INTERACTION OF 84.26 keV PHOTONS BY BOUND ELECTRONS

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ABSTRACT

The energy spectra of 84.26 keV photons scattered incoherently by zirconium, tin, tantalum, gold and uranium have been measured at scattering angles 40°, 60°, 75°, 90° and 110° employing a high resolution 35 c.c. coaxial Ge(Li) detector in a compact geometry. The incoherent scattering functions are determined by comparing the singly differential degraded photon spectra of the scatterer with that of an equivalent aluminium foil. The results are compared with the non-relativistic HFS model calculations data. A general agreement is noticed at large scattering angles in all elements while there is considerable disagreement at 40° in Sn and Zr.

INTRODUCTION

THE Compton scattering interaction is adequately described by Klein and Nishina[1] based on the assumption of free and stationary character for the scattering electron. But due to the atomic binding effects of the electrons at low incident photon energies and the small scattering angles, appreciable deviations from the Klein–Nishina law[1] are expected. These deviations due to tightly bound electrons can be used to study the behaviour of electrons in the atomic shells, effect of the nucleus on the atomic electrons etc. The Compton process for the bound electrons refers only to the incoherent scattering in which the atom absorbs the incident energy. In such a case, the energies of the scattered photons scattered at a particular angle by bound electron are not uniquely determined as in the case of free electrons scattering, but have a certain distribution from zero energy to a maximum energy.

Information on the incoherent scattering by bound electrons has been obtained mainly from the calculations of non-relativistic wave functions. These calculations furnish estimates of a correction factor for the Klein–Nishina formula[1] and they are expressed in terms of an incoherent scattering function $S(x, z)$ which gives the probability that an atom will absorb the energy and be raised to an excited state when an incident photon transfers momentum x to any of the atomic electrons. This is expressed as

$$\frac{d\sigma B}{d\Omega} = \frac{d\sigma e}{d\Omega} \cdot Z \cdot S(x, z) \qquad (1)$$

Where $d\sigma e/d\Omega$ is the usual Klein–Nishina differential collision cross-section per electron, and $d\sigma B/d\Omega$ is the bound electron cross-section. Cromer and Mann[2] and Cromer[3] carried out non-relativistic calculations of $S(x, z)$ based on Hartree–Fock–Slater model for $Z = 2$ to 100 and x from 0.005 to 80 A^{0-1} and were presented by Hubbell *et al*[4]. Shimizu *et al*[5] calculated the incoherent scattering function S_k exclusively for K-shell electron scattering, using non-relativistic hydrogen-like wave functions. According to these models the incoherent factor $S(x, z)$ may range between 0 and 1 on normal grounds of reasoning, the former corresponds to a negligible momentum transfer while the later corresponds to a large momentum transfer.

A few experimental attempts were made to determine the effect of binding by K-shell electrons by a coincidence method of measuring K x-rays and Compton scattered photons[5-20]. Avoiding the difficulties encountered in these experiments the scattering cross-section profiles (electron momentum distribution) were measured[21, 22] in singles mode, employing a high resolution Ge(Li) detector. The data was also compared with theoretical calculations based on the relativistic form-factor approximation from second order perturbation theory. But they found discrepancy at the high energy ends. All these experiments were intended to stuy the differential cross-section for Compton scattering by 662 keV photons. However, there is no theory of incoherent scattering which is valid in this region to predict the trends observed in the experiments.

Experimental investigations at low incident photon energies, *i.e.*, below 500 keV[12, 14, 17, 19, 22, 24] are very

few; in particular no experimental data have been reported so far at 84 keV. As all the experiments are concerned with the determination of deviations from the Klein–Nishina formula[1], a knowledge of incoherent scattering functions is desirable since the Compton scattered quanta are, in general, the largest portion of the background spectra. Further, the experimental accuracy may probably be improved by measuring the singly-differential scattered spectra with a high resolution detector. In view of the basic interest in the process as well as its application for electron momentum distribution studies (popularly known as Compton profiles), it is considered worthwhile to carry out extensive experimental investigations to study the effect of electron binding for low photon energies in a wide range of elements. This is carried out in elements like zirconium, tin, tantalum, gold and uranium using 84.26 keV photons at scattering angles 40° to 110° employing a high resolution 35 c.c. Ge(Li) detector in singles configuration. The incoherent scattering functions are determined experimentally by comparing the degraded photon energy spectra with those for equivalent aluminium foils. The results are discussed in the light of recent theoretical values of $S(x, z)$ tabulated by Hubbell *et al*[4].

