COMPRESSIBLE THREE-DIMENSIONAL STAGNATION POINT FLOW OVER A MOVING WALL

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ABSTRACT

The steady laminar compressible three-dimensional stagnation point flow over a moving wall with mass transfer has been studied and three-dimensional analog of two-dimensional separation has been presented. Both nodal and saddle point regions have been included in the analysis. The equations governing the flow have been solved numerically using an implicit finite-difference scheme. The effect of moving wall, mass transfer, compressibility and wall temperature on the flow field is found to be appreciable.

INTRODUCTION

In recent years, there have been several studies[1-8] to generalize Prandtl's criterion for separation for steady two-dimensional flows to steady three-dimensional flows and to unsteady two and three-dimensional flows. It has been recognized by several authors[2-5] that there is a relationship between unsteady boundary-layer separation and steady separation over a moving wall. Subsequently, Libby[9] has considered the steady three-dimensional stagnation point flow over a moving wall for incompressible fluids and has presented the three-dimensional analog of two-dimensional separation.

Here, we have studied the steady compressible boundary-layer flow over a moving wall with variable gas properties and mass transfer. Both nodal ($0 \leq c \leq 1$) and saddle point ($-1 \leq c < 0$) regions have been considered. Also the analysis pertaining to the three-dimensional separation on the moving wall is presented.

GOVERNING EQUATIONS

Following the analysis of Rott[2], Libby[9,11], and Davey[10], the equations governing the steady compressible boundary-layer flow with variable gas properties ($\rho \propto T$, $\mu \propto T^{\omega}$, $\mathrm{Pr}=0.7$) at the three-dimensional stagnation point with moving wall and also with mass transfer normal to the surface in dimensionless form are

$$(Nf'')' + (f+cs)f'' + (g-f'^2) = 0, \quad (1a)$$

$$(Ns'')' + (f+cs)s'' + c(g-s'^2) = 0, \quad (1b)$$

$$\mathrm{Pr}^{-1}(Ng')' + (f+cs)g' = 0, \quad (1c)$$

$$(NF')' + (f+cs)F' - f'F = 0, \quad (1d)$$

$$(NS')' + (f+cs)S' - cs'F = 0. \quad (1e)$$

The appropriate boundary conditions are

$$\text{at } \eta=0: f=f_w,\ s=f'=s'=0,\ g=g_w,\ F=S=1$$
$$\text{at } \eta\to\infty: f'=s'=g=1,\ F=S=0 \quad (2)$$

where

$$u = u_w F + axf',\ v = v_w S + caxs',$$
$$c = b/a = (dv_e/dy)/(du_e/dx) \quad (3)$$
$$N = \rho\mu/\rho_e\mu_e = g^{\omega-1},\ u_e = ax,\ v_e = by$$

Here the surface ($z=0$) is considered to be moving with velocity components u_w and v_w; and F and S are the dimensionless velocity components of the fluid in the x and y directions, respectively, due to the moving wall. x, y and z are the principal, transverse and normal directions, respectively; u, v and w are the dimensionless velocity components in the x, y and z diretions, respectively; f' and s' are the dimensionless velocity components of the fluid in the x and y directions, respectively, due to the stationary wall; g is the dimensionless enthalpy; f_w and g_w are, respectively, the surface mass transfer and wall temperature; $c(=b/a)$ is the ratio of velocity gradients in the y and x directions at the edge of the boundary-layer; η is the independent variable; ρ, μ and ω are, respectively, density, viscosity and index in the power-law variation of viscosity; N is the ratio of the density-viscosity product across the boundary layer; and T and Pr are temperature and Prandtl number, respectively. The prime denotes the derivatives with respect to η and the subscripts e and w denote the conditions at the edge of the boundary layer and on the surface, respectively.

RESULTS AND DISCUSSION

Equations (1a) to (1c) represent steady compressible boundary-layer flows at a three-dimensional stagnation point of a stationary wall which have been thoroughly investigated in the literature[11]. Hence,

