POSITRON SCATTERING BY HYDROGEN, HELIUM AND LITHIUM ATOMS

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ABSTRACT

Differential scattering cross-sections for elastic scattering of positrons by some light atoms, (hydrogen, helium and lithium) are calculated in the incident energy (E) range 100 to 800 eV. These studies are made within the framework of high energy higher order Born and Glauber eikonal series approximations. The results are in good agreement with other similar studies.

INTRODUCTION

Studies on positron scattering by atoms are as important as electron scattering by atoms in 'atomic physics'. Unfortunately, studies on the cross-sections for positron scattering by atoms are scarce and only a very limited number of approximations are applied for this study, particularly for helium and lithium atoms. We have therefore attempted to extend our studies\(^1\)-\(^10\) for the present elastic process of positron scattering by H, He and Li atoms. All deficiencies in the Glauber\(^11\) and Glauber eikonal series\(^12,13\) approximations are superseded in the present approximations. The advantages of HHOB\(^14\) approximation are (i) all the integrals in the scattering amplitudes are finite (ii) the integrals obtained are in the closed forms (iii) computational procedure is simpler compared to other approximations (iv) results obtained by this approximation are in good agreement with the compared data. We have therefore applied HHOB\(^14\) and GES\(^12\) approximations for the present study. After the formulation of scattering amplitudes the DCS\(^+\) (differential scattering cross-section for positrons) results at a few incident energies for H, He, Li atoms are given in table 1 along with other results\(^15,16\). Finally the present DCS\(^+\) results are discussed with similar results\(^15,16\).

THEORY

Let us consider the nonrelativistic scattering of a positron by a neutral atom of atomic number \(Z\) (= 1, 2, 3 in the present study). Atomic units are used throughout and correspondingly \(K_i, K_f, \) and \(q = K_i - K_f\) denote, respectively, the initial and final momenta of the scattering positron, and the momentum transferred to the target as a result of the elastic collision. Choose the nucleus of the atom as the origin of our coordinate system. Let \(r_0\) and \(r_j\) \((j = 1, 2, \ldots, Z)\) be the coordinates of the incident positron and of the atomic electrons. We also use \(X, +\) and \(-\), to represent the target coordinates and corresponding terms for positron and electron scattering by atoms.

The basic idea for the present study is the nature of interactions between the incident positron (or electron) and the target atom. These interactions, can be given as

\[
V^+(r_0, X) = \frac{Z}{r_0} - \sum_{j=1}^{Z} \frac{1}{|r_0 - r_j|} (1)
\]

\[
V^-(r_0, X) = \frac{Z}{r_0} + \sum_{j=1}^{Z} \frac{1}{|r_0 - r_j|} - V^+(r_0, X), (2)
\]

The right side of (1) shows that the nuclear and electrostatic terms are repulsive and attractive interactions respectively. This situation is quite opposite in the case of electron interaction with atom (2). Now the first Born amplitude can be obtained as

\[
f^{+(1)}[q, y_n] = \frac{1}{2\pi} \int dr_0 \exp(iq.r_0) V^+(r_0)
\]

\[
= -f^{-(1)}[q, y_n] (3)
\]

where

\[
V^+(r_0) = \langle \Psi_f (X)|V^+(r_0, X)|\Psi_i (X) \rangle
\]

and \(\Psi_f (X), \Psi_i (X)\) are the final and initial state wavefunctions\(^1,4,6\) of the target H, He, Li atoms, \(y_n\)'s are exponential parameter of the target wavefunctions\(^1,4,6\). The real and imaginary parts of the second Born approximations remain the same for both electron and positron scattering. A general form of these terms can be written as

\[
\text{Re} f^{+(2)}_{\text{HEA}}[q, y_n, B] = \text{Re} f^{-(2)}_{\text{HEA}}[q, y_n, B], (4)
\]

\[
\text{Re} f^{+(2)}_{\text{HEA}}[q, y_n, B] = \text{Re} f^{-(2)}_{\text{HEA}}[q, y_n, B], (5)
\]
Table 1. DCS* (a₀²Sr⁻¹) results for elastic scattering of positrons by hydrogen, helium and lithium atoms.