EXPERIMENTAL

The experimental arrangement is essentially same as that used by Schumacher[21], and Rullhusen and Schumacher[22] with suitable modifications as shown in figure 1. A collinearity geometry set-up is developed for the measurement of incoherently scattered photon spectra by different metals. The system consists of a detector, a source holder with a provision to adjust the source position, and a target holder with a provision to vary the orientation of scatterer.

The detector, a 35 c.c. coaxial Ge(Li) crystal with a resolution of 3.7 keV, at 662 keV, mounted in a lead housing and connected to a ND 1100 multichannel analyser system, is used for the detection of the incoherently scattered photons. The shielding between the source and the detector is sufficient to reduce the intensity of the source at the location of the detector to a negligible amount at all angles of study. Compact geometry is obtained with a distance of 28 cm between the detector and target, and 12 cm between the source and target. The angular resolution of the detector is found to be 2 degrees with this geometrical arrangement.

A radioactive source of ^{170}Tm with an intensity of 75 mCi, (obtained from the Bhabha Atomic Research

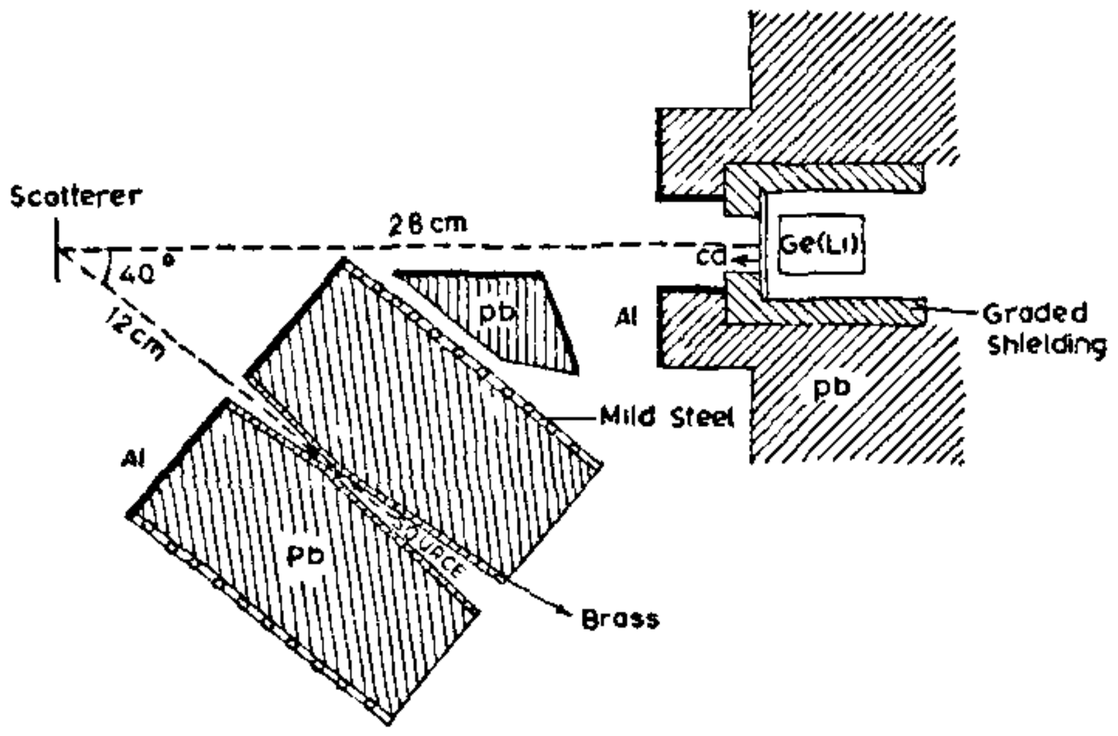

Figure 1. Experimental arrangement.

Centre, Bombay) is carefully inserted into the source holder. This source holder is a lead filled mild steel cylinder of about 1 ton weight with an axial hole of 1 cm dia. The radioactive source (in a radiographic capsule) is fixed at one end of a lead-filled brass tube that moves in the axial hole of the source holder to place the source at any desired position. The source holder is mounted on a vertical stand with casters to enable it to rotate about the centre of the target, keeping the detector at a fixed position throughout the experiment.

