYMMETRICAL WEDGE-LIKE BODY AND ITS INTERPRETATION USING FOURIER TRANSFORM

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ABSTRACT

Inversion of potential field data through Fourier transform is popular in geophysical exploration. Gravity field inversion through modeling in this context is however, mostly confined to symmetrical shapes. Here we present the gravity expression over a two-dimensional asymmetrical wedge-shaped model and propose a technique of interpretation through Fourier transform. The results obtained on a synthetic model have been found to be of high quality.

INTRODUCTION

As an aid to geophysical exploration, mathematical modelling of geological situations is in wide practice. In the analysis of gravity and magnetic fields the method of Fourier transformation has proved to be of great advantage[1-4]. Frequency analysis of asymmetrical models serve generalising geological situations and therefore is of interest[5-10]. Wedge-shaped models being frequently invoked to represent vein-type mineral deposits of uniformly increasing or decreasing thickness, we present here the gravity expression for such a model of infinite horizontal extent. Resorting to frequency transformation of the gravity data, we also suggest here a method for working out the parameters of such a body.

THEORY

Heiland[11] suggests deriving formulae for gravitational attractions over simple two-dimensional objects. Let us consider a two-dimensional asymmetrical wedge-shaped model of top width d and depths Z_1 and Z_2 as shown in figure 1. The angle θ between the horizontal and the sloping edges depends upon the width. Assuming that the body has an excess density of σ with respect to its surroundings, the expression for the gravity anomaly $\Delta g(x)$ at any point x along a profile across the strike can be obtained from fundamentals as

$$\Delta g(x) = 2\gamma\sigma\left[\frac{x}{2}\ln\frac{x^2+z_2^2}{x^2+z_1^2} - z_1\tan^{-1}\frac{x-d}{z_1} - z_1\tan^{-1}\frac{x}{z_1}\right.$$

$$-\frac{1}{2}\{(x-d)\sin\theta - z_1\cos\theta\}\sin\theta\ln\frac{x^2+z_2^2}{(x-d)^2+z_1^2}$$

$$\left.+\cos\theta\left\{\tan^{-1}\frac{x}{z_2} - \tan^{-1}\frac{x-d}{z_1}\right\}\right], \quad (1)$$

where γ is the universal gravitational constant. The Fourier transform $G(\omega)$ of the gravity anomaly is

$$G(\omega) = R(\omega) + iX(\omega)$$

$$= \int_{-\infty}^{+\infty} \Delta g(x)\cdot\exp(-i\omega x)\cdot dx \quad (2)$$

where $R(\omega)$ and $X(\omega)$ are the real and imaginary components respectively. Substituting for $\Delta g(x)$ from (1) and solving the integral, we get

$$R(\omega) = 2\pi r\sigma\left[\frac{\exp(-\omega z_2)}{\omega^2}\sin\theta\cos\theta\right.$$

$$\left.-\frac{\exp(-\omega z_1)}{\omega^2}\sin\theta\cos\theta(\theta+\omega d)\right] \quad (3)$$

