MHD UNSTEADY HELE-SHAW FLOW OF VISCO-ELASTIC FLUID

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ABSTRACT

In the present paper, an exact and general solution of Hele-Whaw flow of visco-elastic (Kuvshinskii model) fluid past an elliptic cylindrical post between two parallel horizontal planes in the presence of uniform transverse magnetic field has been discussed. It has been observed that the elastic parameter increases the flow whereas the Hartmann number retards the fluid flow.

INTRODUCTION

HELE-SHAW are concerned with flows past a vertical cylindrical post between two horizontal planes. Such flows are important in blood flow. It was suggested[1] that the capillary blood vessels in the alveoli of the lung are best described not as tubes, but as forming a sheet, so that the alveolar blood flow is a flow between two parallel membranes interposed with posts. Many investigations[2-4] have therefore paid attention towards the Hele-Shaw flows, but they only studied the steady state flows. Swaminathan[5] studied unsteady Hele-Shaw flow of viscous fluid, while others[6,7] discussed the Hele-Shaw flow of non-Newtonian fluid past circular cylinder under the influence of some particular forms of pressure function. However, a more complete and exact solution is needed. The present paper is an attempt to obtain this by describing the velocities and the pressure function for the flow of visco-elastic fluid past an elliptic post in the presence of a uniform magnetic field and to observe the effects of Hartmann number (M) and the elastic parameter (T) on the fluid flow.

MATHEMATICAL ANALYSIS

We consider the flow of a visco-elastic fluid past an elliptic cylinder

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \; -h \le z \le h$$

confined between two parallel plates $z = \pm h$. A uniform magnetic field B_0 is applied parallel to the axis of cylinder. The physical model of the problem is shown in figure 1.

