ATMOSPHERIC TIDAL OSCILLATIONS
PART 1. HISTORICAL DEVELOPMENT*

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ABSTRACT

The invention of the barometer in the seventeenth century led to the discovery of atmospheric tidal oscillations with a pronounced solar semi-diurnal component. The resonance theory proposed by Kelvin to explain these oscillations had to be abandoned around the 1950's when knowledge about the vertical thermal structure of the atmosphere became available through rocket soundings. The current theory known as thermo-tidal theory shows that atmospheric tides have their origin in the absorption of solar radiation by water vapour and ozone in the atmosphere.

INTRODUCTION

The radiant energy received from the sun drives the atmospheric and oceanic circulations which control terrestrial weather and climate through a complex system of energy transformations and feed-back mechanisms. Besides its over-all thermal action on the earth-atmosphere system, the sun and the moon also exert gravitational attraction on the ocean and the atmosphere. The rotation of the earth on its axis gives rise to periodic diurnal variations of the gravitational and thermal effects at any given point on the earth's surface which generate oscillations of the oceans and the atmosphere known as tides or tidal oscillations. If T is the period of a solar/lunar day, the tidal oscillations have periods equal to T/n where $n = 1, 2, 3, \ldots$.

SOME FEATURES OF OCEANIC TIDES

The oceanic tides manifested as the rise and fall of sea level at coastal stations twice a day have been known from ancient times. The association between oceanic tides and the meridional transists of the moon had also been noticed. However, the physical mechanism remained unknown until Newton[1] showed that the oceanic tides are a consequence of the gravitational attraction of the moon and the sun on the oceanic waters. The dynamical theory of oceanic tides was developed in great detail by Newton's successors, in particular by Laplace[2].

The essential features of the oceanic tides which are caused by the gravitational attraction of the moon and the sun can be understood from simple considerations. Let us consider a spherical non-rotating earth covered by a shallow ocean of uniform depth, and the sun as the attracting body. The orbital motion of the earth round the sun is governed by the balance between the centrifugal force and the gravitational attraction of the sun. Referring to figure 1 this balance can be symbolically represented by the equation

$$\omega^2 r_s E = GSE/r_s^2 \tag{1}$$

or

$$\omega^2 = GS/r_s^3 \tag{1a}$$

where ω = angular velocity of the earth in its orbital motion, r_s is the central sun-earth distance, G is the gravitational constant and E, S are

