SHORT COMMUNICATIONS

NEW METHODS FOR THE MEASUREMENTS OF MAGNETIC FIELD GRADIENTS

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ONE of the important parameters that need to be accurately known for the measurement of diffusion coefficients using the NMR spin-echo method[1] is the magnetic field gradient. Various techniques have been used for this purpose to date, namely (i) actual measurement of the magnetic field at different points using a field probe, (ii) experiment on a sample with a known diffusion coefficient (iii) analysis of the shape of the free induction decay and (iv) calculation of the field gradient using the Biot-Savart law. Each of these methods suffers from drawbacks as discussed in detail by Murday[2]. In this paper, we present alternative methods for the purpose, which we believe are simple, accurate and free from the disadvantages of the other methods. We believe that it is for the first time that these methods are being proposed, though the principles underlying them have been known and been applied elsewhere for quite some time.

Two different types of magnetic field gradients have been in vogue: (i) the steady field gradients[1] and (ii) the pulsed field gradients[3]. The latter method is better suited for the measurements of very small diffusion coefficients since it enables one to use large gradient values without the accompanying degradation of the echo. However, we present our method for the measurement of the steady gradients first and then develop the method for pulsed gradients since for the latter method it is necessary to measure a steady gradient as well.

MEASUREMENT OF STEADY GRADIENTS

A continuous wave (CW) NMR technique is used in this method with field modulation and phase sensitive detection. A conventional Robinson Oscillator is used as the r.f. source. The central idea of the method is a split-pair sample holder. It is a teflon cylinder, 1.5 cm long, 1.5 cm dia. with two very thin (~ 0.4 mm) rectangular, grooves, 1 cm deep and 1 cm apart from each other. These grooves are filled with glycerine. The tank coil of the Robinson oscillator is wound on this sample cell and the ^1H resonance of the sample is recorded after phase sensitive detection with a PAR lock in amplifier. Typical signals are shown in figure 1.

As is seen from the figure, in the absence of a gradient, the two samples give rise to a single signal. However, when a steady gradient is applied, the two samples will experience two different magnetic fields and will give rise to a two-component signal. On increasing the current through the coil, the separation between the two signals goes on increasing. Since the two samples are 1 cm apart within the magnetic field, the separation between the two signals recorded on a calibrated chart paper directly gives the gradient in gauss per cm.