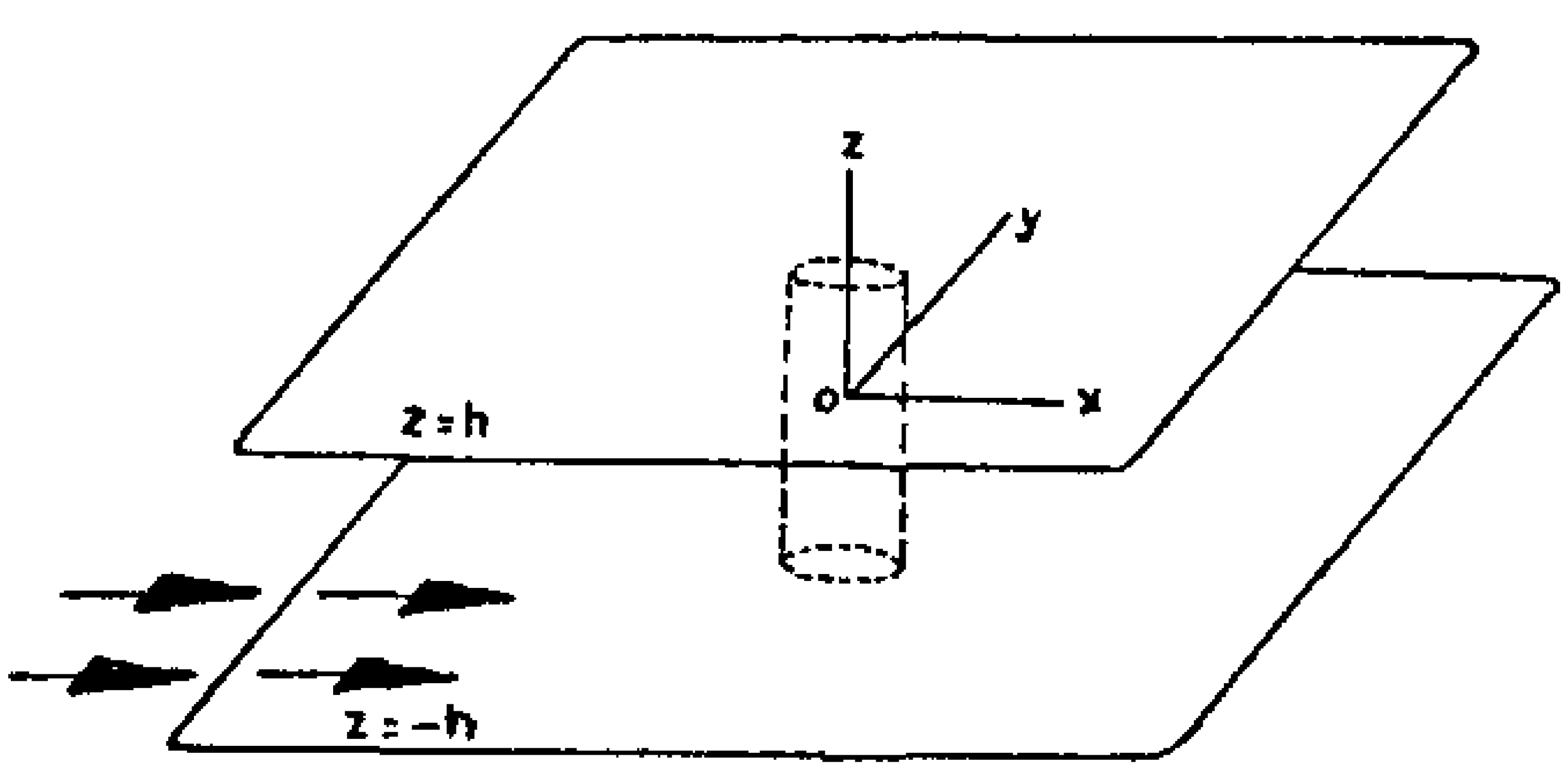

Figure 1. Flow past an elliptic cylinder.

The modified equations in non-dimensional form, describing the Hele-Shaw flow of the conducting viscoelastic fluid[7] in the presence of uniform magnetic field (O, O, B_0) are

$$u + T u_{tt} = -(p_x + T p_{xt}) + u_{zz} - M^2 \times (u + T u_t), \quad (1)$$

$$v_t + T v_{tt} = -(p_y + T p_{yt}) + v_{zz} - M^2 \times (v + T v_t), \quad (2)$$

$$0 = p_z, \quad (3)$$

$$0 = u_x + v_y, \quad (4$$

where the suffixes denote the respective partial derivatives, $M = h\ B_0(\sigma/\mu)^{1/2}$ = Hartmann number, $T = \lambda_0 \nu / h$ is the non-dimensional visco-elastic parameter and the other symbols have their usual meanings. The fluid is initially at rest and at infinity it flows parallel to the major axis.

The initial and boundary conditions accordingly are:

(i) $u = 0 = v$ for $t \le 0$,
(ii) $u = 0 = v$ at $z = \pm 1$ for $t > 0$, (5)
(iii) normal velocity at the surface of the cylinder is zero
(iv) $v = 0$ at $|x|, |y| \to \infty$.

Now using the transform technique and the method of separation of variables, (1) to (4) can be solved as

$$u = 2\left[\sum_{r=0}^{\infty} \frac{(-1)^{r+1}}{q_r} U(r,t) \cos q_r z\right] F(\xi, \eta), \quad (6)$$

$$v = 2\left[\sum_{r=0}^{\infty} \frac{(-1)^{r+1}}{q_r} U(r,t) \cos q_r z\right] f(\xi, \eta), \quad (7)$$

and

$$p = \cos\eta \left[c \cosh\xi + b\left(\frac{a+b}{a-b}\right)^{\frac{1}{2}} \exp(-\xi)\right] g(t) + \text{const.}, \quad (8)$$

where $U(r,t) = T\,g(0)\,H(r,t) + \int_0^t H(r,\lambda)\, G(t-\lambda)\, d\lambda$,

$$H(r,t) = \sum_i \frac{\exp(\alpha_i t)}{2T\alpha_i + (1+TM^2)},$$

α_i being a root of $Ts^2 + (1+TM^2)s + (M+q_r^2) = 0$,

$$q_r = \frac{(2r+1)\pi}{2},\; G(t) = g(t) + T\frac{d}{dt}g(t),$$

$g(t)$ = an arbitrary function of time,

$$F(\xi,\eta) = 1 - \frac{b}{c}\left(\frac{a+b}{a-b}\right)^{\frac{1}{2}} \frac{\cos 2\eta - \exp(-2\xi)}{\cosh 2\xi - \cos 2\eta},$$

$$f(\xi,\eta) = -\frac{b}{c}\left(\frac{a+b}{a-b}\right)^{\frac{1}{2}} \frac{\sin 2\eta}{\cosh 2\xi - \cos 2\eta},$$

$x + iy = c\cosh(\xi + i\eta)$, $c^2 = a^2 - b^2$.