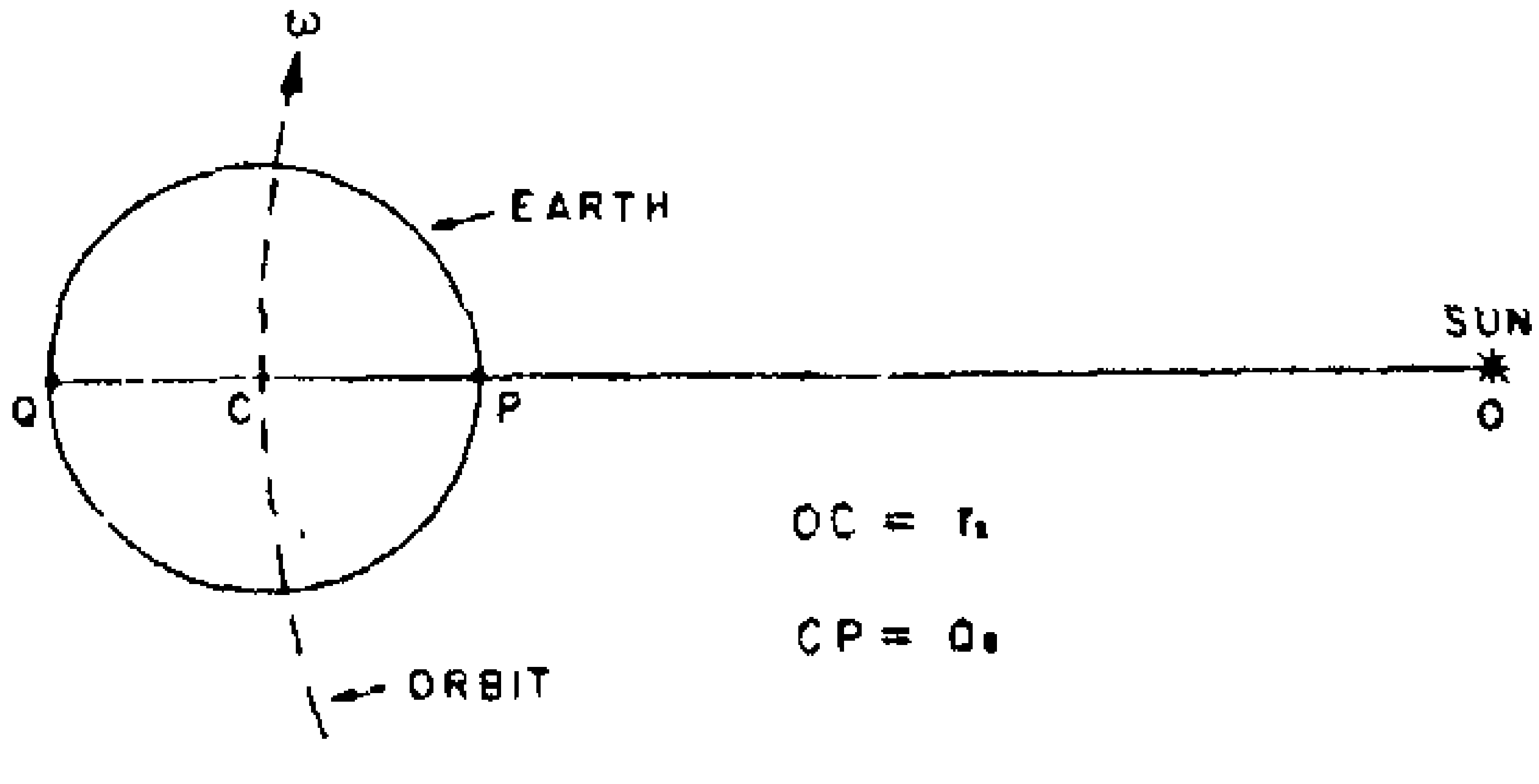

Figure 1. Orbital motion of the earth

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the masses of the earth and the sun respectively.

Let a_e be the radius of the earth. Considering unit mass at P, the centrifugal force (f_1) and the gravitational force of attraction by the sun (f_2) on it are:

$$f_1 = \omega^2(r_s - a_e) \text{ and } f_2 = GS/(r_s - a_e)^2 \quad (2)$$

From the equation (1) we see that $f_1 < f_2$. Hence unit mass at P experiences an unbalanced force ($f_2 - f_1$) towards the sun.

Similarly, considering unit mass at Q the centrifugal force (f'_1) and gravitational force of attraction (f'_2) are

$$f'_1 = \omega^2(r_s + a_e) \quad \text{and} \quad f'_2 = GS/(r_s + a_e)^2. \quad (3)$$

Comparison with (1) shows that in this case $f'_1 > f'_2$. Unit mass at Q experiences an unbalanced force ($f'_1 - f'_2$) directed away from the sun.

The magnitude of the unbalanced force in both cases can be written as ($\omega^2 r - GS/r^2$) where r is the distance of the unit mass from the sun. The gradient of this quantity at $r = r_s$ is the tidal force exerted by the sun and is given by

$$F_s = 3GS/r_s^3. \quad (4)$$

The tidal force is directly proportional to the mass of the tide-raising body and inversely proportional to the cube of its distance from the earth. Since the water particles are free to move, the spherical ocean would assume a spheroidal shape on a non-rotating earth with the major axis of the spheroid along the line OC in figure 1. This is known as the 'solar equilibrium tide'.

Following similar reasoning the tidal force exerted by the moon can be written as:

$$F_m = 3GM/r_m^3 \quad (5)$$

where M = mass of the moon and r_m = the central moon-earth distance.

