SHORT COMMUNICATIONS

MHD FLOW OF A DUSTY GAS THROUGH A HEXAGONAL CHANNEL

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The attention of many research workers in fluid mechanics has been directed towards studying the influence of dust particles on the motion of fluids in the past few years. Recently, the flow of a viscous incompressible gas with uniform distribution of dust particles through a hexagonal channel with arbitrary time varying pressure gradient has been studied.

The present investigation aims at extending the analysis of Gupta and Varshney in the case of a transverse magnetic field.

Formulation and solution of the problem

Using a rectangular cartesian coordinate system (x, y, z) such that z-axis is along the axis of the channel and the cross-section of the channel is formed by the straight lines:

\[ y = \pm \frac{a\sqrt{3}}{2} \quad \text{and} \quad y \pm \sqrt{3}x = \pm a\sqrt{3}. \]

(1)

The gas and particle velocities \(u(x, y, t)\) and \(v(x, y, t)\), respectively, are in z-direction.

Under the transformations

\[ x_1 = y, \quad x_2 = y - \sqrt{3}x \quad \text{and} \quad x_3 = y + \sqrt{3}x, \]

the non-dimensional momentum equations after applying a constant transverse magnetic field of strength \(H_0\) appear in the form:

\[ \frac{\partial u}{\partial t} = f(t) + \left( \frac{\partial^2}{\partial x_1^2} + 4 \frac{\partial^2}{\partial x_2^2} + 4 \frac{\partial^2}{\partial x_3^2} \right)u + 2 \frac{\partial^2}{\partial x_1 \partial x_2} + 2 \frac{\partial^2}{\partial x_1 \partial x_3} - 4 \frac{\partial^2}{\partial x_2 \partial x_3} \right)u + \beta(u - u) - MU, \]

(2)

\[ \frac{\partial v}{\partial t} = L(u - v), \]

(3)

where \(\beta = (fr) = N_0 K a^2 / \rho v, \quad f = N_0 m / \rho, \quad r = m v / K a^2, \quad L = 1/\gamma, \quad M = \sigma B^2 a^2 / \rho v\) (magnetic field parameter) are dimensionless constants and \(-\partial p / \partial z = f(t)\), an arbitrary function of time, \(B_0(=\mu_0 H_0)\) the component of electromagnetic induction, \(\sigma\) the electrical conductivity, \(\mu_0\) the magnetic permeability and other symbols have their usual meanings.

Non-dimensional initial and boundary conditions are:

(i) \(t \leq 0, \quad u(x_1, x_2, x_3, t) = 0 = v(x_1, x_2, x_3, t)\),

\(\text{every where in the channel}, \)

(4)

(ii) \(t > 0, \quad u(x_1, x_2, x_3, t) = 0 = v(x_1, x_2, x_3, t)\)

at \(x_1 = \pm \sqrt{3}/2, \quad x_2 = \pm \sqrt{3}\) and \(x_3 = \pm \sqrt{3} \).

(5)

Due to symmetry of motion about \(x_1 = 0, \quad x_2 = 0\) and \(x_3 = 0\), we shall consider the motion in the region \(x_1 \geq 0, \quad x_2 \geq 0\) and \(x_3 \geq 0\) and accordingly the boundary conditions (5) become

\[ u(x_1, x_2, x_3, t) = 0 = v(x_1, x_2, x_3, t), \]

at \(x_1 = \sqrt{3}/2, \quad x_2 = \sqrt{3}\) and \(x_3 = \sqrt{3}, \)

(6)

and

\[ \frac{\partial u}{\partial x_1} = 0 = \frac{\partial v}{\partial x_1} \text{ at } x_1 = 0; \]

\[ \frac{\partial u}{\partial x_2} = 0 = \frac{\partial v}{\partial x_2} \text{ at } x_2 = 0; \]

\[ \frac{\partial u}{\partial x_3} = 0 = \frac{\partial v}{\partial x_3} \text{ at } x_3 = 0. \]

(7)

To solve the problem by using the technique of integral transforms, we use finite Fourier cosine transforms and inversion formula. Multiplying (2) and (3) by \([\cos (P_1 x_1) \cdot \cos (Q_2 x_2) \cdot \cos (R_3 x_3)]\) and integrating within the limits \(0\) to \(\sqrt{3}/2\), \(0\) to \(\sqrt{3}\) and \(0\) to \(\sqrt{3}\) and using conditions (6) and (7), also using finite Fourier cosine transforms, we get

\[ \partial U / \partial t = \frac{(-1)^{p+r}}{P_1 Q_2 R_3} f(t) - b_{pr} U + \beta(V - U) - MU, \]

(8)

\[ \partial V / \partial t = L(U - V), \]

(9)

where \(U, \quad V, \quad b_{pr}, \quad P_1, \quad Q_2, \quad R_3\) have the same values as in Gupta and Varshney.