Thin foils of scatterers of 12.5 mm diameter are fixed to the target holder made of thin perspex ring. These foils are made from 99.99% pure metals. The target assembly is fixed at the centre of the system. The target is turned to an angle in a plane determined by the source, target and the detector such that the absorption of scattered photons are minimized.

The scattered spectra for different foil thicknesses of experimental and aluminium targets are recorded on ND 512 channel analyser at angles 40°, 60°, 75°, 90°, and 110°. In recording the spectra the time of collection is so adjusted that the statistical errors are less than 1%. A typical scattered photon spectrum of 2.48 mg/cm^2 tantalum at angle 75° is shown in figure 2. It is observed that the spectrum extends over a wide range of energies in the case of a single element. Well-resolved elastic and inelastic peaks along with the characteristic K x-ray peaks of the scatterer can be seen clearly in figure 2. The background spectrum is measured in each angle after removing the scatterer, and is substracted from the data both in the cases of experimental and aluminium scatterers. The background corrected spectrum is again corrected for Compton contribution due to coherent scattering of photons. The resultant spectrum is finally corrected for

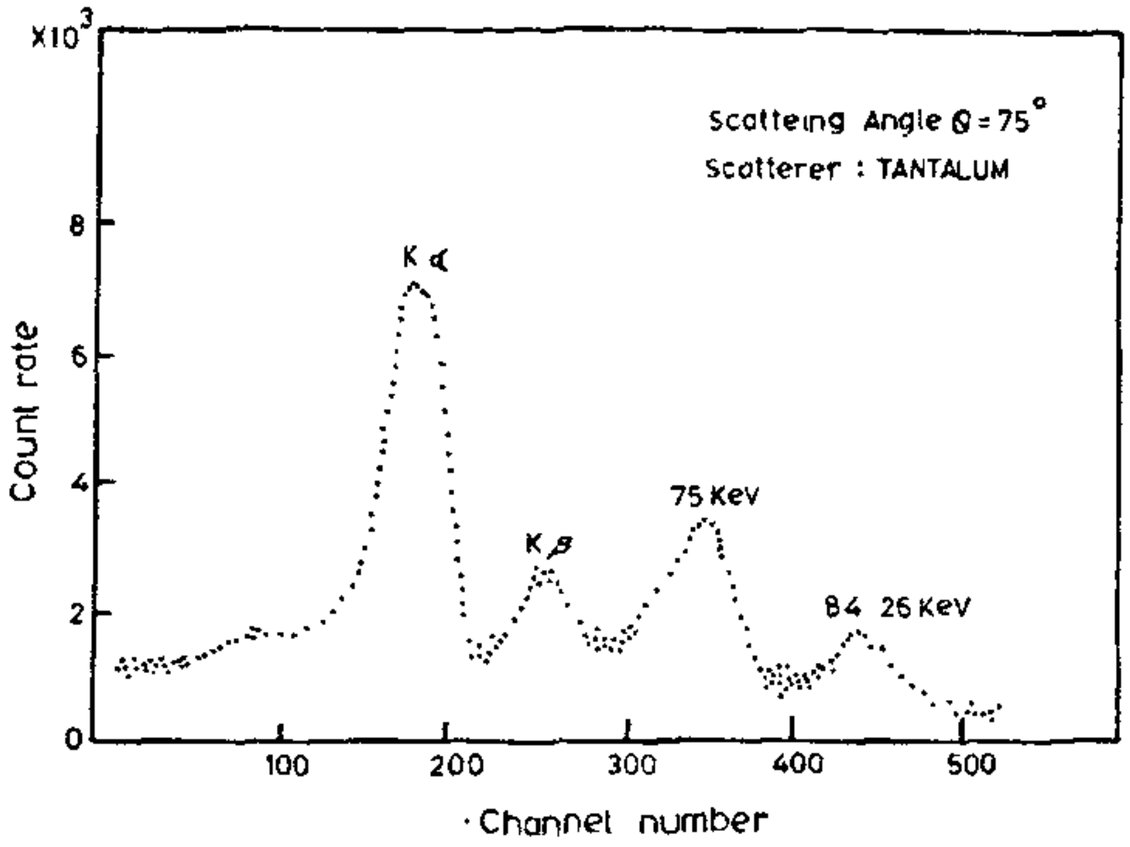

Figure 2. Scattered photon spectrum of the primary energy 84.26 keV photons corresponding to 2.48 mg/cm² Ta scatterer at 75°.

finite energy resolution by properly choosing mono-energetic sources in the scattered photon energy range of 84 keV.