The ratio of the tidal forces exerted by the moon and the sun is given by

$$\begin{aligned} F_m/F_s &= M/S\,(r_s/r_m)^3 \\ &= [\,(a_m/r_m)\,(r_s/a_s)]^3\,(\rho_m/\rho_s) \quad (6) \end{aligned}$$

where a_m, a_s are the radii of the moon and sun respectively and ρ_m, ρ_s the corresponding mean densities of the two astronomical bodies. Since the angular diameter of the moon and the sun are nearly the same as seen from the earth, $a_m/r_m \approx a_s/r_s$. Hence

$$F_m/F_s \approx \rho_m/\rho_s = 3.34/1.41 = 2.37. \quad (7)$$

Despite its much smaller mass compared with that of the sun, the proximity of the moon to the earth renders its gravitational tidal effect about 2.4 times that of the sun. The oceanic tides are, therefore, primarily controlled by the moon. At the times of the new moon and full moon when the sun and moon are nearly in a line with the earth, the solar and lunar gravitational effects reinforce each other with high tidal range (spring tides). At half moon when the moon and sun have an angular separation of about 90°, the tidal range is smaller (neap tides).

On a rotating earth, every point will experience two high tides and two low tides in the course of a day. Since a lunar day is about 50 min longer than a solar day, the times of high and low tides get progressively delayed from one day to the next completing a cycle in the course of a lunar month. This simplified picture gives some of the basic ideas; the quantitative aspects of the actual oceanic tides are more complex because of the continental barriers, the non-uniform depth of the oceans, etc.

OBSERVATIONAL FEATURES OF ATMOSPHERIC TIDES

The existence of tidal pressure oscillations in the atmosphere came to be recognised after the invention of the barometer by Torricelli in 1643. At tropical latitudes where the day to day changes of pressure are small, the barometer shows a systematic rise and fall of pressure in the course of the day, with maxima around 1000 and 2200 hr and minima around 0400 and 1600 hr local mean solar time (LMT) This feature is illustrated by figure 2 which shows the barometric oscillations at four Indian stations—Trivandrum, Kodaikanal, Pune and Jodhpur—for the months of January and July. The coordinates of the stations and the 24-hourly mean pressure values are also indicated in the diagram. At mid-latitudes these oscillations are generally masked by the much