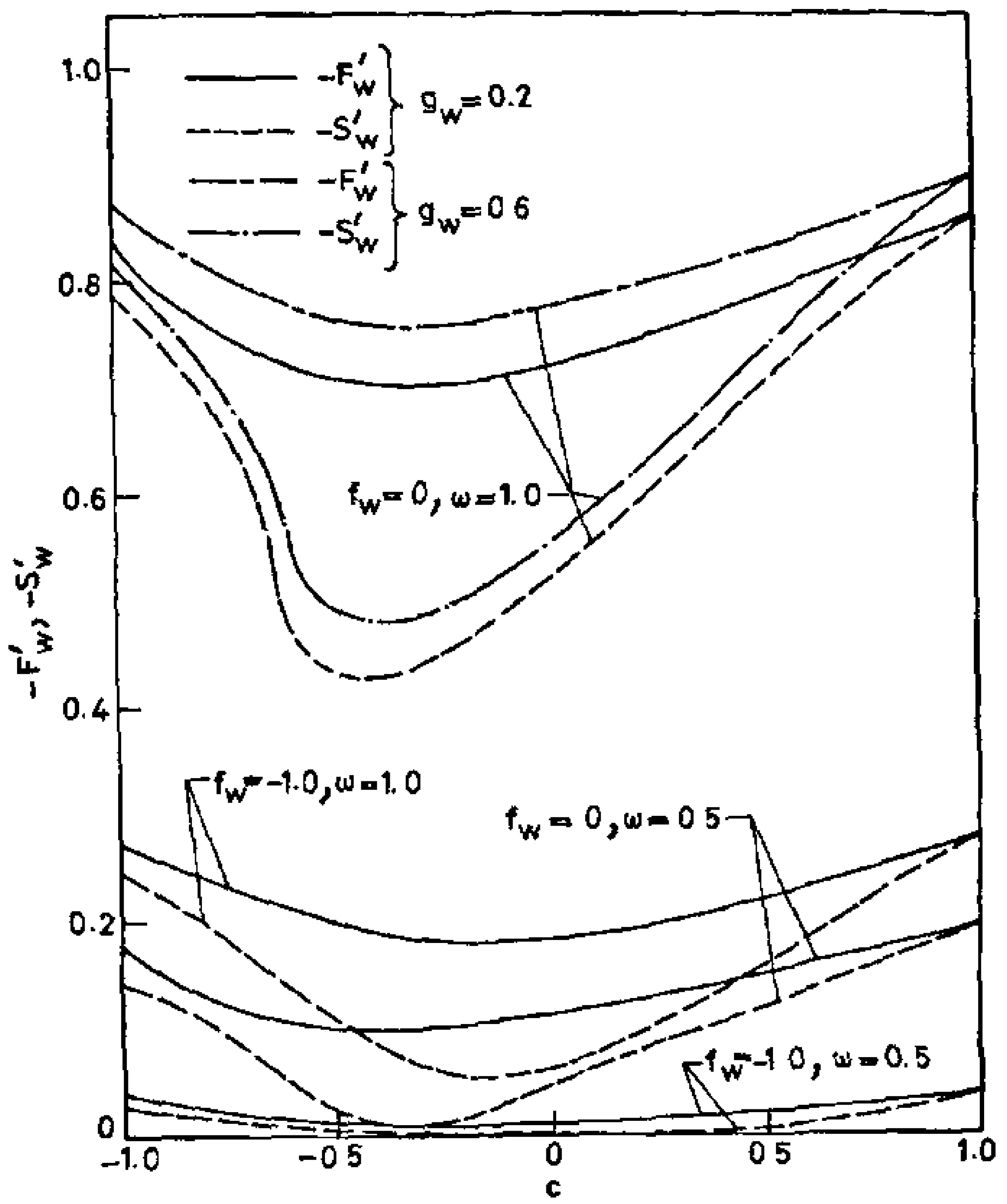

FIG.1

Figure 1. Skin-friction parameters.

they are not presented here. However, (1d) and (1e) are new and they represent the flow associated with the moving wall. Also, they are linear unlike those of stationary-wall case (*i.e.*, (1a) to (1c)) which are nonlinear. Equations (1a) to (1c) have to be solved first as they are independent of (1d) and (1e). Subsequently, (1d) and (1e) have to be solved as they require the results of (1a) to (1c). Hence, we have solved (1a) to (1e) under boundary conditions given in (2) numerically using an implicit finite-difference scheme[12]. However, the results of only (1d) and (1e), which represent moving wall case, are presented in figure 1. It is found that the skin-friction parameters in the x and y directions ($-F'_w$, $-S'_w$) continuously decrease as c decreases in the range $c^* \leq c \leq 1$ ($c^* \simeq -0.5$) and beyond this value they increase with c, whereas for stationary wall the skin-friction parameter in the x direction (f''_w) has trend similar to the moving wall case, but the skin-friction parameter in the y direction (s''_w) continuously decreases with c. The effect of injection $f_w (f_w < 0)$ is to reduce $-F'_w$ and $-S'_w$ whatever may be the values of c, ω and g_w. Also, for a given f_w, $-F'_w$ and $-S'_w$ increase as ω decreases form 1 to 0.5 or g_w increases.