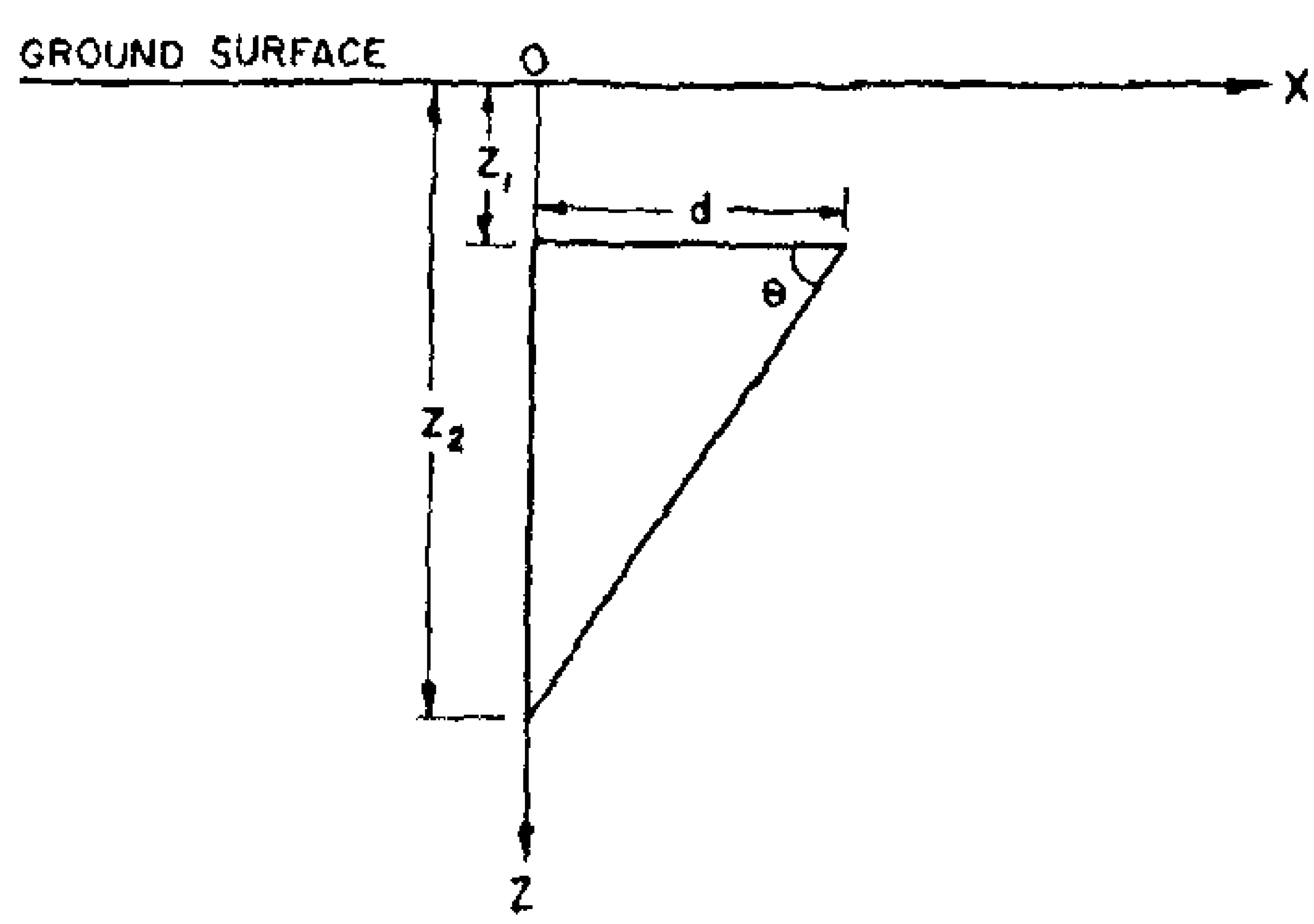

Figure 1. Geometry of the two-dimensional asymmetrical wedge model.

and

$$X(\omega) = 2\pi r\sigma\left[\frac{\exp(-\omega z_1)}{\omega^2}\{1 - \sin\theta\sin(\theta + \omega d)\} - \frac{\exp(-\omega z_2)}{\omega^2}\cos\theta\right]. \quad (4)$$

Therefore the amplitude spectrum

$$A(\omega) = \sqrt{R^2(\omega) + X^2(\omega)}$$
$$= 2\pi r\sigma\left[\left(\frac{\exp(-\omega z_1)}{\omega^2}\right)^2\{1 + \sin^2\theta - 2\sin\theta\sin(\theta + \omega d)\} + \left\{\frac{\exp(-\omega z_2)}{\omega^2}\right)^2\cos\theta\right]^{1/2} \quad (5)$$

and the phase spectrum

$$\phi(\omega) = \tan^{-1}\frac{X(\omega)}{R(\omega)}$$
$$= \tan^{-1}\left\{\frac{1 + \sin\theta\cdot\sin(\theta + \omega d)}{\sin\theta\cdot\cos(\theta + \omega d)}\right\}. \quad (6)$$

Equation (1) above is useful for the forward problem in geophysics *viz* generating the gravity profile over a wedge-shaped two-dimensional model and (3) to (6) help analysing the frequency response of the gravity function.