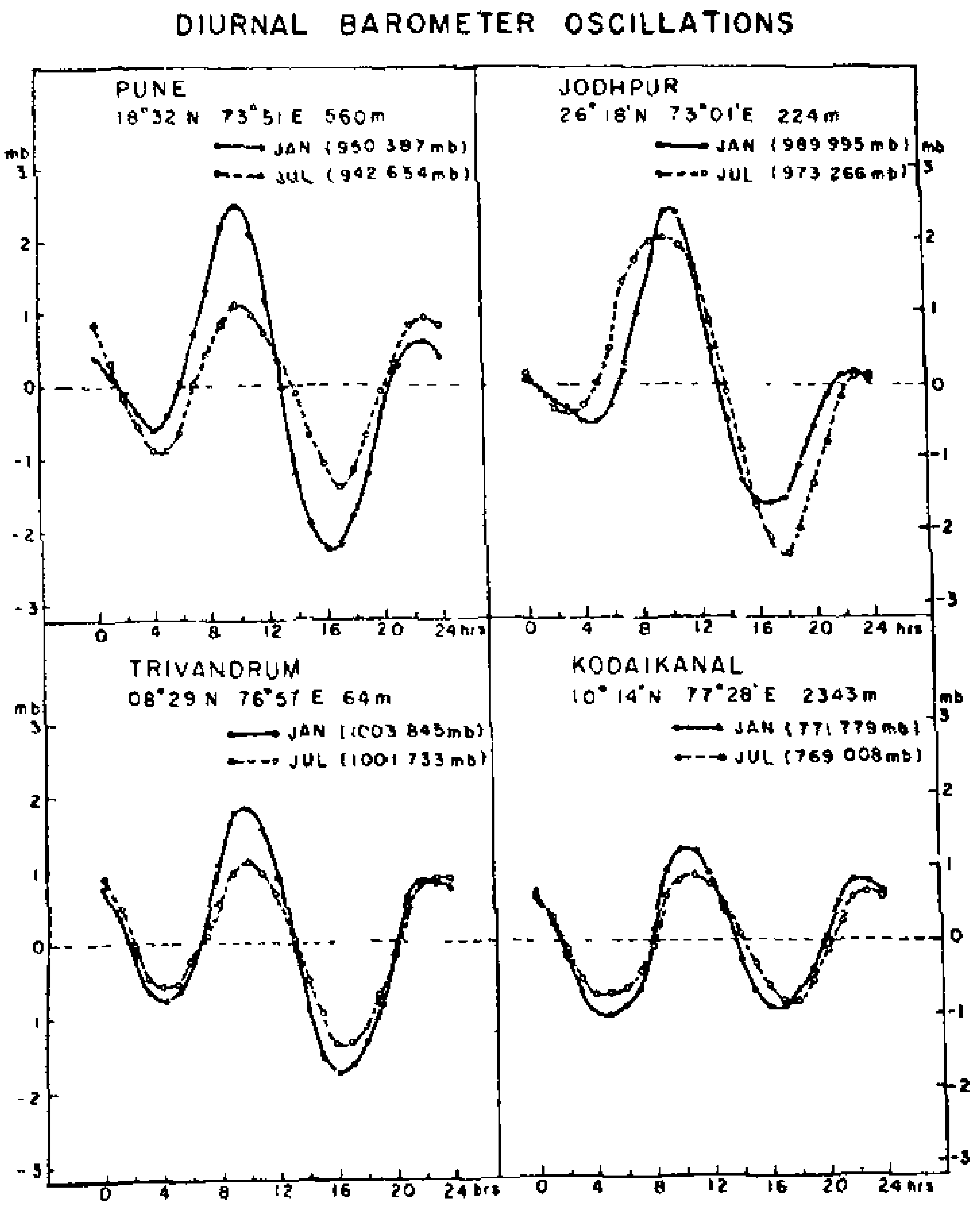

Figure 2. Diurnal variation of pressure at 4 stations

larger synoptic scale pressure changes associated with migratory cyclones and anti-cyclones. Nevertheless, analysis of the hourly pressure data, averaged over a long period to eliminate the irregular pressure fluctuations, reveals the existence of the semi-diurnal barometric oscillations at these latitudes also but with lesser amplitude than in the tropics. This shows that these oscillations are global in nature.

An important and striking difference between the atmospheric tidal pressure oscillation and its oceanic counterpart is that in the case of the atmosphere, the times at which the maxima and minima occur remain practically unchanged from one day to the next. This shows that the atmospheric tidal oscillation is not gravitationally excited. Otherwise, the lunar control should be more than the solar and the times of maxima and minima should follow the lunar day as in the case of oceanic tides. Thus, it became clear from the very beginning that the barometric oscillations are primarily of solar origin and also that gravitational attraction is not the exciting agency. The other periodic influence of the sun on the earth-atmosphere system is the diurnal heating cycle. Hence it was inferred that atmospheric tides are thermally excited oscillations.

KELVIN'S RESONANCE THEORY

The thermal excitation theory of barometric oscillation was, however, confronted with a serious difficulty stemming from the results of the analysis of the observational data. Harmonic analysis of the hourly pressure values shows that in general, the second harmonic of the pressure oscillation denoted by $S_2(p)$ has larger amplitude than the first harmonic denoted by $S_1(p)$. Similar analysis of the hourly temperature values shows that the first harmonic $S_1(T)$ is far more dominant than the second harmonic $S_2(T)$. If the barometric oscillation is of thermal origin, one should expect $S_1(p)$ to be stronger than $S_2(p)$ which is not the case.

