LETTERS TO THE EDITOR

A NOTE ON THE MECHANICS OF MUSCLE

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SOME biomathematical aspects of the muscles are presented here. The elastic nature of the muscle is discussed. The torsional waves through muscle tissue covered with a thin elastic membrane are separately considered.

The basic mechanics of muscles is that it develops a force at constant length whereas it contracts and does work when weighed down. The total muscle weight is expresed as $M \equiv \Sigma\ m_j$, where m_j are the masses of the individual fibre. Contraction of the muscle results in shortening and/or development of tension. The total amount of tension that a muscle can exert under optimal conditions is $T = f(N)$, where N = total number of fibres. If all our muscles ($\simeq 2.7 \times 10^8$ individual fibre) exert their combined tension in the same direction, they can develop a force of at least 25 tons[1]. The absolute muscle force, which is defined as the total amount of tension in kg per cm² of physiologic cross section is about 4 kg/cm² in man. Again, body weight is a function of the muscle weight.

Vertebrate striated muscle consists largely of filaments, which do not change their length as the muscle extends and contracts, but slide past each other.

A muscle is shortened to do external work which depends on the fibre's length as well as on its cross-section. In man this length varies from 5 mm for the shortest bundles of the multifidus to more than 400 mm for the sartorius muscle. Though the behaviour of the muscle is viscoelastic, for mathematical simplicity, the stretched—spring model of the muscle is generally accepted and we assume that it obeys Hooke's law.

Initial energy produced by a muscle is given by[2] E_i = activation heat + shortening heat + work done. If we compare muscles of the same shape, it is found that the muscle strength is approximately proportional to muscle volume (L^3).

Let x be the length of the muscle at any time t during shortening; x_0, the initial (resting) length before contraction starts and x_f the length of complete contraction.

$$\text{The restoring force } F = f(q)\ , \tag{1}$$

where $q = x - x_f$

We assume that the muscle model is equivalent to a spring embedded in a plastic or highly viscous mass, and the spring mass system to be critically damped[3]. So, the muscle motion can be mathematically analysed.

Writing the speed of shortening v as

$$v = K(x - x_f)\ , \tag{2}$$

where K is a constant, then integrating and imposing the condition that

$$\text{at } t=0,\ x = x_0\ , \tag{3}$$

we get

$$x = x_f + (x_0 - x_f)\exp(-kt)\ , \tag{4}$$

x_0 being the initial or starting length.

Therefore,

$$v = K(x_0 - x_f)\exp(-kt) \tag{5}$$

The fraction shortened at any time t is given by

$$\delta\left(\equiv \frac{x_0 - x}{x_0 - x_f}\right) = 1 - \exp(-kt)\ , \tag{6}$$

K is known here as the shortening constant. The larger the load m, the smaller is the shortening constant K.

We now consider a non-linear restoring force which is represented by

$$P = aQ + bQ^3 + \ldots\ldots\ldots\ldots,\quad Q = x_f - x\ .$$

In this case, we write (7)

$$v = \alpha S + \beta S^3 + \ldots\ldots\ldots\ldots, \tag{8}$$

α, β are constants, $S = x_f - x$.

Integrating and imposing the condition (3), we get

$$\frac{Z}{Z+\alpha} = B\exp(-2\alpha t)\ , \tag{9}$$

where

$$B \equiv \frac{\beta(x_f - x_0)^2}{\beta(x_f - x_0)^2 + \alpha}$$

$$Z \equiv \beta(x_f - x)^2\ .$$

From (9), we get $\quad x = x_f - \gamma\exp(-\alpha t)$

where

$$\gamma = \left[\frac{\alpha(2x_f\ x_0 - x_f^{\ 2} - x_0^2)}{\beta(x_f^{\ 2} + x_0^2 - 2x_f\ x_0) + \alpha} \right]^{\frac{1}{2}} \qquad (10)$$

and $v = \gamma\alpha \exp(-\alpha t)$ (11)

This gives the speed at any time t.