If the post interposed between the plates is a circular cylinder of radius a, then $b \to a$,

$$F(\xi,\eta) \to 1 - \frac{a^2(x^2-y^2)}{(x^2+y^2)^2},$$

and $f(\xi,\eta) \to \dfrac{-2a^2 xy}{(x^2+y^2)^2}$,

and $p \to \left(x + \dfrac{a^2 x}{x^2+y^2}\right) + \text{const}$.

The integrals appearing in the results can easily be evaluated for the known forms of $g(t)$ and consequently various flows can be discussed for various of $g(t)$. Particularly when $g(t)$

$g(t) = -a_0 \exp(-b_0 t)$, we have

$$\int_0^t H(r,\lambda)\, G(t-\lambda)\, d\lambda = a_0(b_0 T - 1) \times \sum_i \frac{\exp \alpha_i t \quad \exp(-b_0 t)}{(\alpha i + b_0)\{(1+TM^2) + 2T\alpha_i\}}$$

If pressure function is independent of time, then $b_0 = 0$

If $M = O$, $b_0 = O$ and $b \to a$, the results are identical to those obtained by Gupta *et al*[7].

Discussion

Expressions (6) to (8) are the most general and exact solutions which describe the Hele-Shaw flow past an elliptic post in the presence of transverse magnetic field. If $T = 0$, the viscous fluid flow is immediately obtained. Figures 2 and 3 represent the nature of the

