PROPAGATION OF FINITE AMPLITUDE WHISTLER WAVES THROUGH THE IONOSPHERE

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ABSTRACT

The nonlinear correction to the ionospheric dielectric constant induced by heating is compared to that due to ponderomotive force of the whistler wave and the results have been used to study the self-focusing of whistlers in the ionosphere. It is found that ponderomotive force causes self-focusing of whistler waves for frequencies less than the half local electron gyrofrequency.

INTRODUCTION

The amplitude-dependent dielectric constant of a plasma causes self-focusing, filamentation and self-trapping of the propagating wave. These phenomena have been studied both theoretically and experimentally mostly for high power laser beams. The self-focusing of radio waves propagating through the ionosphere has been studied in the context of ionospheric modification experiments in overdense plasma[1-4]. Gurevich[5] provides an excellent review of the physics of self-focusing instability of radio waves in the ionosphere. Perkins and Goldman[6] developed the theory of self-focusing of radio waves in an underdense ionosphere. All these works refer to high power and high frequency radio waves from man made transmitters to study ionospheric striations. Stenzel in a series of papers observed the filamentation of high power whistlers in a laboratory plasma[7-10]. Sodha and Tripathi[11] provided theoretical explanations for Stenzel's observations. Compared to these, the investigations of self-focusing instability of natural VLF waves have not been taken seriously although it was first pointed out by Litvak[12] in 1970. Recently, Das and Singh[13,14] have demonstrated the possibility of self-focusing of naturally occurring whistlers in the ionosphere and magnetosphere. This brief report presents a more detailed numerical study of self-focusing of finite amplitude, naturally occurring whistler waves propagating through the ionosphere.

WAVE PROPAGATION AND ITS SELF-FOCUSING IN IONSPHERIC PLASMA

The finite amplitude wave propagation is governed by the wave equation

$$\nabla^2 \vec{E} - \nabla(\vec{\nabla}\cdot\vec{E}) = -\frac{\omega^2}{c^2}\varepsilon\cdot\vec{E}, \qquad (1)$$

where dielectric tensor ε of the medium is written as

$$\varepsilon_{\parallel} = \varepsilon_{\parallel 0} + \varepsilon^1_{\parallel p,h};\ \varepsilon_{\perp} = \varepsilon_{\perp 0} + \varepsilon^1_{\perp p,h}. \qquad (2)$$

The subscript 0 refers to the small amplitude limit. The subscript p or h in the nonlinear correction terms indicates the source of nonlinearity: ponderomotive or heating induced by collisions. The elements of the dielectric tensor as given by Das and Singh[14] are

$$\varepsilon_{\parallel} = 1 - \frac{\omega_{p0}^2}{\omega^2};$$

$$\varepsilon_{\perp 0} = 1 + \frac{\omega_{p0}^2}{\omega(\omega_H - \omega)}, \qquad (3a)$$

$$\varepsilon^1_{\parallel p} = \frac{\omega_{p0}^2}{\omega^2}[1 - \exp(-\alpha_p E_0^2)];$$

$$\varepsilon^1_{\perp p} = \frac{\omega_{p0}^2}{\omega(\omega - \omega_h)}[1 - \exp(-\alpha_p E_0^2)], \qquad (3b)$$

$$\varepsilon^1_{\perp h} = \frac{\omega_{p0}^2}{\omega^2}\left[1 - \frac{1}{(1+\alpha_h E_0^2)}\right];$$

$$\varepsilon^1_{\perp h} = \frac{\omega_{p0}^2}{\omega(\omega - \omega_h)}\left[1 - \frac{1}{(1+\alpha_h E_0^2)}\right], \qquad (3c)$$

$$\alpha_p = \frac{e^2}{16 m K_B T_0 \omega^2}\frac{1}{(1-\omega_H/\omega)};$$

$$\alpha_h = \frac{M e^2}{24 m^2 K_B T_0 \omega^2}\frac{1}{(1-\omega_H/\omega)^2}, \qquad (3d)$$