The force-velocity relationship, as given in[3], is

$$(F + a)(v + b) = (F_{max} + a)b = \text{constant}\ , \qquad (12)$$

where a and b are positive constants. The speed v of shortening decreases with increasing force. Therefore,

$$\frac{x - x_0}{t} = F_m - F)b/(F + a) \qquad (13)$$

Equation (13) assumes constant velocity. This hyperbolic relationship is obeyed by a wide variety of muscle and muscle system.

The intrinsic speed of the muscle is defined by

$$v_s = \frac{v_{max}}{L}\ , \qquad (14)$$

where L is the length of the muscle, v_s varies among muscles of the same animal and betweeen different animals.

The problem of muscular movement has been discussed in[4]. The velocity of sound waves through muscles is given by $c = (c_{11}/\rho)^{1/2} \simeq 1570$ metres/sec. We now want to consider the problem of torsional waves in muscle fibres. To consider a mathematical picture of the muscle, we generally assume the number of muscle fibres in a motor unit and take it to be cylindrical. These fibres are covered with thin elastic membrane. We assume that the muscles are anisotropic *i.e.*, have a preferential direction of orientation.

The stresses for a transversely anisotropic elastic medium can be written as

$$\begin{aligned} T_1 &= C_{11}S_1 + C_{12}S_2 + C_{13}S_3 \\ T_2 &= C_{12}S_1 + C_{22}S_2 + C_{12}S_3 \\ T_3 &= C_{13}S_1 + C_{12}S_2 + C_{11}S_3 \\ T_4 &= C_{44}S_4, T_5 = (C_{11} - C_{12})S_5, T_6 = C_{44}S_6\ , \end{aligned} \qquad (15)$$

where S_j are the strain components and c_{ij} are elastic stiffnesses. For torsional wave propagation through mucle, we assume that the displacements

$$u = w = 0. \qquad v = V(r)\exp[i(\gamma z + pt)] \qquad (16)$$

In this case, the only non-vanishing equation of motion will reduce to

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\ \frac{\partial V}{\partial r} + (\alpha^2 - \frac{1}{r^2})\ V = 0\ , \qquad (17)$$

where

$\alpha^2 = \dfrac{\rho p^2 - C_{44}\gamma^2}{C_{44}}$, ρ is the density of the muscle.

The velocity of torsional waves through muscle is given by $(c_{44}/\rho)^{1/2}$. The solutions for the two different regions (muscle and membrane) can now be written as

$$\begin{aligned} V/a &= AJ_1(\alpha_1 r) \\ V/b &= BJ_1(\alpha_1 r) + CY_1(\alpha_1 r)\ , \end{aligned} \qquad (18)$$

the surfaces are $r = a$ and $r = b$ $(b > a)$.

Here

$$\begin{aligned} \alpha_1^2 &= (\rho_a\ p^2 - C_{44}Y^2)/C_{44} \\ \alpha_2^2 &= (\rho_b\ p^2 - C_{44}^1 Y^2)/C_{44}^1\ , \end{aligned} \qquad (19)$$

J and Y are Bessel functions of the first and second kind respectively.

The boundary conditions for the system are:

$$\begin{aligned} &\text{at}\quad r = a,\ V/a = V/b \text{ and } T_6/a = T_6/b\ , \\ &\text{at}\quad r = b,\ T_6/b = 0\ , \end{aligned} \qquad (20)$$

and

$$T_6 = C_{44}\left(\frac{\partial V}{\partial r} - \frac{V}{r}\right)\exp[i(\gamma z + pt)]$$

$$\begin{vmatrix} J_1(\xi_1) & -J_1(\xi_2) & -Y_1(\xi_2) \\ F_1(\xi_1) & F_2(\xi_2) & F_3(\xi_2) \\ 0 & F_2(\xi_3) & F_3(\xi_3) \end{vmatrix} = 0, \qquad (21)$$

where

$\xi_1 = \alpha_1 a, \xi_2 = \alpha_2 a, \xi_3 = \alpha_3 b,$

$F_1(\xi) = C_{44}[\xi_1 J_0(\xi_1) - 2J_1(\xi_1)]$

$F_2(\xi_2) = C'_{44}[-\xi_2 J_0(\xi_2) + 2J_1(\xi_2)]$.