<table>
<thead>
<tr>
<th></th>
<th>Hydrogen, E = 200 eV</th>
<th>Hydrogen, E = 400 eV</th>
<th>Helium, E = 400 eV</th>
<th>Lithium, E = 400 eV</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>HHOB</td>
<td>GES</td>
<td>Others¹⁵</td>
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</tr>
<tr>
<td>θ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
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<td>2.82</td>
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<td>9.74</td>
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<td>10</td>
<td>8.62</td>
<td>1.03</td>
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<td>3.96</td>
<td>1.52</td>
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<td>1.67</td>
<td>2.05</td>
<td>—</td>
<td>5.52</td>
</tr>
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<td>110</td>
<td></td>
<td></td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

The symbol a ± b denotes a × 10 ± b

\[
\text{Im}^+ f^{(2)}_{\text{HHOB}}(q, y, B) = \text{Im}^- f^{(2)}_{\text{HHOB}}(q, y, B),
\]

(6)

B is average excitation energy in the above terms (4), (5) and (6) and the third second Glauber eikonal series terms can be written as

\[
f^{+^{(3)}}_{\text{GES}}(q, y) = -f^{-^{(3)}}_{\text{GES}}(q, y),
\]

(7)

and

\[
f^{\pm^{(2)}}_{\text{GES}}(q, y) = \text{Im}^\pm f^{(2)}_{\text{HHOB}}(q, y, B),
\]

\[
\therefore \text{Im}^\pm f^{(2)}_{\text{HHOB}}(q, y, B).
\]

(8)

Using these modified scattering amplitudes (3) to (8) for hydrogen¹, helium⁴ and lithium⁶ atoms, the DCS* through order \((K_l^{-1})\) can be obtained in HHOB and GES approximations from the following direct scattering amplitudes.

\[
\sigma^{d}_{\text{HHOB}} = [f^{+^{(1)}}_{\text{HHOB}}(q, y) + \text{Re} f^{(2)}_{\text{HHOB}}(q, y, B)]^2 + \text{Re}^2 f^{(2)}_{\text{HHOB}}(q, y, B) + f^{(3)}_{\text{GES}}(q, y) + \text{Im}^+ f^{(2)}_{\text{HHOB}}(q, y, B)\]

(9)

\[
\sigma^{d}_{\text{GES}} \simeq |f^{+^{(1)}}_{\text{GES}}(q, y) + f^{+^{(3)}}_{\text{GES}}(q, y)|^2 + \text{Im}^+ f^{(2)}_{\text{HHOB}}(q, y, B)^2.
\]

(10)

Using (9) and (10), the DCS* results are calculated for hydrogen, helium and lithium atoms.

RESULTS AND DISCUSSIONS

Table 1 shows the present DCS* results for H, He and Li atoms at incident energies 200 and 400 eV in the angular range \(\theta \leq 110\). These results for H, and He are compared with the available results of optical¹⁵ and eikonal Born Series (EBS)¹⁶ methods. DCS* values of the HHOB approximation are found to be smaller than GES values and nearer the compared¹⁵,¹⁶ results. The HHOB approximation, DCS*, for hydrogen atom is in good agreement with the compared¹⁶ results in the small and intermediate angular regions and the results for helium atom are satisfactory with the compared¹⁵ results in this angular region. It is difficult to comment on the DCS* of lithium atom because of the unavailability of data for comparison.

The present results could be further improved if we consider the third Born term¹⁴ instead of the third GES term¹² in the present DCS* calculations. The approximations employed can be easily extended to the study of DCS* for inelastic process⁴,¹⁰.

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(a) His/Her age should be 35 years or below on the date of commencement of the conference.
(b) His/Her paper should be accepted.
(c) His/Her local hospitality has to be borne by the organisers or some other agency.

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