where m and e are the mass and charge of the electron while M is the mass of heavy scatterer. K_B and T are the Boltzman constant and plasma temperature respectively. Expression (3) is valid for the WKB, paraxial and weak nonlinearity approximations (*i.e.* $\alpha_{p,h}E_0^2 \ll 1$)[15]. Considering a right hand circularly polarised whistler wave $(E_x = iE_y = E_0 \exp[i(\omega t - kz)],\ E = E_x + iE_y)$ and neglecting the second order derivative in axial direction $(\partial^2/\partial z^2)$ equation (1) is rewritten as

$$-2ik\frac{\partial E}{\partial z} + \alpha_{D\perp}\nabla_\perp^2 E + \frac{\omega^2}{c^2}\alpha_{p,h}\varepsilon_{\perp 0}|E|^2 E = 0 \qquad (4)$$

where $k^2 = \frac{\omega^2}{c^2}\varepsilon_{\perp 0}$ and $\alpha_{D\perp} = \frac{1}{2}\left(1 + \frac{\varepsilon_{\perp 0}}{\varepsilon_{\parallel 0}}\right)$. For an isotropic medium $\alpha_{D\perp} = 1$ and expression (4) becomes a parabolic equation with weak non-linearity[15]. In the presence of magnetic field $\alpha_{D\perp} \neq 1$ and the rays diffuse in a transverse direction either more rapidly or more slowly than in the isotropic medium depending upon the relative magnitude of ω_{p0}/ω and ω/ω_H. Using Gaussian intensity distribution of the wave in transverse direction as $EE^* = E_0^2 \exp(-r^2/r_0^2)$ in equation (4) following expressions are obtained

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{f^3}\left[\frac{1}{Z_d^2} - \frac{1}{Z_f^2}\right] = -\frac{1}{f^3 Z_{fd}^2}, \qquad (5a)$$

$$\frac{\partial \phi}{\partial z} = \frac{r_0}{\alpha_{D\perp} f^2}\left[-\frac{1}{Z_d^2} + \frac{1}{Z_f^2}\right]$$

$$= \frac{r_0}{\alpha_{D\perp}}\frac{1}{f^2 Z_{fd}^2}, \qquad (5b)$$

where f, ϕ and r_0 are the beam width parameter, phase factor and radius of the beam respectively.

$$Z_d = kr_0^2/\alpha_{D\perp}$$

and

$$Z_f = \left[\left(\frac{\varepsilon_{\perp 0}}{1 - \varepsilon_{\perp 0}}\right)\frac{1}{\alpha_{p,h}\alpha_{D\perp}}\right]^{1/2}\frac{r_0}{E_0}$$

are defined as diffraction scale length and the focal length due to self-focusing. When $Z_f \approx Z_d$ the diffraction effect is almost balanced by the self-action of the wave and $Z_{fd} \to \infty$ giving $f = 1$ for all values of z showing that the wave is propagating with constant radius. This is usually called as self-channeling or self-trapping of the wave beam with equilibrium radius

$$r_{ost} = \frac{c}{\omega}\left[\frac{\alpha_{D\perp}}{(1 - \varepsilon_{\perp 0})\,\alpha_{p,h}E_0^2}\right]^{1/2}.$$

The wave beam with $r_0 > r_{ost}$ would converge with the increasing distance and the wave will finally be self-focused. The critical value of the wave electric field for self-trapping is given as

$$E_{ocr} \sim \left(\frac{\varepsilon_{\perp 0}}{1 - \varepsilon_{\perp 0}}\right)\frac{\alpha_{D\perp}}{\alpha_{p,h}k^2 r_0^2} \qquad (6)$$

RESULTS AND DISCUSSION:

For the numerical study of self-focusing of whistler waves the ionosphere is modelled as horizontally stratified with simple Chapman layer[17]. The value of $\delta = 2m/M$ and the ionospheric temperature T_0 are chosen corresponding to *D*-, *E*- and *F*-regions[18] as given in table 1.