As could be seen in figure 2, the gravity anomaly profile has a minimum of characteristic features (like the peak value) while the resolution is high in the amplitude and phase spectra. Therefore analysis in the frequency domain helps solving the inverse problem of geophysics with greater reliability.

METHOD OF INVERSION

In the higher frequency range the amplitude spectrum (5) modifies to

$$A(\omega) = \frac{2\pi r\sigma}{\omega^2}[1 + \sin^2\theta - 2\sin\theta\cdot\sin \times (\theta + \omega d)]^{1/2}\exp(-\omega z_1). \quad (7)$$

Now, we define the modified amplitude spectrum $A_m(\omega)$ as

$$A_m(\omega) = \omega^2 A(\omega) = 2\pi r\sigma[1 + \sin^2\theta - 2\sin\theta\cdot\sin \times (\theta + \omega d)]^{1/2}\exp(-\omega z_1) \quad (8)$$

The graph between $A_m(\omega)$ and ω, as could be seen in a typical case (figure 2b), will have damped oscillations

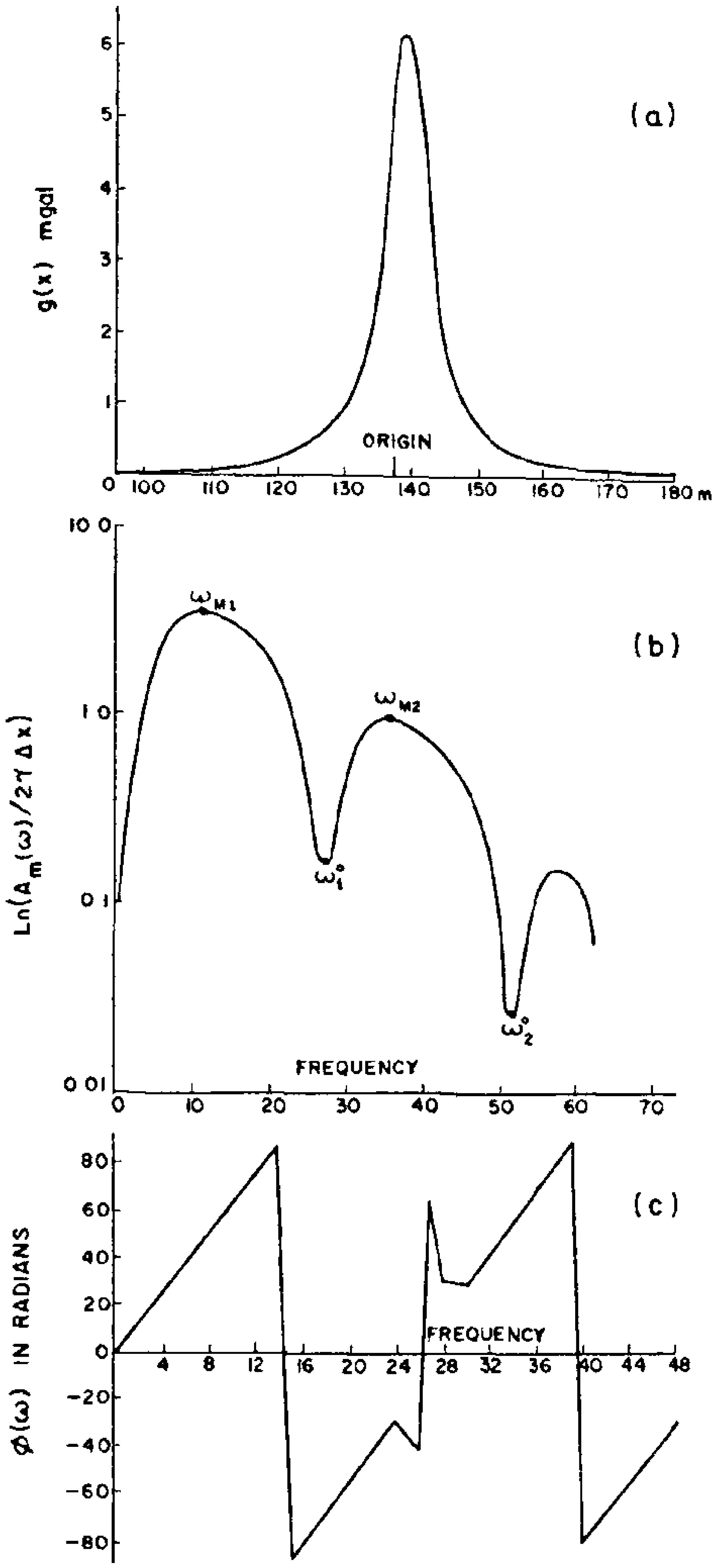

Figure 2. Gravity profile (a) and its log amplitude (b) and phase (c) asymmetrical wedge model (synthetic case).

gradually decaying at higher frequency ranges. Also $A_m(\omega)$ vanishes when

$$[1 + \sin^2\theta - 2\sin\theta\sin(\theta + \omega d)] = 0. \quad (9)$$

If ω_1^0 and ω_2^0 are two consecutive frequencies where $A_m(\omega)$ vanishes from (9), we have

$$d = 2\pi/(\omega_2^0 - \omega_1^0) = 2\pi/(\omega_n^0 - \omega_{n-1}^0). \quad (10)$$

Table 1 *Results of interpretation of the synthetic model*

Parameter	Z_1 (m)	Z_2 (m)	d (m)	θ	Distance of origin	σ g/c.c.
Assumed values	1	10	5.20	60°	135.2	1
Estimated values	0.99	10.22	5.33	59.9°	136.0	0.92

and therefore