Here we present the analysis for three-dimensional separation on the moving wall which depends on the nature of stagnation point There are three regions to be considered: (I) $0 \leq c \leq 1$ (nodal point region), (II) $c^* \leq c < 0$, (III) $-1 \leq c < c^*$ (No. (II) and (III) correspond to saddle point region ($-1 \leq c < 0$)). Since the effect of compressibility does not change the results qualitatively, the analysis presented by Libby[9] for nodal point flows ($0 \leq c \leq 1$) holds good in the present case also for the nodal point region (case I). However for the sake of completeness, it is briefly presented here.

We consider a line $y(x)$ such that the wall shear is tangent everywhere. Then y is the solution of the equation[9]

$$dy/dx = \tau_y/\tau_x = (v_w S'_w + cays''_w)/(u_w F'_w + axf''_w), \tag{4a}$$

where τ_x and τ_y are the shear stresses in the x and y directions, respectively. The solution of the above equation can be written in the form[9]

$$Y - \alpha = C(\beta\xi - a)^{1/\beta} \tag{4b}$$

where

$$Y = ayu_w, \quad \alpha = -v_w c S'_w/(u_w s''_w), \quad \xi = ax/u_w$$
$$a = -F'_w/(cs''_w), \quad \beta = f''_w/(cs''_w) \tag{4c}$$

Here C is a constant of integration. We now consider the case when the body is moving in the x direction only (*i.e.*, $u_w > 0$, $v_w = 0$), then $\alpha = 0$. When $\xi < 0$, $\beta > 0$ and $a < 0$. The condition $a > 0$ implies a reverse flow in the x direction, *i.e.*, $u > 0$ close to the wall and $u < 0$ beyond this region. The line corresponding to $\xi = a/\beta = -F'_w/f''_w$ divides the surface into two regions: To the left of this line, the shear at the wall is to the right while to the right of this line, this shear is to the left[9].

Next, we consider the saddle point flow in the region $c^* < c < 0$ ($c^* (c^* < 0)$ is the value of c for which $s''_w = 0$) for which (4a) and (4b) are also valid. As before, we consider that the body is moving in the x