Table 1

Region	δ	T_0 (°K)
D	2.10^{-3}	3.10^2
E	1.10^{-3}	3.10^2
F	1.10^{-4}	1.10^3

Figure 1 shows variation of $\chi = |(\varepsilon^1_{\parallel,\perp p}/\varepsilon^1_{\parallel,\perp h})|$ with normalised frequency. χ is found to decrease with frequency because the whistler waves propagating through the ionosphere satisfy the condition $\omega \ll \omega_H < \omega_{p0}$ and hence χ varies as $1/\omega$. Further as we have assumed $\alpha_{p,h}E_0^2 \ll 1$, χ becomes independent of wave amplitude. Since for ionospheric whistlers $\omega/\omega_H \sim 10^{-2} - 10^{-1}$, the main contribution to the non-linear dielectric constant comes from the heating effect.

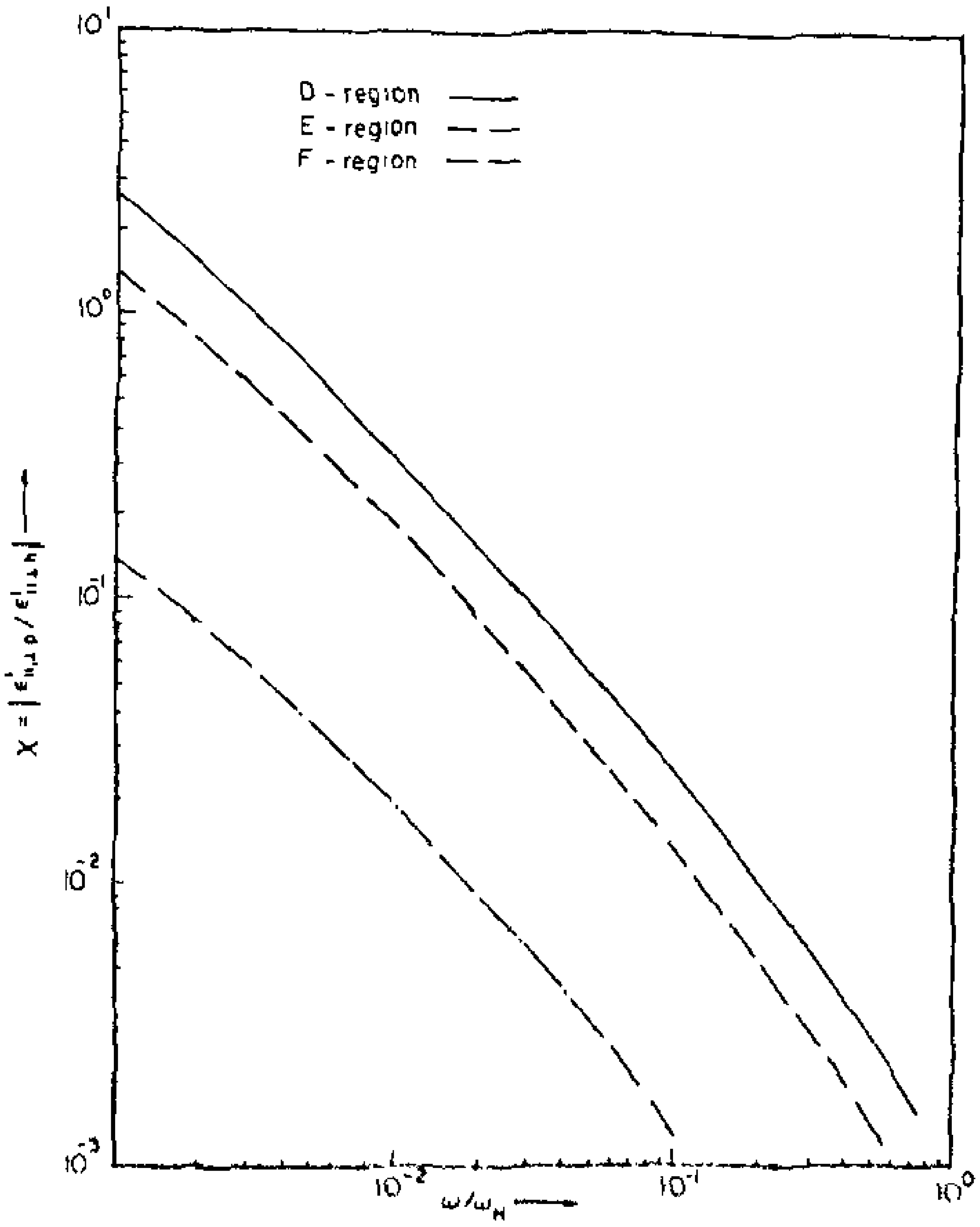

Figure 1. Variation of χ with normalised frequency.

Also, as one moves up in the ionosphere the magnitude of the nonlinear correction decreases.

Variation of $\alpha_{D\perp}$ with normalised frequency (shown in figure 2) is found to be independent of plasma density. This is because in the ionosphere $\omega_{p0}^2 \gg \omega^2, \omega_H^2$ and $\varepsilon_{\parallel 0}/\varepsilon_{\perp 0}$ becomes independent of ω_{p0}. Also, $\alpha_{D\perp}$ changes its sign at $\omega_H/2$. By taking an exact form of f and ϕ, Litvak[12] has shown that $\alpha_{D\perp} > 0$ and $\alpha_{D\perp} < 0$