In order to determine the incoherent scattering cross-section at any angle, the fractional number of gamma rays at that angle must be obtained. If S is the source strength, N_x the number of electrons in the scatterer material situated at a distance r from the source point, and C_x the number of incoherently scattered photons detected per unit time, then

$$C_x = \frac{S}{4\pi r^2} \cdot N_x \cdot \frac{d\sigma x}{d\Omega}(\theta) \cdot \Omega \cdot \varepsilon_\gamma \cdot \exp(-\mu X_T) \quad (2)$$

where $\theta(d\sigma x/d\Omega)$ is the differential incoherent scattering cross-section of gamma rays per electron of the scatterer, Ω is the solid angle subtended by the detector at the target, ε_γ is the intrinsic efficiency of detection, and $\exp(-\mu X_T)$ is the self-absorption correction factor for the scatterer material.

In the present investigation the primary source intensity determination is avoided by conducting auxiliary experiments with aluminium foils in which the binding effects are negligible compared to the incident photon energy and Klein–Nishina Law[1] could be employed. The degraded scattered spectra of aluminium foils are used for comparison with (2). The number of incoherently scattered events C_{Al} by free and stationary electrons of the aluminium target at the same defined angle as that of the experimental scatterers are given by

$$C_{Al} = \frac{S}{4\pi r^2} \cdot N_{Al} \cdot \frac{d\sigma Al}{d\Omega}(\theta) \cdot \Omega \cdot \varepsilon_\gamma \cdot \exp(-\mu X_{T_{Al}} \quad (3)$$

The measurements C_{Al} are made before and after each cycle of observation of experimental scatterers. From the above two equations, the incoherent scattering cross-section ratio is

$$\frac{d\sigma x}{d\sigma Al} = \frac{C_x}{C_{Al}} \cdot \frac{N_{Al}}{N_x} \cdot \frac{\exp(-\mu X_{T_{Al}})}{\exp(-\mu X_T)}. \quad (4)$$

The cross-section ratios $d\sigma x/d\sigma Al$ are evaluated for each element with thin foils of different thicknesses and the minimum foil thickness being 8.68 mg/cm² (Zr), 5.62 mg/cm² (Sn), 2.48 mg/cm² (Ta), 2.68 mg/cm² (Au), and 46.36 mg/cm² (U). Self-absorption correction is experimentally determined by least square fit method to obtain true incoherently scattered events for unit thickness of a scatterer element. The overall error in the determination of the cross-section ratio is the root mean square value of the individual errors in C_x/C_{Al} and other errors, and is found to be less than 3%.

RESULTS AND DISCUSSION

The experimental values are presented in tables 1–4. The recent theoretical incoherent scattering functions $S(x, z)$, compiled by Hubbell *et al*[4] are also furnished in the same tables for comparison. The experimental value of the ratio vary from 0.668 in uranium ($z = 92$) at 40° to 0.952 in zirconium ($z = 50$) at 75°. The ratio decreases with an increase in atomic number for a

Table 1 *Incoherent scattering functions $S(x, z)$ of 84.26 keV photons by uranium*

Angle	Experimental value $d\sigma x/d\sigma Al$	Hubbell *et al* value $S(x, z)$
40°	0·668 ± 0·033	0·720
60°	0·771 ± 0.032	0 813
75°	0·845 ± 0.032	0.854
90°	0.878 ± 0.031	0.882
110°	0.915 ± 0.031	0.905

Table 2 *Incoherent scattering functions $S(x, z)$ of 84.26 keV photons by gold*

Angle	Experimental value $d\sigma x/d\sigma Al$	Hubbell *et al* value $S(x, z)$
40°	0 691 ± 0.028	0.743
60°	0.789 ± 0 030	0.837
75°	0.851 ± 0.031	0 877
90	0 886 ± 0 031	0.902
110°	0 902 ± 0 032	0 923

Table 3 *Incoherent scattering functions $S(x, z)$ of 84 26 keV photons by tantalum*