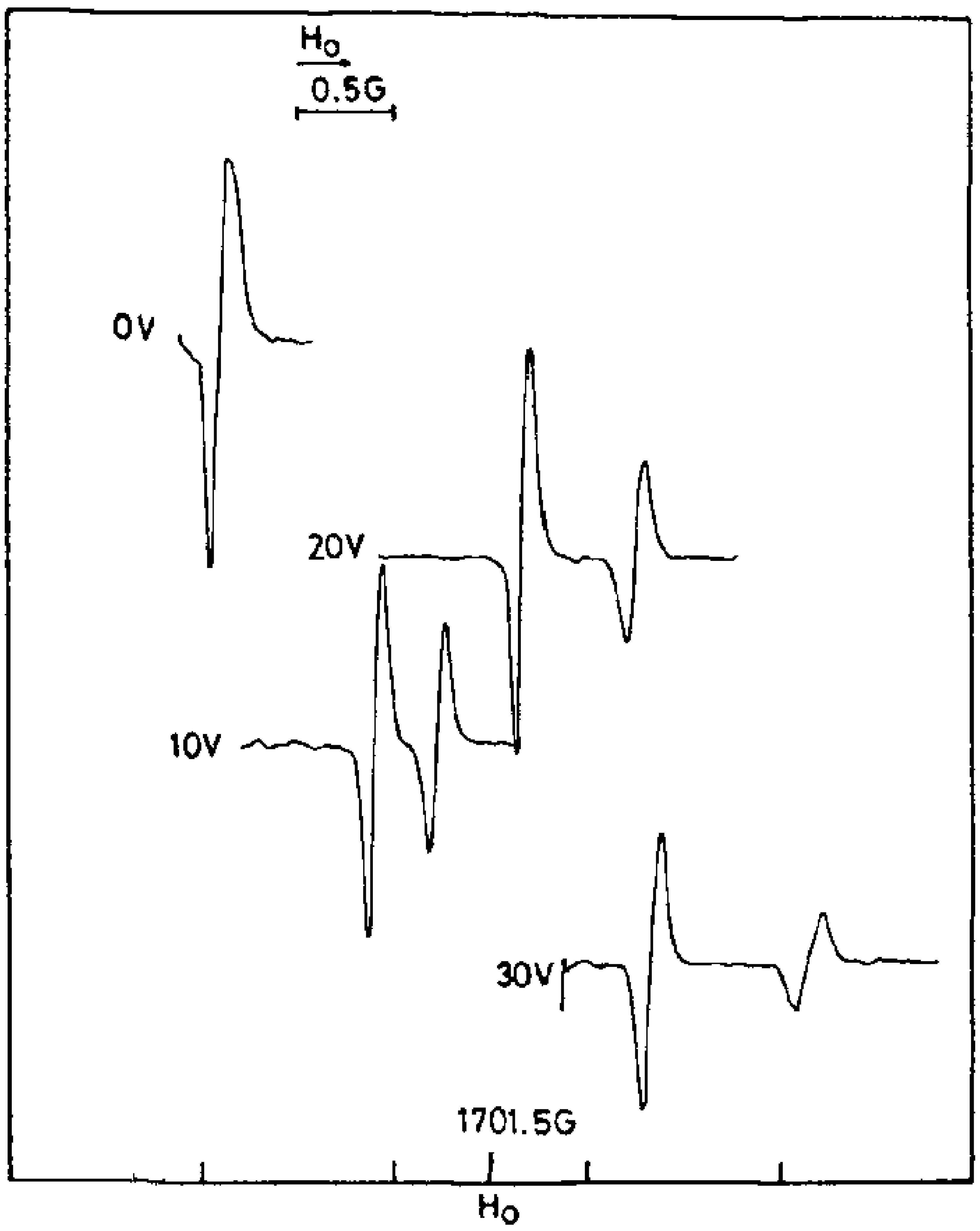

Figure 1. ^1H NMR signals with the split pair sample holder for different values of magnetic field gradients. The numbers by the side of the signals indicate voltages across the gradient coil of resistence 144Ω and correspond to gradients of (i) 10 V → 0.33 Gauss/cm (ii) 20 V → 0.58 Gauss/cm and (iii) 30 V → 0.8 gauss/cm.

We believe that this technique will be useful for the measurement of most of the steady gradients encountered in the laboratory. As shown in the figure, we have covered a range of gradient values starting from $g_{oz} = 0$ gauss/cm to 0.8 gauss/cm, the upper limit having been decided by the available power supply. The signals indicate a small amount of broadening for the highest values of gradients used, its origin most probably being the imperfect machining of the sample grooves. It is possible to further decrease the width of these grooves, thus obtaining still narrower individual lines.

The method also adapts itself very conveniently to the determination of the homogeneity of the gradient. By moving the oscillator along the gap, signals can be recorded at different positions and the gradients at different locations can be compared. It is also possible to make the planar samples perpendicular to the static magnetic field direction by rotating either the field or the sample to obtain the smallest linewidth possible. Though we have used a CW technique to record the signals, one can as well use a pulsed technique, and then Fourier transform the free induction decay after the 90° pulse to obtain the NMR spectrum. It is even possible to measure the gradient from the beat frequency on the free induction decay.

MEASUREMENT OF PULSED MAGNETIC FIELD GRADIENTS

The measurement of pulsed magnetic field gradients is slightly more difficult and the method proposed here uses an auxiliary steady gradient to accomplish it. The scheme is illustrated in figure 2. With a suitably chosen steady gradient, spin echoes are observed with a 90° $-\tau-$ 180° pulse sequence. The echo occurs in a time τ after the 180° pulse. Now a second experiment is done where the pulsed gradient of width δ to be calibrated is applied after the 180° pulse (before the echo appears) at a time t such that $t \ll \tau$. This gradient pulse will have the effect of focusing the isochromats faster and the echo will occur at a time $t+\delta+\tau' < \tau$. By measuring τ' and τ the pulsed gradient g can be calculated as follows: Referring to the spin phase graphing[4] of the isochromats as shown in figure 2, in the presence of only the steady gradient the change in the total phase until the 180° pulse, of the isochromats as a result of the gradient is

$$\varphi = \gamma g_0 z \tau$$

For the echo to occur after the 180° pulse, the rephasing (in contrast with dephasing) should occur to