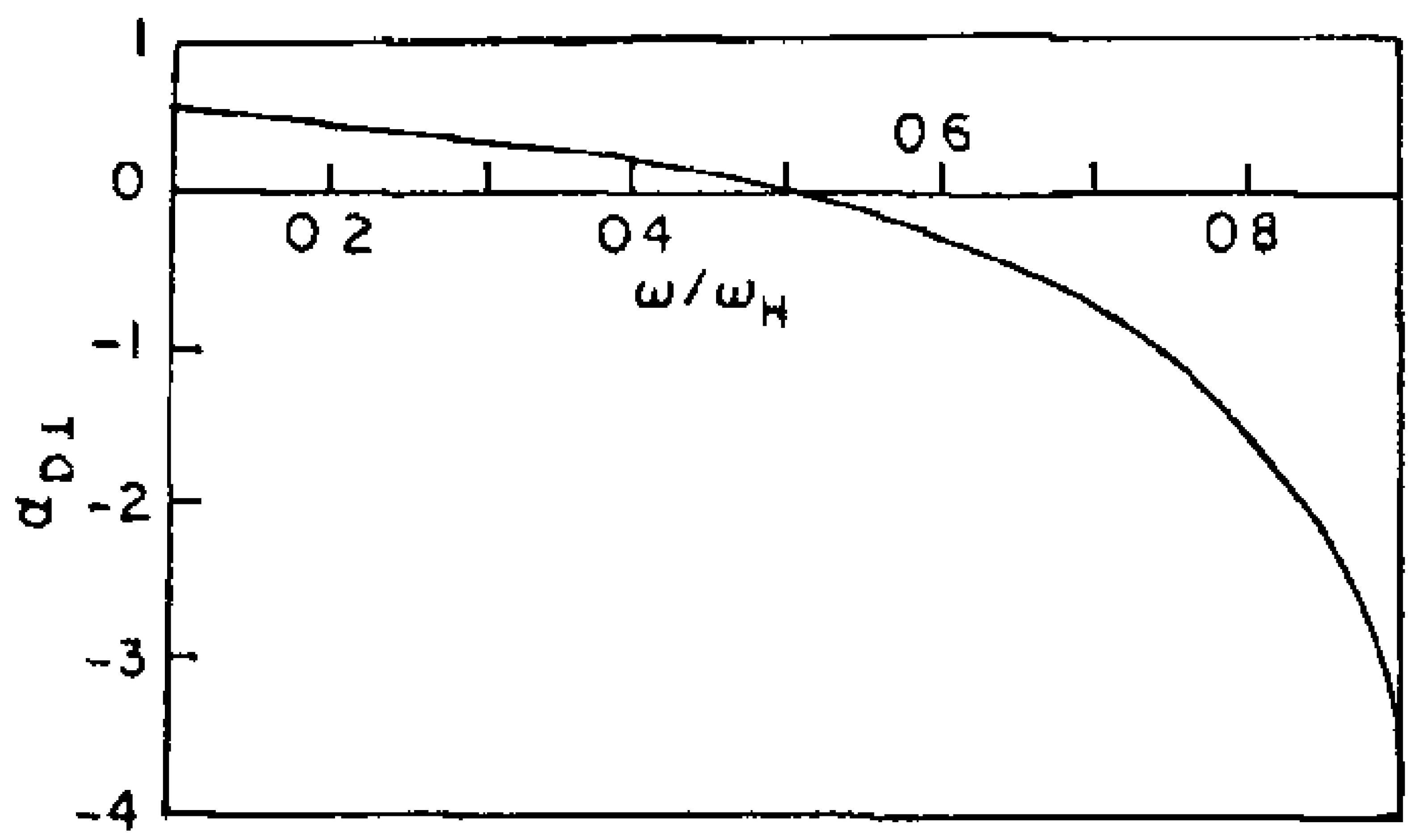

Figure 2. Variation of $\alpha_{D\perp}$ with normalised frequency.