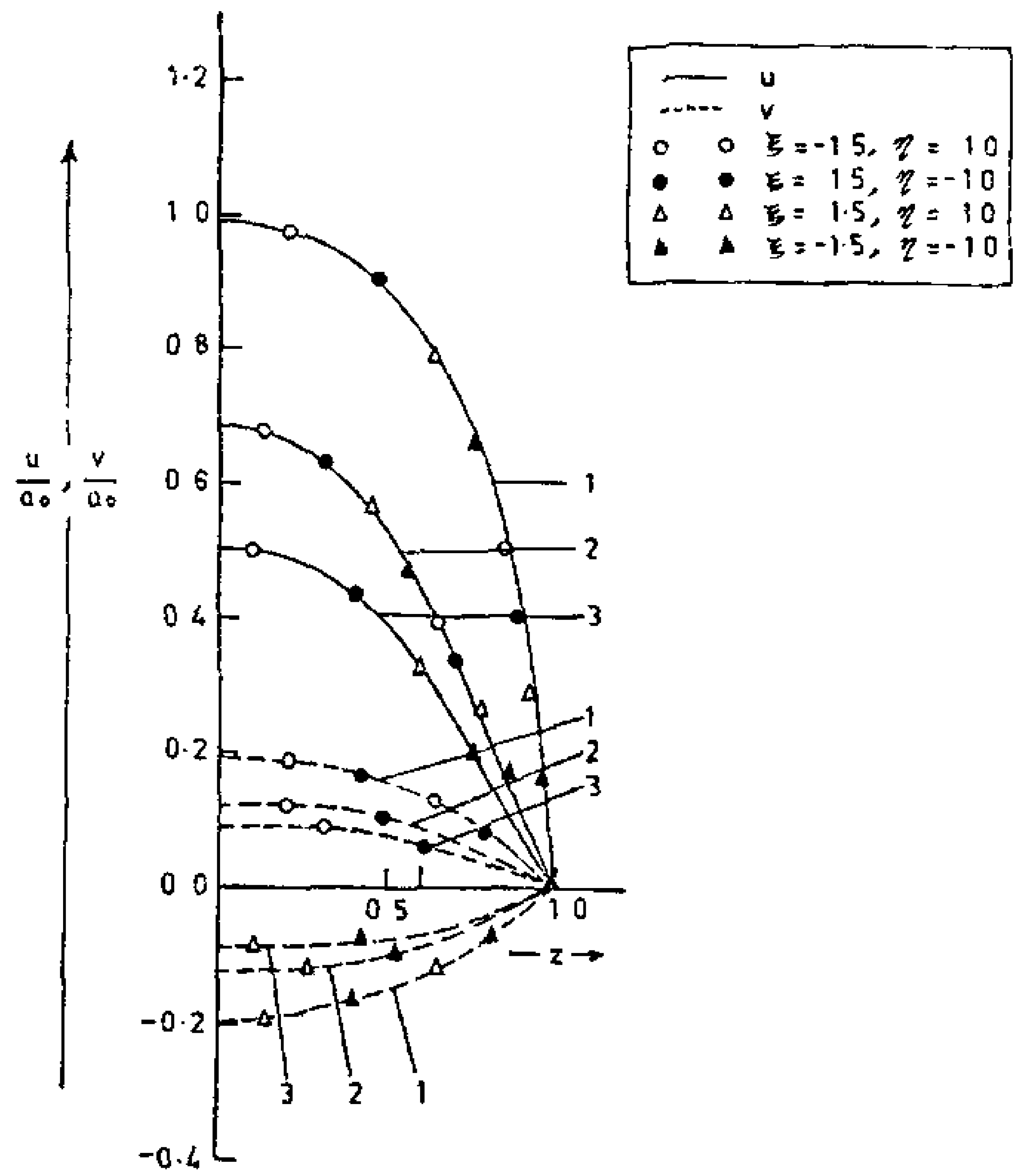

Figure 2. Velocities plotted against z at $t = 1.0$ when $g(t) = -a_0$, $a = 1.414$, $g = 1.0$ for (1) $T = 1.0$, $M = 0$(2), $T = 1.0$, $M = 1.0$ and (3) $T = 0$, $M = 0$.