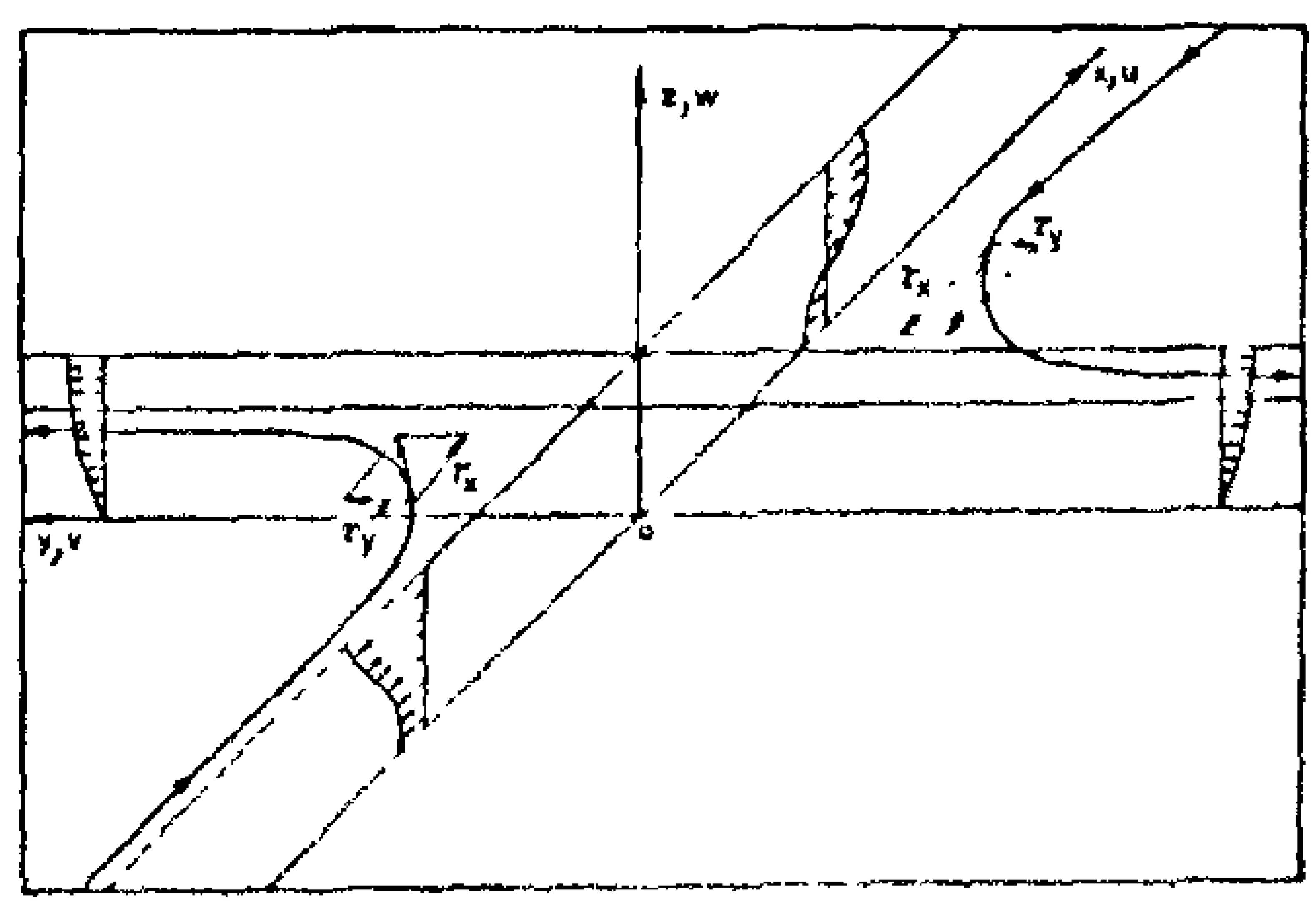

FIG 2

Figure 2. Schematic representation of the flow ($-0.5 < c < 0$) with moving wall in x direction.

direction only (*i.e.*, $u_w > 0$, $v_w = 0$) which implies that $\alpha = 0$. For $\xi > 0$, $\beta < 0$ and $a < 0$. This means that there is a reverse flow in the x direction and the line $\xi = a/\beta$ divides the line into two regions: To the left of this line, the wall shear is to the left and to the right of it, the same shear is to the right. The value of $c^*(c^* < 0)$ for which $s''_w = 0$ depends on the parameters ω, f_w, and g_w. Some of the features of this case are shown schematically in figure 2.

Now we consider the surface shear lines in the region $-1 \le c < c^*$. As in earlier cases, $u_w > 0$, $v_w = 0$, hence $\alpha = 0$. Also for $\xi < 0$, $\beta > 0$ and $a < 0$. Thus it reduces to the case I ($c > 0$) described earlier. Also, the direction of the shear lines is the same as that for $c > 0$. This is due to the reverse flow in y component of the velocity which occurs at $c \simeq -0.5$ independent of the wall which is moving in the x direction. Thus it can be concluded that for both nodal and saddle point flows ($-1 \le c \le 1$), the line $\xi = a/\beta$ is a locus of surface shear lines dividing the flow into "upstream" and "downstream" regions and is analogous to a three-dimensional separation line without the other manifestations of separation.

Conclusions

Here, we have extended the analysis of Libby for the steady three-dimensional stagnation-point flow over a moving wall for incompressible fluid to compressible fluid case with mass transfer. The effect of compressibility and mass transfer on the flow field is found to be significant. Also the three-dimensional analog of two-dimensional separation is shown.

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POLAROGRAPHIC STUDY OF MIXED LIGAND COMPLEXES OF COPPER WITH SOME CARBOXYLIC ACIDS AND PROPYLENEDIAMINE

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Abstract

Equilibrium studies on the formation of the ternary complexes with homo donors of the type Cu XY, where X is propylenediamine and Y succinic, maleic or malonic acid, has been studied by polarography. The overall formation constants have been evaluated at 25° C and at an ionic strength of 1.0 M (KNO_3) using Schaap and McMasters' method. The mixed ligand stabilisation constant, clearly indicates the preferred formation of copper (II) mixed ligand complexes with nitrogen donors over oxygen donors. The other driving forces leading to the favoured formation of mixed ligand complexes are also discussed. The stability constants of complex species are reported.

Introduction

Mixed-Ligand Complexes of metal ions have been extensively studied in recent years since these play an important role in biological process[1-3] Ternary complexes of various metal ions involving aminoacids, diamines and carboxylic acids have been studied by pH meteric technique[4-6] Research