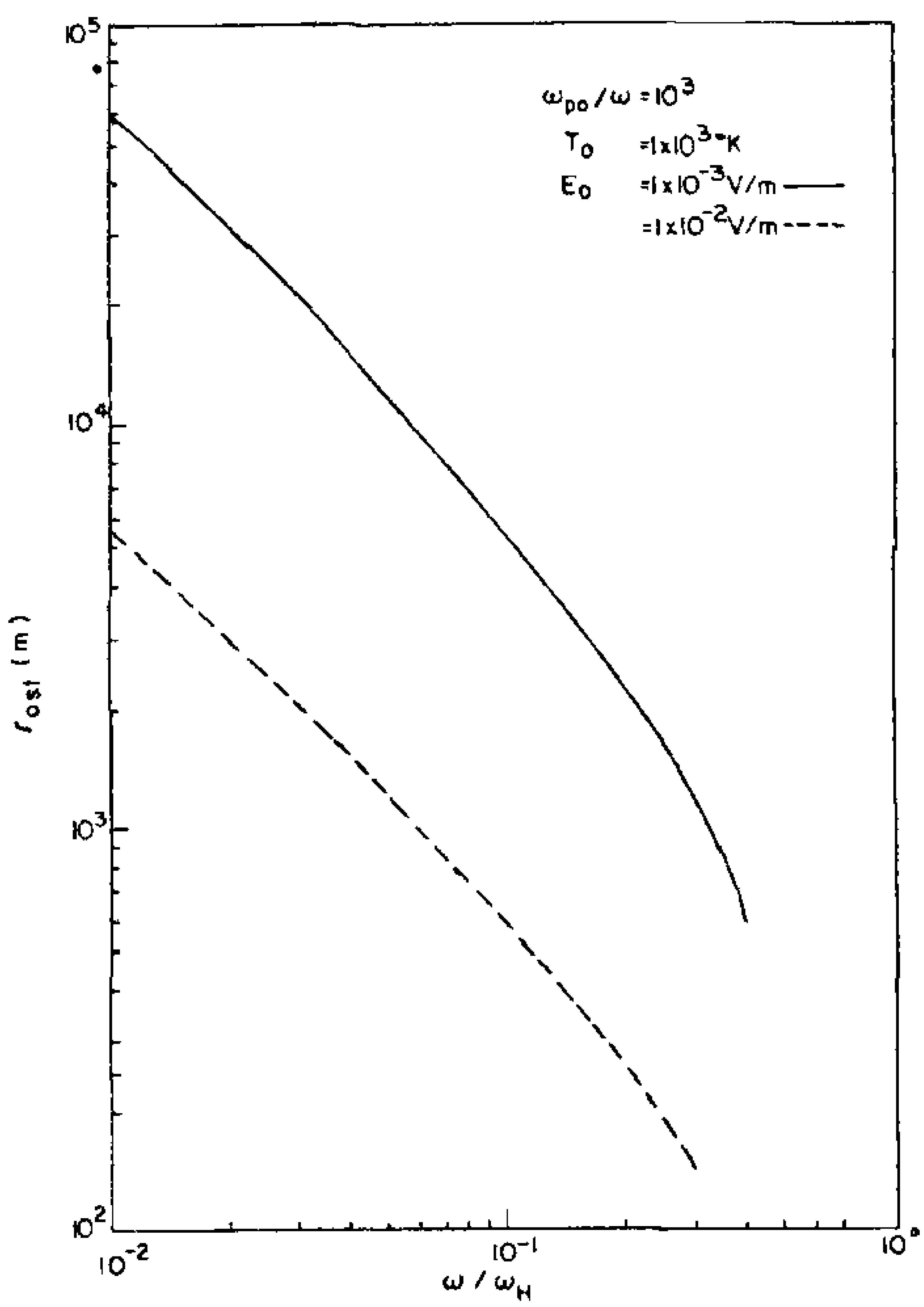

Figure 3. Variation of r_{ost} with normalised frequency.

corresponds to self-focusing of the wave beam with negative and positive values of the radius of curvature of the phase front respectively.

Figure 3 gives variation of the minimum radius r_{ost} required for self-focusing with normalised frequency. The radius of the self-trapped beam decreases with the increasing frequency or wave amplitude. For a wave of a given intensity and frequency, an increase in plasma frequency results into a decrease in r_{ost}. Thus, in a dense plasma medium wave of comparatively smaller radius can also be self-focused. The value of r_{ost} in the ionosphere is of the order of a few kms. Equation (6) along with (5) shows that the wave is not self-focussed due to heating induced nonlinearity which is consistent with the results of Sodha *et al*[19].

Variation of normalised self-focusing distance Z_{fd}/r_0 with frequency is shown in figure 4. It is found that Z_{fd}/r_0 increases slowly at lower frequency and rapidly at higher frequency becoming infinite at $\omega_H/2$. Thus, self-focusing for $\omega > \omega_H/2$ is not possible. Further, with the increasing wave amplitude, Z_{fd}/r_0 decreases which is in conformity with the results of Sodha and Tripathi[11].