We apply Laplace transforms to (8) and (9) under the transform initial conditions \(U = 0 = V\) at \(t = 0, \) we get

\[ vU = \frac{(-1)^{p+r}}{P_1 Q_2 R_3} f(s) - b_{pr} U + (V - U) - MU, \]

(10)

\[ \delta V = L(U - V), \]

(11)
where $U$, $V$ and $T(s)$ are Laplace transforms of the respective quantities.

Now to obtain $u$ and $v$ in the case of constant pressure gradient, first solve (10) and (11) for $U$ and $V$ and then invert Laplace transforms by convolution theorem and after that applying inversion formulae for finite cosine transforms and then putting $f(t) = C$, where $C$ is an absolute constant, and on simplifying, we get

$$u = \frac{16C}{3\sqrt{3}} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} (-1)^{p+q+r} \frac{P_{p}Q_{q}R_{r}(b_{pq} + M)}{(x_{1} - x_{2}) \left\{ (x_{1} + M + b_{pq}) \exp(x_{1}t) 
- (x_{2} + M + b_{pq}) \exp(x_{2}t) \right\}} \times \cos(P_{p}x_{1}) \cos(Q_{q}x_{2}) \cos(R_{r}x_{3}), (12)$$

$$v = \frac{16C}{3\sqrt{3}} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} (-1)^{p+q+r} \frac{P_{p}Q_{q}R_{r}(b_{pq} + M)}{(x_{1} - x_{2}) \left\{ (x_{1} + M + b_{pq}) \exp(x_{1}t) 
- (x_{2} + M + b_{pq}) \exp(x_{2}t) \right\}} \times \left( \frac{1 - \alpha_{1} \exp(x_{1}t) - \alpha_{2} \exp(x_{2}t)}{\alpha_{1} - \alpha_{2}} \right) \times \cos(P_{p}x_{1}) \cos(Q_{q}x_{2}) \cos(R_{r}x_{3}), (13)$$

**Figure 2.** Velocity profiles of particles for a constant pressure gradient $C$, for different values of $M$ and $T$.

**Figure 1.** Velocity profiles of gas for constant pressure gradient $C$, for different values of $M$ and $T$.

**Deduction**

When magnetic field parameter $M \rightarrow 0$, the problem reduces to that considered by Gupta and Varshney.

**Conclusion**

We have plotted $u$ and $v$ for different values of $t$ and $M$ with $r = 0.8$ and $f = 0.2$ under constant pressure gradient, as shown in figures 1 and 2. It is clear that $u$ and $v$ decrease with the increase of $M$. Thus the implementation of a magnetic field is a device for laminar flow control, while $u$ and $v$ increase with the lapse of time and have a maximum on the axis of the channel. It is also evident from the figures that the gas moves faster than the particles.

Authors are highly thankful to the learned referee for his critical comments and suggestions.

31 March 1984; Revised 17 July 1984


**LIGHT SCATTERING FROM (NH₄)₂ZnBr₄ IN THE INCOMMENSURATE PHASE**

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(NH₄)₂ZnBr₄ belongs to the (NH₄)₂ZnCl₄ family of crystals which exhibit successive phase transitions and incommensurate phases. X-ray structural studies have shown that this crystal has an incommensurate phase in the range 395-432 K. Cs₂ZnCl₄, another member of the family, also shows a high temperature phase transition which is probably to an incommensurate phase. This paper reports light scattering intensity and Raman spectroscopic studies of the high temperature incommensurate phase in (NH₄)₂ZnBr₄. Preliminary Raman spectroscopic studies, particularly in the low temperature ferroelectric phase, have been reported elsewhere.

Polycrystalline samples of (NH₄)₂ZnBr₄ were grown at room temperature (20-25°C) by slow evaporation of the aqueous solution of NH₄Br and ZnBr₂ in stoichiometric proportions. Unpolarised light scattering intensity and Raman spectroscopic measurements were made, using techniques described in an earlier communication.

Figure 1 shows the total intensity of light scattered at 90° from the sample as function of temperature. A very large increase in the vicinity of 400 K shows striking evidence of the incommensurate phase.

Figures 2 and 3 show the results of Raman spectroscopic studies on the same sample. The peak heights of three different Raman lines (figure 2), corresponding to internal modes of the ZnBr₂⁻ ion, versus temperature show a large increase in the region corresponding to the incommensurate phase and the integrated areas under two Raman bands (figure 3) show a similar behaviour.

The incommensurate phase is characterised by a complex order parameter with amplitude fluctuations (amplitudons) and phase fluctuations (phasons). Amplitudons are generally observable by Raman spectroscopy, whereas phasons contribute to the Central Peak. A soft amplitude mode was observed in