Angle	Experimental value $d\sigma x/d\sigma Al$	Hubbell *et al* value $S(x, z)$
40°	0.706 ± 0.034	0.759
60°	0.809 ± 0 032	0.849
75°	0 899 ± 0.033	0.886
90°	0 909 ± 0 035	0.910
110°	0 898 ± 0 036	0.930

Table 4 *Incoherent scattering functions $S(x, z)$ of 84 26 keV photons by tin*

Angle	Experimental value $d\sigma x/d\sigma Al$	Hubbell *et al* value $S(x, z)$
40°	0.768 ± 0 030	0.813
60°	0.897 ± 0.032	0.890
75°	0 929 ± 0 032	0.921
90°	0.941 ± 0.033	0.939
110°	0.927 ± 0 037	0 954

given angle. At forward scattering angles, the ratio is far less than one, while for larger scattering angles, the ratio approaches unity. Similar variation was also observed in the earlier investigations for incident photons of 320 keV, 279 keV and 145 keV[12, 14, 17, 19, 23]. However, the present experimental values cannot be compared with other experimental results because no experimental data exist for 84 keV in any of these elements. The incoherent scattering functions with uranium and gold scatterers are of special interest in view of the fact that the degraded photon spectrum in no case extends up to the full energy as expected from the kinematics of free electron scattering. The binding effects are, therefore, expected to be large in uranium and gold scatterers. This situation is less severe in low atomic number element zirconium, and intermediate in tantalum as the binding energy of the K-electron decreases towards Low-Z elements.

The results are compared with the recent theoretical values calculated by Cromer and Mann[2], and Cromer[3] for average shell electrons based on non-relativistic Hartree–Fock wave functions[4]. It is observed that the theoretical values at all angles are less than unity and these values systematically increase when the scattering angle increases in a single element. The values also increase with decrease of atomic number Z with respect to an angle. As can be seen from the tables 1–5,

Table 5 *Incoherent scattering functions $S(x, z)$ of 84 26 keV photons by zirconium*

Angle	Experimental value $d\sigma x/d\sigma Al$	Hubbell *et al* value $S(x, z)$
40°	0.797 ± 0.035	0 849
60°	0.934 ± 0.034	0.913
75°	0 952 ± 0.033	0.939
90°	0.938 ± 0 034	0.955
110°	0 939 ± 0 038	0.967

the results agree with the theoretical data within experimental errors at large scattering angles (75°, 90° and 110°) in all elements. The experimental values at forward angles 60° and 40° in high-Z elements (U, Au, Ta) are less than the corresponding theoretical values. Disagreement with theory is noticed at 40° in low-Z elements, tin and zirconium. The discrepancy in all elements at forward angles may be due to the large binding effects of the electrons, as the incident photon energy is very low. This behaviour shows the influence of percentage of innermost shell electrons participating in the interaction process and the binding effects are expected to be more significant even in low-Z elements. The agreement in all elements at large photon scattering angles may be due to the validity of the non-relativistic HFS wave functions. The variation of the experimental values from 0.668 to 0.952 may be attributed to the importance of L and higher shell electron contributions to the incoherent scattering function. Since the K-shell binding energy of uranium is greater than the incident photon energy, the binding effects in this case are essentially due to the L and higher shell electrons. Hence, the discrepancy with the theory at forward angles in elements ranging from $z = 50$ to 92 indicates the need for rigorous relativistic approach in the evaluation of binding effects of electrons.

The variation of the incoherent scattering functions $S(x, z)$ with momentum transfers for each element is shown in figure 3. The momentum transfer is

$$x = (\sin\theta/2)/\lambda(\text{Å}) \qquad (5)$$

where λ is the wavelength of the incident photon radiation expressed in angstrom units with the value of $\lambda = 12398.520/E$ (eV). Here E is the energy of the incident photon radiation. The cross-section ratios, as can be seen from the figure 3, are less than unity and approach zero as the momentum transfer decreases to zero. This situation arises as the momentum transfer to