Towards the end of the last century this problem attracted the attention of Lord Kelvin[3] and he advanced a tentative explanation which came to be known as the *resonance theory*. Kelvin argued as follows:—

"The cause of the semi-diurnal variation of barometric pressure cannot be the gravitational tide-generating influence of the sun because if it were there should be a much larger lunar influence of the same kind while in reality the lunar barometric tide is insensible or nearly so. It seems, therefore certain that the semi-diurnal variation of the barometer is due to temperature. Now, the diurnal term in the harmonic analysis of the variation of temperature is undoubtedly much larger in all, or nearly all places than the semi-diurnal. It is then very remarkable that the semi-diurnal term of the barometric effect of the variation of temperature should be greater, and so much greater as it is than the diurnal. The explanation probably is to be found by considering the oscillations of the atmosphere, as a whole, in the light of the very formulas which Laplace gave in his *Méchanique Céleste* for the ocean, and which he showed to be also applicable for the atmosphere. When thermal influence is substituted for the gravitational, in the tide generating force reckoned for, and when the modes of oscillation corresponding respectively to the diurnal and semi-diurnal terms of the thermal influence are investigated, it will probably be found that the period of the free oscillation of the former agrees much less nearly with 24 hr than that of the latter with 12 hr; and that, therefore, with comparatively small magnitude of the tide generating force, the resulting tide is greater in the semi-diurnal term than in the diurnal".

RISE AND DECLINE OF THE RESONANCE THEORY

The basic idea of the resonance theory is that the atmosphere as a whole has a natural period of oscillation very close to 12 hr which gets amplified by resonance even though the tide-producing effect of $S_2(T)$ is much less than that of $S_1(T)$. The problem, therefore, became one of demonstrating by numerical calculations based on relevant data that the global atmosphere has a natural period of oscillation equal to or very nearly equal to 12 hr.

From the time that Kelvin proposed the resonance theory till about the 1950's a number of scientists sought its verification in the absence of an alternate theory. Among those who devoted attention to this problem are Lord Rayleigh[4], Margules[5-7], Lamb[8,9], Chapman[10], Bartels[11], Taylor[12,13], Pekeris[14], Weekes and Wilkes[15], and Wilkes[16]. Theory shows that for the atmosphere to have a natural period of oscillation of 12 hr to produce resonance amplification of $S_2(p)$ it should have an "equivalent depth" of 7.85 km. The equivalent depth is a concept from Laplace's tidal theory. It is the depth of an ocean covering the earth and capable of free oscillation of a given period and geographical distribution. The numerical value of the equivalent depth depends on the vertical structure of the atmosphere of which knowledge was limited in the earlier years, so that various plausible assumptions were made by different workers. The existence or otherwise of an equivalent depth of 7.85 km for the actual atmosphere of the earth became a controversial issue as more and more knowledge of the vertical structure of the atmosphere accumulated. Nevertheless, till about the 1950's the resonance theory generally held the field, despite doubts expressed by workers from time to time. In particular, this theory gained support from the work of Pekeris[14] who derived an equivalent depth of about 8 km in addition to the earlier value of about 10 km derived by Bartels[12] which had led to serious questioning of the resonance theory around the 1930's.

From about the 1950's, rocket soundings of the upper atmosphere furnished for the first time the actual thermal structure of the upper stratosphere and the mesosphere, which showed that the observed temperatures around 50 km were more than 50°C lower than what had been assumed by Pekeris[14] in his calculations on which the equivalent depth of 8 km was based. Fresh calculations based on the new data by Jacchia and Kopal[17] showed conclusively that the atmosphere has only one equivalent depth around 10 km. This led to the abandonment of the theory of resonance amplification as the cause for the semi-diurnal atmospheric tidal pressure oscillation, nearly 70 years after it was proposed by Kelvin. The large number of studies undertaken during this period to confirm or disprove Kelvin's hypothesis, nevertheless, contributed a great deal to the theory of atmospheric oscillations.