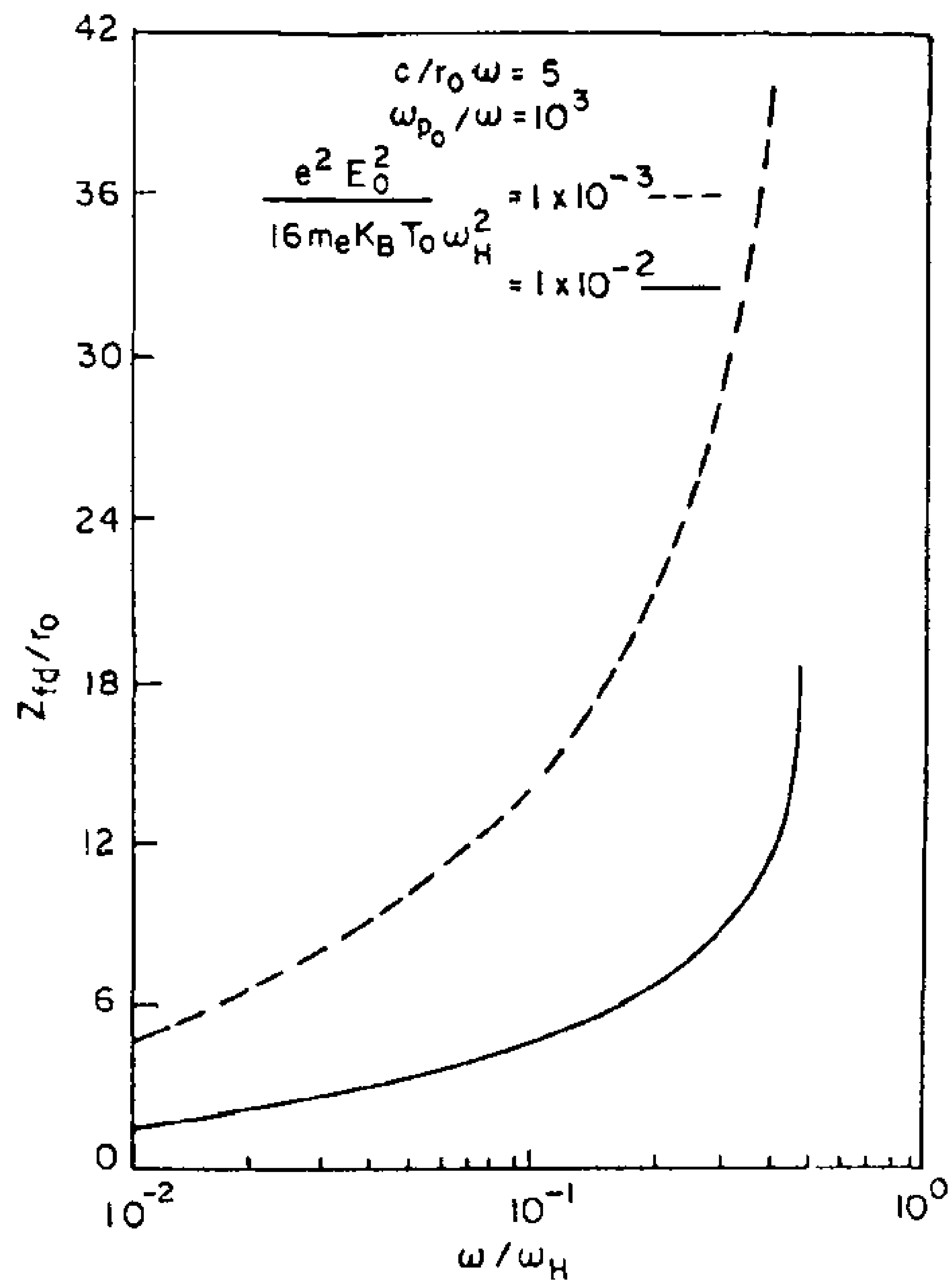

Figure 4. Variation of Z_{fd}/r_0 with normalised frequency.