$$\theta = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{3\pi - 2\omega_1^0 d}{2}\right). \qquad (11)$$

Again $A_m(\omega)$ has a maximum value at $\theta + \omega d = -\pi/2$. Then for two consecutive frequencies (ω_{m1} and ω_{m2}) at which $A_m(\omega)$ depicts peak values,

$$\frac{A_m(\omega_{m1})}{A_m(\omega_{m2})} = \exp\{z_1(\omega_{m1} - \omega_{m2})\} \qquad (12)$$

Taking logarithms on both sides of (12) we get

$$z_1 = \frac{\ln A_m(\omega_{m1}) - \ln A_m(\omega_{m2})}{\omega_{m2} - \omega_{m1}}, \qquad (13)$$

and therefore

$$Z_2 = (Z_1 + \mathrm{d}\tan\theta). \qquad (14)$$

Thus four parameters d, θ, Z_1 and Z_2 of the source can be obtained from (10), (11), (13) and (14) respectively.

While the slope of the phase spectrum (6) represents the distance of the true origin (surface point vertically above the vertical edge) from the arbitrary reference point, substitution of all these values in (1) at any x yield the excess density.

EXAMPLE

To demonstrate the method proposed a gravity profile was considered assigning values to the various parameters of the two dimensional asymmetrical wedge-shaped model. With a digitization interval (Δx) of 1 meter the profile data were Fourier-transfored. The gravity anomaly profile in the space domain, and its amplitude and phase spectra are shown in figures 2a, b and c respectively. The method of interpretation suggested above revealed results which are very close to the true values. The values estimated are furnished in table 1 along with those assumed for a comparison.

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