THERMO-TIDAL THEORY

The resonance theory of $S_2(p)$ had all along taken into consideration only the heating of the atmosphere due to the upward transport of heat from the ground by turbulent convection. It was tacitly assumed that the ground is the only effective absorber of solar radiation and that the atmosphere is heated from the bottom. In the absence of resonance amplification, this heating could not explain the observed large amplitude of the semi-diurnal oscillation which is a pressure wave travelling from east to west. When the resonance theory was found untenable, alternative sources of thermal excitation were looked for. It was discovered that selective absorption of the incoming solar radiation by some of the atmospheric constituents which takes place over a large depth of the atmosphere is a far more powerful source of thermal excitation than the surface heat exchange. The two important constituents in this context are water vapour in the troposphere and ozone in the stratosphere-mesosphere. Siebert[18-20] was the first to draw attention to the importance of water vapour and ozone absorption for the excitation of atmosphereic tides. However, by using unrealistic temperatures above the tropopause (isothermal at a temperature of 160°K) he under-estimated the importance of ozone. This was rectified by Butler and Small[21] who showed that the heating of the ozone layer due to absorption of solar ultraviolet radiation accounts for a substantial part of $S_2(p)$ at the ground. Since 1966 major contributions to the theory of atmospheric tides have been made by Lindzen[22-28] in a series of papers. These and other studies relating to atmospheric tides have been discussed in a monograph by Chapman and Lindzen[29].

The new theory, known as *thermo-tidal theory*, which does not involve resonance amplification, is able to account for many of the observed features of atmospheric tides. It is found that the gravitational excitation of atmospheric tides by the sun is much weaker than the thermal excitation, even though there is no way of isolating solar gravitational tides from solar thermal tides in the observational data. Lunar tides can be distinguished because of the difference in period between a lunar day and a solar day. By analysis of hourly pressure data of several stations over the globe, Chapman and Westfold[30] showed that the amplitude of the annual mean lunar semi-diurnal air-tide in barometric pressure is about 5% of the amplitude of the corresponding solar tide. Despite its small amplitude, the lunar atmospheric tide is of interest since the mechanism of excitation (gravitational) is exactly known.

Harmonic analysis of hourly pressure data of individual stations shows that $S_1(p)$ and $S_2(p)$ account for most of the variance in the time series. The residual variance is almost completely accounted for by $S_3(p)$ and $S_4(p)$ whose amplitudes are an order of magnitude less than those of the first two harmonics. The amplitudes of the latter are comparable at tropical stations but in general the semi-diurnal wave is the more dominant one. All the four harmonic components show local and seasonal variations.

The most outstanding feature of the atmospheric tidal pressure oscillation is the strength and regularity of the solar semi-diurnal tide. As stated earlier, the equivalent depth of the atmosphere for the most important mode associated with this oscillation is 7.85 km. The thermo-tidal theory shows that for this equivalent depth, the mode of oscillation has long vertical wavelength of the order of 150 km and it responds to the excitation with high efficiency. Calculations indicate that the contribution of the Ozone excitation to $S_2(p)$ is about 3 times greater than that of water vapour excitation.

The thermo-tidal theory also accounts for the suppression of $S_1(p)$ in the surface pressure oscillation despite the fact that the exciting mechanism has periodicity of a solar day. The equivalent depths of the atmosphere corresponding to the modes of the solar diurnal tide are very small. These modes propagate with shorter wavelengths of the order of 10 to 20 km. Some of the modes get trapped and cannot propagate downwards; there is also destructive interference between waves excited at different levels. All these account for the lesser amplitude and irregular behaviour of the diurnal component compared to the semi-diurnal in the surface pressure oscillation, particularly at high latitudes. However, the diurnal excitation is strong at the levels of absorption and the modes propagate upward to mesospause levels without energy trapping. Theory indicates that diurnal thermo-tidal fields will dominate the semi-diurnal fields in the upper atmosphere over the tropics and sub-tropics with large diurnal oscillations in the winds and temperatures. Rocket measurements of winds and temperatures in the mesosphere and doppler radar wind measurements above 80 km by tracking the drift of ionised meteor trails lend observational support to these theoretical predictions.