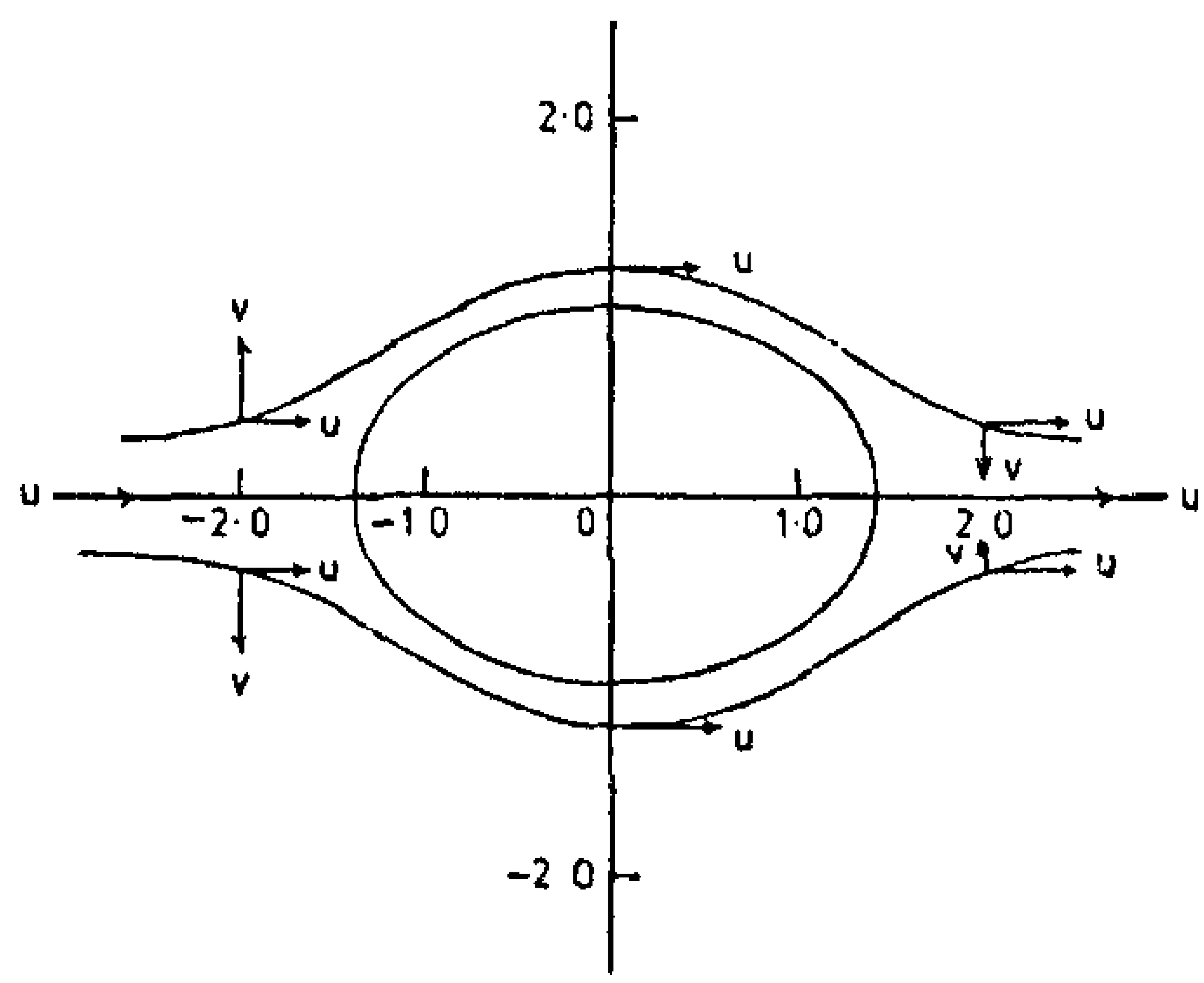

Figure 3. Directions of velocities around the cylinder when $a = 1.414$ and $b = 1.0$.

velocity profiles and the effects of M and T upon the fluid flow when pressure function does not depend on time. From these it is observed that (i) the flow is symmetric about the central plane $z = 0$ (Expressions 6 and 7), (ii) is maximum in it for all M and K (figure 2) and (iii) an increase in M decelerates the flow while and increase in T accelerates the flow (figure 2). The flow directions in the four quadrants around the cylinder are indicated in figure 3.

1. Sobin, S. S. and Fung, Y. C., *Physiologist*, 1967, **10**, 308.
2. Thompson, B. W., *J. Fluid Mech.*, 1968, **31**, 379.
3. Lee, J. S. and Fung, Y. C., *J. Fluid Mech.*, 1969, **37**, 657.
4. Buckmaster, J., *J. Fluid Mech.*, 1970, **41**, 523.
5. Swaminathan, K., Ph.D. Thesis, Agra Univ., Agra, 1975.
6. Johari, A. K., *Acta Ciencia Indica*, 1976, **2**, 377.
7. Gupta, M., Kant, R. and Sharma, H. S., *J. Math. Phys. Sci.*, 1979, **13**, 189.

SOME MIXED COORDINATION COMPOUNDS OF TITANIUM(IV): A NEW SYNTHESIS FOR CYCLOPENTADIENYL AND INDENYL TITANIUM(IV) DERIVATIVES-II

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ABSTRACT

When the mixed ligand complexes of titanium(IV) with polyhydric phenols and acid amides are refluxed with cyclopentadiene or indene in tetrahydrofuran-alcohol mixture, new mixed coordination componds of the composition $(D)_2Ti(phenoxy)_2(amide)_2$ are obtained. The acid amide molecules are lost on heating the complexes to leave compounds of the composition $(D)_2Ti(phenoxy)_2$. (D is cyclopentadienyl, C_5H_5 or indenyl, C_9H_7 group.) The complexes are characterized by their IR and UV spectra and other measurements.

INTRODUCTION

It was observed by Rastogi[1] that the mixed coordination complexes of TI(IV) with polyhydric phenols and acid amides were soluble in hydroxylic solvents. It was also observed that the complexes decompose when their alcoholic solutions are kept for a long time (3–4 days) probably due to the disruption of polyhydroxy phenolic linkages which finally lead to the formation of a gelatinous mass. The fact that polyhydric phenol linkages are susceptible to attack has been used in the present work for preparing the cyclopentadienyl and indenyl derivatives. This communication describes a new method of synthesis of cyclopentadienyl and indenyl titanium (IV) derivatives. derivatives.

EXPERIMENTAL

All the experimental procedures and precautions have been described in the previous communication[1]

Preparation of dicyclopentadienyl bis(formamide) titanium(IV) diquinolate-IV

About 1 g of bis(formamide)titanium(IV) diquinolate was refluxed with freshly-distilled cyclopentadiene in the molar ratios 1:2.5 in a 1:1 mixture of alcohol and tetrahydrofuran (THF) for about 24 hr and then left for 12 hr. The reaction mixture was then filtered and the solvent from the dark coloured filtrate was removed by evaporation under reduced pressure. A waxy residue was obtained, which on extraction with hot petroleum ether left behind brown coloured crystalline solid. It was recrystallised from THF and washed with petroleum ether and dried. Analogous complexes with other diphenoxy diamide (dimethyl formamide and urea) complexes were similarly obtained. It was observed that the yield and ease of preparation of the complexes from formamide compounds were higher as compared to other amide complexes. Diindenyl derivatives were also obtained in a similar way. The complexes are yellowish brown to dark brown in colour. They are soluble in alcohol and THF but insoluble in benzene and chloroform. They are hydrolysed by dilute acids, alkalies and hot water. The compounds start decomposing on heating (~175° C).