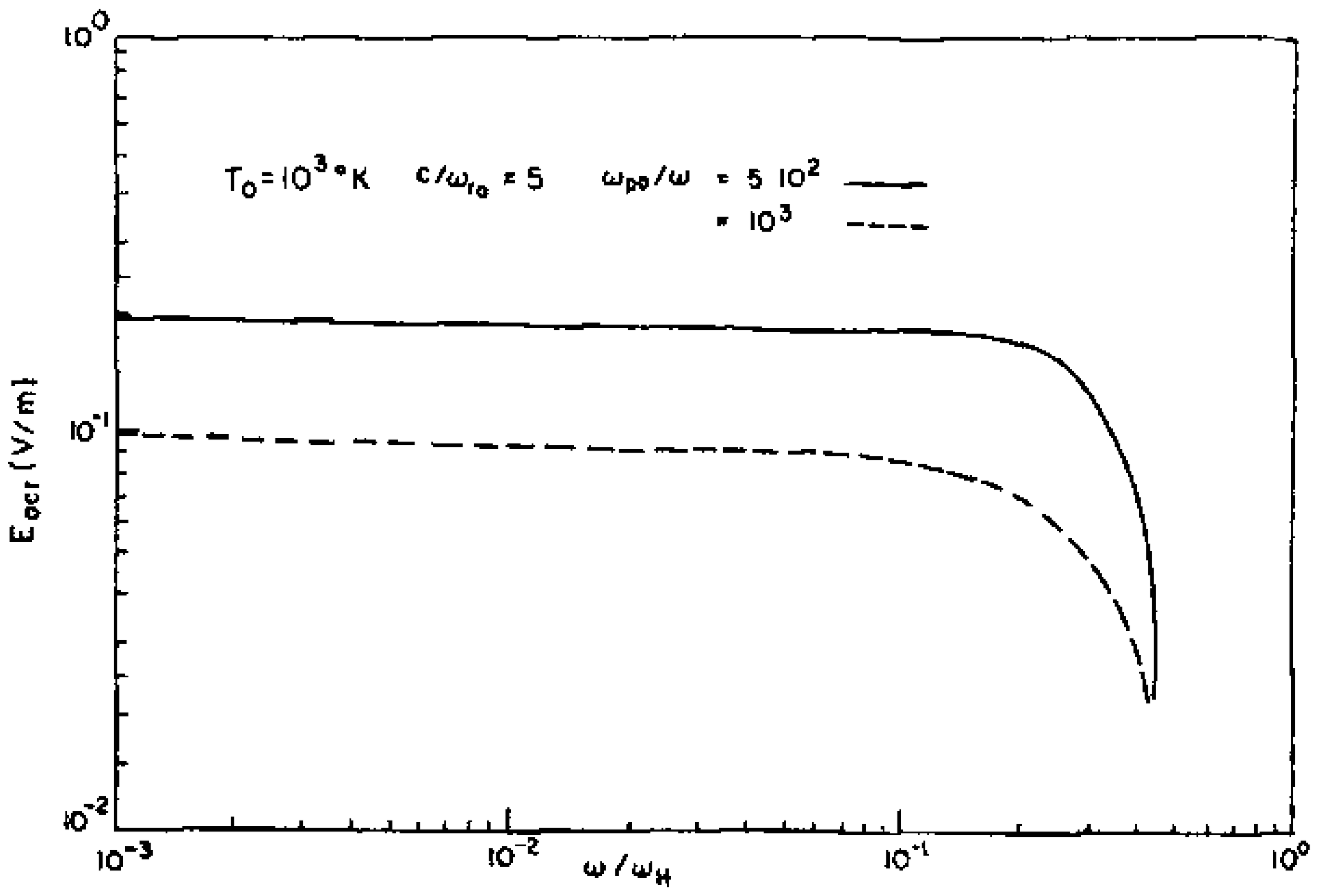

Figure 5. Variation of E_{ocr} with normalised frequency.

Figure 5 gives the variation of E_{ocr} with frequency. It is found that E_{ocr} decreases as frequency increases. The value of E_{ocr} is found to be of the order of a few mV/m which is well within the experimentally observed range of whistler wave amplitudes[20,21]. It is clear that for a fixed frequency, value of E_{ocr} is lower in a denser medium. The present results also closely agree with those of Taniuti and Washimi[22] and Washimi[23].

To detect and identify self-focusing of whistlers in space plasma due to ponderomotive force, experiments using high intensity whistler waves transmitted from a powerful transmitter with varying frequency should be conducted. Due to self-focusing, the wave beam splits into various filaments in its transverse direction which may be used to explain the banded structure of recorded whistler sonograms. Since the filamentation and self-focusing of different frequencies will occur at different points along the wave path, one can expect concentrations of energy at different points on the recorded sonograms which may look like banded whistlers. It is tentatively suggested that the detailed theory on this line should be worked out.

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