COMPARISON OF THEORY AND OBSERVATION

The thermo-tidal theory of Lindzen involves a number of simplifying approximations. The atmosphere is assumed to be in hydrostatic equilibrium and local thermodynamic equilibrium. The land-sea distribution, continental tropography, and viscous dissipative forces are ignored. The basic fields of pressure, temperature and density are assumed to be independent of latitude and longitude so that the basic wind field is zero. The effect of diurnal variation of clouds and release of latent heat, which can be important particularly in the tropics, is not considered. Infra-red radiative transfer is not taken into account and non-linear effects are ignored. The last approximation implies that the excitations are sufficiently small.

Despite these approximations the theory predicts very well the average behaviour of the atmospheric tides below 100 km. However, the theory does not properly predict the local effects and seasonal variations of atmospheric tides. There are also a few discrepancies between theory and observation. For example, the theory predicts that the maximum of $S_2(p)$ should occur around 0900 hr LMT whereas the observed maximum is around 1000 hr LMT. The theory also under-estimates the amplitude of $S_1(p)$ at low latitudes. A reversal in the phase of the semi-diurnal tidal component of the zonal wind $S_2(u)$ at a height of about 30 km is predicted by the theory which is not supported by observation.

Since the 1970's several studies in atmospheric tides have been addressed towards refinement of the original thermo-tidal theory of Lindzen (now known as the "classical" theory) to explain the discrepancies between theory and observation. Lindzen and Blake[31] examined the effect of inclusion of viscosity. Lindzen and Hong[32] studied the effects of taking into account mean winds and horizontal temperature gradients in the atmosphere for winter/summer seasons. It was found that the phase reversal of $S_2(u)$ was now shifted to greater heights in summer at middle and high latitudes for which there is some observational support. However, the phase discrepancy of about 1 hour in the maximum of $S_2(p)$ at the surface persisted. Subsequently Lindzen[33] proposed latent heat release in the atmosphere as an additional source of excitation for $S_2(p)$. By harmonic analysis of the hourly rainfall of a large number of stations he found that the phase of the semi-diurnal component in the rainfall was around 0330 hr LMT with an amplitude of about 1 mm day^{-1}. This could provide a plausible explanation for the phase shift in $S_2(p)$. A later study by Hamilton[34] provides some support for this result. Other recent theoretical studies are those of Walterscheid and Venkateswaran[35,36], Walterscheid *et al*[37] and Walterscheid and de Vore[38]. These studies employed a spectral model with the inclusion of motion field and improved heating functions. The phase reversal of $S_2(u)$ in the classical model was removed but the discrepancy in the phase of $S_2(p)$ persisted.

Part 2 will deal with the results of the observational study of atmospheric tidal surface pressure oscillations over India.

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NEWS

COMPUTER CRIME

Recent news stories have featured a student in the United States charged with using an international data communications network to break into some 200 computer accounts at 14 sites around the world; another student who penetrated the central memory of a company computer and erased data from it; a bank employee who opened a fictitious account in a subsidiary abroad and credited it with a small sum of money each week by manipulating a computer programme.

These are examples of what has come to be called computer "crime", though the term is sometimes an exaggeration since the act may be carried out with criminal intent but may equally be a mere show of technological prowess or even be caused by negligence or operator error. Because intent is so hard to prove and because of the novelty of the "crime", national legal systems are ill equipped to cope with this type of crime which, as computer systems are increasingly interlinked through telecommunications networks, has taken on an international dimension.

The legal issues that arise in this new field and the way in which OECD countries are tackling them are explored in an interesting article "*Computer Crime*" by Martine Briat, OECD Directorate for Science, Technology and Industry, Division of Information, Computer and Communications Policy, OECD Information Service, Chateau de la Muette, 2 rue Andre-Pascal, F 75775 Paris Cedex 16 France, (The *Oecd Observer*, No. 127, March 1984, p. 16).