HALL EFFECTS ON COMBINED FREE AND FORCED CONVECTIVE HYDROMAGNETIC FLOW THROUGH POROUS MEDIA

MUKESH GUPTA* AND OM PRAKASH VARSHNEY

Department o Mathematics, Agra College, Agra 282 002, India.

*Postal Address: 28/284, Gokulpura, Agra 282 002, India.

ABSTRACT

The effects of Hall current and permeability of the porous medium on combined free and forced convective hydromagnetic flow in parallel plate channel have been studied, when there is a uniform axial temperature variation along the channel walls. The induced magnetic field and heat transfer characteristics in the flow are determined. Expressions for the shearing stress components have also been sought. The effects of porous medium and Hall parameter on the velocity, the induced magnetic field and shearing stress is interpreted with the aid of graphs and a table.

INTRODUCTION

FLOWS through porous media are very much prevalent in nature and therefore, the study of flows through porous media has become of principal interest in many scientific and engineering applications. A general equation of motion for the flow through porous media has been derived[1] and the results have been applied to some basic flow problems. The effect of buoyancy forces on a forced convective flow of an electrically conducting fluid in a horizontal channel with a linear axial temperature variation along the wall under the influence of transverse magnetic field has been investigated[2]. Gupta's problem has also been studied by taking Hall effect into account[3]. Recently, the flow of a rarefied gas through a channel with Hall effect has also been studied[4]. In all the above studies, the effect of normal density fluid was considered but attempt to analyse the effect of Hall currents in case of a flow through porous medium does not seem to have attracted any attention.

Mazumder *et al.*[3] did not consider the flow through porous media. So the purpose of the present study is to investigate the Hall effects in porous media. The combined effect of them gives rise to an interesting phenomenon which is consistant with the physical situations of the problem.

MATHEMATICAL FORMULATION OF THE PROBLEM AND ITS SOLUTION

We take x- and y-axes and transverse to the parallel horizontal plates coinciding with the planes $y = \pm L$. A uniform strong magnetic field H_0 is imposed parallel to y-axis. Let (u, v, w) and (H_x, H_y, H_z) be the components of the velocity $\bar{q}$ and the magnetic field $\bar{H}$ respectively. At a large distance from the entry section, the flow will be fully developed and in the steady state, all the physical quantities (except pressure) depend on y only.

The equations of momentum and magnetic induction[3] for fully developed steady flow through a porous medium of permeability K in rationalised MKS units are reduced to:

$$O = -\frac{\partial p}{\partial x} + \mu \frac{d^2u}{dy} + \mu_e H_0 \frac{dH_x}{dy} - \mu \frac{u}{K}, \tag{1}$$

$$O = -\frac{\partial p}{\partial y} - \rho g - \mu_e \left(H_x \frac{dH_x}{dy} + H_z \frac{dH_z}{dy} \right), \tag{2}$$

$$O = \mu \frac{d^2w}{dy^2} + \mu_e H_0 \frac{dH_z}{dy} - \mu \frac{w}{K}, \tag{3}$$

$$-\frac{d^2H_x}{dy^2} + m \frac{d^2H_z}{dy^2} = \sigma \mu_e H_0 \frac{du}{dy}, \tag{4}$$

$$-\frac{d^2H_z}{dy^2} - m \frac{d^2H_x}{dy^2} = \sigma \mu_e H_0 \frac{dw}{dy}, \tag{5}$$

where μ is the coefficient of viscocity, μ_e the magnetic permeability, ρ the fluid density, σ the fluid conductivity and $m = \omega\tau$ (Hall parameter), ω the cyclotron frequency, τ the electron collision time. If we assume uniform axial temperature variation along the channel walls, we may take the temperature in the flow as

$$T - T_0 = Nx + \Phi(y), \tag{6}$$

Where N is a constant.
Using (6) and the equation of state

$$\rho = \rho_0 [1 - \beta (T - T_0)] \tag{7}$$

in (2) and integrating, we get

$$p = -\rho_0 g y + \rho_0 g \beta N x y + \rho_0 g \beta \int \Phi \, dy$$

$$- \frac{\mu_e}{2} (H_x^2 + H_z^2) + F(x) \tag{8}$$

where β is the coefficient of thermal expansion and ρ_o the density of a reference state.