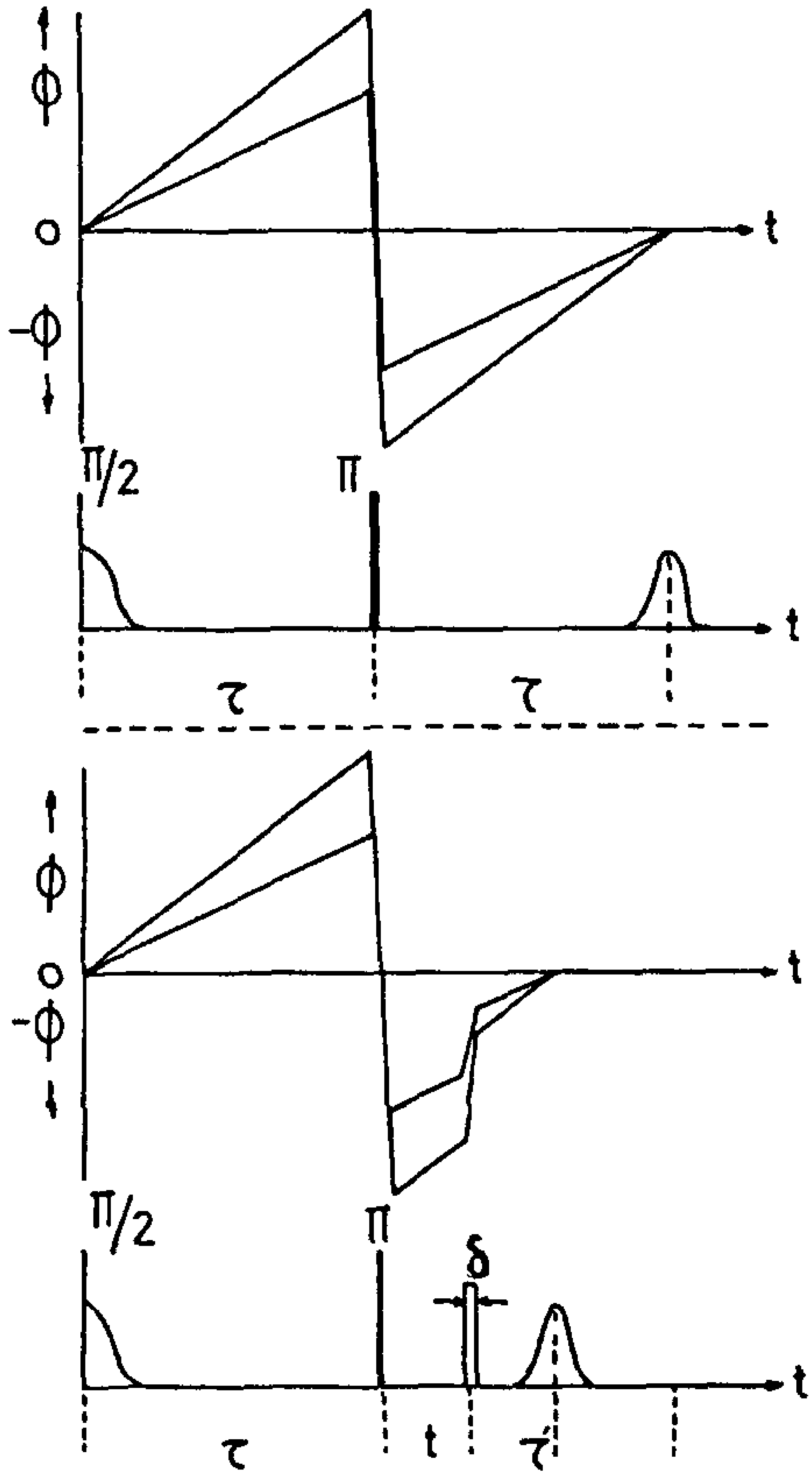

Figure 2. Spin phase graphing of post $-$ 180° gradient pulse method.

the same extent. This is done in three steps (i) upto t under the steady gradient $g_0 = \gamma g_0 z t$ (ii) an amount $\gamma(g+g_0)z\delta$ during the gradient pulse of strength g and width δ and (iii) upto the time of echo formation τ', under the steady gradient g_0.

i.e. $\qquad \varphi' = \gamma g_0 z t + \gamma g z \delta + \gamma g_0 z \delta + \gamma g_0 z \tau'$

Echoes will occur when $\varphi' = \varphi$

i.e. $\qquad \gamma g_0 z \tau = \gamma g_0 z t + \gamma g z \delta + \gamma g_0 z \delta + \gamma g_0 z \tau'$

$\therefore \qquad g_0 \tau = g_0 t + g\delta + g_0 \delta + g_0 \tau'$

or $\qquad g_0(\tau - t - \delta - \tau') = g\delta$

$$\therefore \qquad g = g_0 \frac{\tau - t - \delta - \tau'}{\delta}.$$

Since the times τ, t, τ' and δ and the steady gradient g_0 are already known, the pulsed gradient can straightaway be calculated.