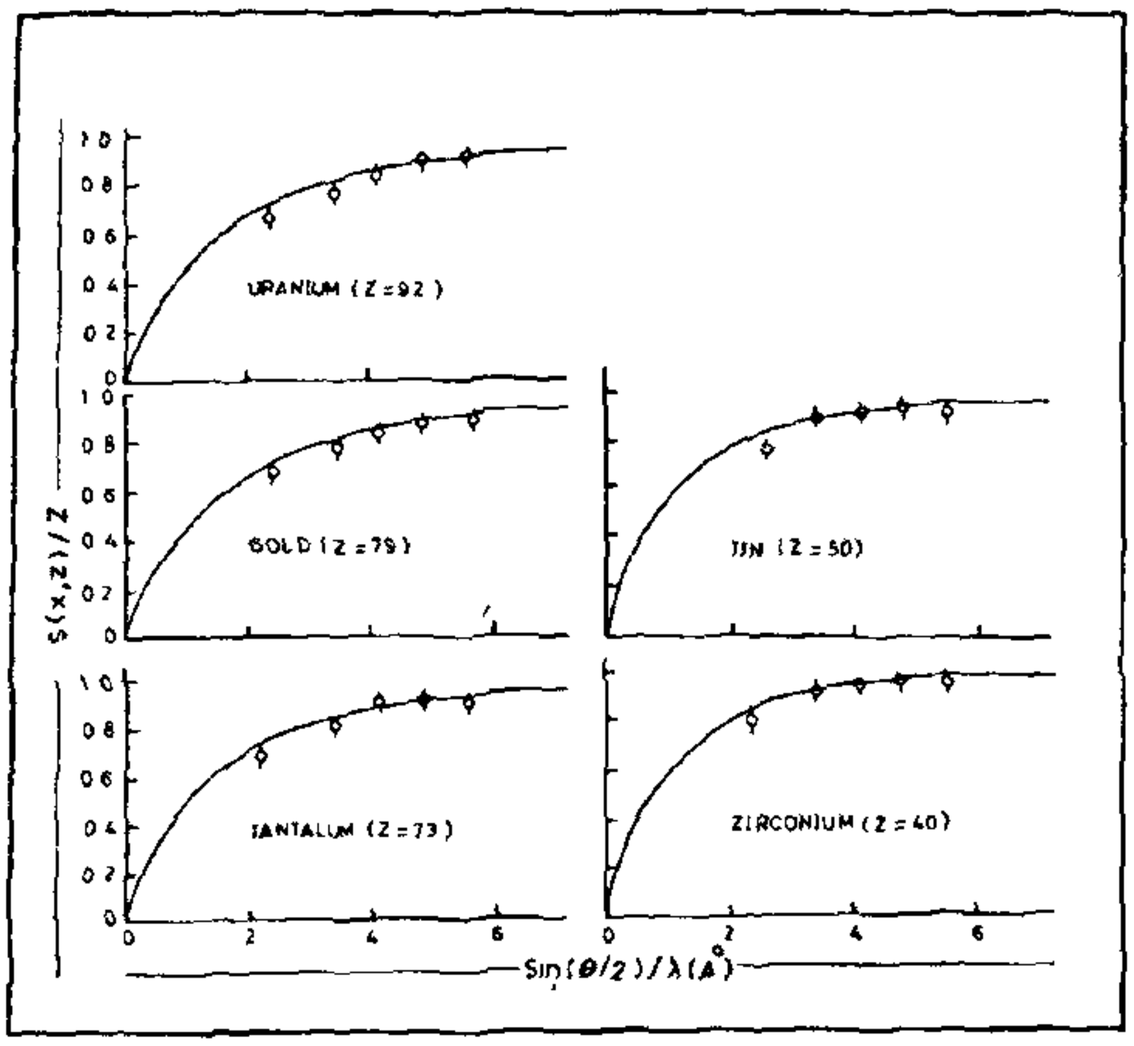

Figure 3. Variation of the incoherent scattering function with momentum transfer of 84.26 keV photons by U, Au, Ta, Sn and Zr. The solid lines indicate the theoretical values based on non-relativistic HFS calculations data of Hubbell *et al*[4]. Open circles indicate the present experimental values.

the scattering electron is lesser than the binding effects of the atomic shell. In this case, the probability for removal of the tightly bound electron is less at forward scattering angles, and so, the momentum distribution of the electron is strongly dependent on the scattering angle. This behaviour can be seen from figure 3 in all elements under investigation. Though the essential behaviour is similar, the actual theoretical and experimental values disagree to some extent. This discrepancy is expected since the scattering of 84.26 keV incident photon energy is completely due to the *L*-shell electrons in very high-*Z* elements (u) and combined effect of *K* and *L*-shell electrons in the other elements and this may be explained in the framework of the relativistic theory. Such a rigorous relativistic approach in the evaluation of electron binding effects for low photon energies is needed.

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