$F_3(\xi_2) = C'_{44}[-\xi r_{20}(\xi_2) + 2Y_1(\xi_2)]$

$F_2(\xi_3) = \xi_3 J_0(\xi_3) - 2J_1(\xi_3)$

$F_3(\xi_3) = \xi_3 Y_0(\xi_3) - 2Y_1(\xi_3)$.

We assume $b = a(1 + \varepsilon)$, ε is small and $C_{ta} \gg C_{tb}$, where $C_{ta} = (C_{44}/\rho_a)^{\frac{1}{2}}$ and C_{tb} $(C'_{44}/\rho_b)^{\frac{1}{2}}$ Since, $\xi_{2,3}$ are very small, the equation (21) can be written in a simplified way as

$$\{6\alpha_2\varepsilon a(1 - N) + 2N(4 - a\alpha_2)\}J_1(\alpha_1 a)$$

$$= N\{4 - a\alpha_2(1 + 3\varepsilon)\}\alpha_1 a J_0(\alpha_1 a)\ , \qquad (22)$$

where $N = C_{44}/C'_{44}$.

In deriving equation (22), we have neglected $\alpha_2^2\epsilon^2$ and have used the expansions of Bessel functions for small arguments[5]. The equation (22) gives the dispersion relation which consists of an infinite number of sets present in it. Since, $\alpha_2 \epsilon$ is very small, the above equation can further be simplified to

$$J_1(\alpha_1 a) - 0.5\alpha_1 a J_0(\alpha_1 a) = 0 \;, \qquad (23)$$

$$\text{where} \quad \alpha_1 a = \left\{ a\gamma \left(\frac{p^2}{\gamma^2 C_{ta}^2} - 1 \right)^{\frac{1}{2}} \right\} \qquad (24)$$

The values of α_1 a are given by[5] 5.1356, 8.4172, 11.6198, 14.7960

Thus the frequency of torsional waves through muscle can be determined.

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PULSED NMR STUDIES OF MOLECULAR REORIENTATION IN CH_3NH_3I

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Mono methyl ammonium halides CH_3NH_3X where X = F, Cl, Br or I form an interesting family of compounds with diverse molecular motions of CH_3 and NH_3 groups. One of the members of this family CH_3NH_3I is reported to undergo structural phase transitions at 295 K[α phase] and 83 K[δ-phase] as indicated by IR studies[1,2]. But ^{127}I NQR investigations[3] show evidence of a phase transition[phase] at 166 K. CH_3NH_3I is bimolecular [Z = 2] with tetragonal structure [a_0 = 5.11 A; C_0 = 8.97 A] having a space group D_{4h}^{1}(P 4/nmm)[4]. In this compound the C-N bond of $CH_3NH_3^+$ coincides with the fourfold crystallographic axes. Since the cations have threefold symmetry C_{3v} along the C—N bond, they must be either orientationally disordered or freely rotating. Torsional and librational modes of the cation have been determined by inelastic neutron scattering studies[5]. We have carried out proton spin-lattice relaxation measurements in CH_3NH_3I and its N-deuterated derivative CH_3ND_3I in the temperature range 77–420 K to obtain possible information about the kinetics of motion of CH_3 and NH_3 molecular groups and their connection to the observed phase transitions. These measurements have been carried out on a home-made solid state pulsed NMR spectrometer working at a Larmor frequency of 10 MHz[6].

The temperature dependence of T_1 in CH_3NH_3I shown in the figure depends very much on the way the sample is cooled. When the sample is slowly cooled from room temperature or slowly warmed up to room temperature from the lowest temperature, the behaviour obtained is the same and is indicated by closed circles. If the specimen is quenched by sudden cooling to liquid nitrogen temperature and allowed to warm up gradually to room temperature, the T_1 variation