There is an additional advantage in this method. Since the echo occurs sooner than before, the attenuation due to diffusion during $\tau - t - \delta - \tau'$ is no longer present. Then the echo intensity would be more than before, thus leading to an increase in the accuracy.

It is also possible to use a single pre-180° pulse gradient pulse instead of the post-180° gradient pulse as shown. In this case, of course, the echo would occur at a later time.

Another variation of the same technique, namely, by using two gradient pulses of different magnitudes on either side of the 180° pulse, would enable one to calibrate a very large range of pulse gradients.

This new scheme of applying the field gradients, namely the hybrid combination of the steady gradient and a pulsed gradient can also be used for the measurement of the self-diffusion coefficient in favourable cases. The details will be presented in a separate publication.

9 March 1984

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ON THE POST-NEWTONIAN EFFECTS IN THE MILLISECOND PULSAR 1937+214

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THE theory of rotating bodies[1] places constraints on the parameters characterizing a millisecond pulsar[2]. Upto a certain critical value of the angular momentum, the only possible equilibrium figure for a selfgravitating rotating body is a Maclaurin spheroid. At this critical value, called a point of bifurcation, nonaxisymmetric, Jacobi ellipsoids also become permissible equilibrium configurations. It is however more convenient to talk in terms of the angular velocity, which is not a monotonically increasing function of the eccentricity e (of the plane containing the rotation axis) as the angular momentum is. The rotational properties are determined by the parameter $\Omega^2/\pi G\rho$.

The Maclaurin sequence becomes secularly unsutable at $\Omega^2/\pi G\rho = 0.374$ (*i.e.* $e = 0.8127$), where the Jacobi sequence branches off. The sequence becomes dynamically unstable at $\Omega^2/\pi G\rho = 0.449$ ($e = 0.9529$) beyond which no equilibrium figure is possible. Note than an inviscid, classical Maclaurin spheroid continues to be stable even beyond the bifurcation point.

However, even if a small viscosity is present, the Maclaurin spheroid becomes unstable beyond the bifurcation point[1]. A Maclaurin spheroid being axisymmetric is not a source of gravitational radiation, but gravitational waves will be emitted during its oscillations. The gravitational radiation reaction also makes the Maclaurin spheroids unstable beyond the bifurcation point[3]. A toroidal magnetic field leaves the bifurcation point unaffected, whereas a field along the axis of rotation pushes it to higher values of eccentricity[4]. To significantly affect the bifurcation point, the ratio $\mathscr{M}_{33}/\pi G\rho I$, where $\mathscr{M}_{33}$ is the axial magnetic energy and I the moment of interia, should be of order unity[5].

For the 1.56 ms pulsar this ratio is only 10^{-18}, making the effect of the magnetic field negligible. In any case, a magnetic field cannot inhibit the instability due to either viscosity[5] or radiation reaction. The two instabilities however operate through different modes, each stabilizing the other. One can indeed construct situations where the two cancel[6]. But the viscosity required to offset the destructive effects of the gravitational radiation reaction is about 10^{13} times greater than that of the neutron star models[6]. Thus a Maclaurin spheroid is not likely to be stable beyond the point of bifurcation. Even if the neutron star is born spinning so rapidly that it is a truly triaxial figure, it will radiate away gravitationally its nonaxisymmery in a matter of a day and become axisymmetric[3].

In other words the shortest period pulsar can have is the one corresponding to the value of $\Omega^2/\pi G\rho$ at the bifurcation point with an appropriate choice of the average mass density.

One can write, for the point of bifurcation $\Omega^2/\pi G\rho = 1.884\, \rho_{14}\, P_{ms}$, where $\rho = \rho_{14} \times 10^{14}\ \mathrm{g\ cm^{-3}}$ is the mean density and $P = P_{ms} \times 10^{-3}$ s the period. The 1.56 ms pulsar would be exactly at the point of bifurcation if its density were $\rho_{14} = 2.07$ which corresponds to a mass of $0.7\, M_\odot$. Obviously the millisecond pulsar cannot have $\rho_{14} < 2.07 (M < 0.7\, M_\odot)^7$. If $M = 1.4\, M_\odot$ ($\rho_{14} = 4.64$) then the millisecond pulsar has an eccentricity $e = 0.56$ and $\Omega^2/\pi G\rho = 0.167$. The bifurcation point now corresponds to