We introduce the following non-dimensional quantities:

$$y^* = y/L, \quad u^* = uL/(\nu P_x),$$

$$P_x = \frac{-L^3}{\rho_o \nu^2} \frac{dF}{dx}, \quad w^* = wL/(\nu P_x),$$

$$H_x^* = \frac{H_x}{\sigma \mu_e H_o \nu P_x}, \quad H_z^* = \frac{H_z}{\sigma \mu_e H_o \nu P_x},$$

$$K^* = K/L^2, \quad M^2 = \frac{\mu_e^2 H_o^2 L^2 \sigma}{\rho_o \nu},$$

$$G = \frac{\beta g N L^4}{\nu^2 P_x}. \tag{9}$$

Eliminating p from (1) and (8) and introducing the non-dimensional quantities, after dropping the stars, we have

$$\frac{d^2u}{dy^2} + M^2 \frac{dH_x}{dy} - \frac{u}{K} - G y = -1, \tag{10}$$

Equation (6) shows that positive or negative values of N correspond to heating or cooling along the channel walls. Considering $P_x > 0$, it follows from the definition of G given by (9) that $G \gtrless 0$ according as the channel walls are heating or cooling in axial direction.

Further (3) reduces to, after dropping the stars,

$$\frac{d^2w}{dy^2} + M^2 \frac{dH_z}{dy} - \frac{w}{K} = 0 \tag{11}$$

Equation (11) multiplied by $i (= \sqrt{-1})$ when added to (10) gives

$$\frac{d^2U}{dy^2} + M^2 \frac{dh}{dy} - \frac{U}{K} - G y = -1, \tag{12}$$

where $U = u + iw$, $h = H_x + i H_z$. (13)

Similarly combining (4) and (5) and using (9), after dropping the stars, we have

$$\frac{d^2h}{dy^2} = - \frac{1}{(1 + im)} \frac{dU}{dy}. \tag{14}$$

The no-slip conditions at the plates $y = \pm 1$ are

$$U(\pm 1) = 0 \tag{15}$$

and since the plates are assumed electrically non-conducting, the magnetic boundary conditions are

$$h(\pm 1) = 0 \tag{16}$$

Solutions of (12) and (14) satisfying (15) and (16) are:

$$U(y) = \frac{G}{K_1^2 \sinh K_1} (\sinh K_1 y - y \sinh K_1)$$

$$+ \frac{1}{K_1} \Big\{ (\cosh K_1 - \cosh K_1 y) /$$

$$\Big[K_1 \cosh K_1 - (M_1^2/K_1^2)(K_1 \cosh K_1$$

$$- \sinh K_1) \Big] \Big\} \tag{17}$$

and

$$h(y) = \frac{G M_1^2}{M^2 K_1^2} \Big\{ \frac{1}{2} (y^2 - 1) + \frac{1}{K_1 \sinh K_1}$$

$$\times (\cosh K_1 - \cosh K_1 y) \Big\} + \frac{M_1^2}{M^2 K_1^2}$$

$$\times \Big\{ (\sinh K_1 y - y \sinh K_1) /$$

$$\Big[K_1 \cosh K_1 - (M_1^2/K_1^2)(K_1 \cosh K_1$$

$$- \sinh K_1) \Big] \Big\} \tag{18}$$

where

$$M_1^2 = \frac{M^2}{(1+im)} \text{ and } K_1^2 = M_1^2 + \frac{1}{K}. \quad (19)$$

Separating into real and imaginary parts, Eqs. (17) and (18) readily give the expressions of the x and z components of velocity and induced magnetic field.

The dimensionless shear stress for the primary and secondary flows at the upper and lower plates can be obtained by using the values in

$$\left(\frac{du}{dy}\right)_{y=\pm 1} \quad \text{and} \quad \left(\frac{dw}{dy}\right)_{y=\pm 1}$$

When buoyancy forces are absent (G 0), the shear stress components due to the primary and secondary flow do not vanish on either of the plates. Thus we arrive at the interesting conclusion that in the absence of the buoyancy forces, the primary and cross-flows do not show incipient flow reversal in case of a flow through porous medium. On the other hand, the cross-flow due to Hall currents shows incipient flow reversal although the primary flow does not when G 0 and $K=\infty$ (non-porous medium). The incipient reversed flow for the primary and cross-flows at the upper and lower plates takes place corresponding to that values of G at which the dimensionless shear stress vanishes at the plates.

Heat Transfer

The equation of energy including viscous and Ohmic dissipation is

$$u\frac{\partial T}{\partial x} = K_2\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left[\left(\frac{du}{dy}\right)^2 + \left(\frac{dw}{dy}\right)^2\right] + \frac{1}{\rho c_p \sigma}\left[\left(\frac{dH_x}{dy}\right)^2 + \left(\frac{dH_z}{dy}\right)^2\right] \quad (20)$$

where K_2 is the thermal diffusivity, c_p is the specific heat and the temperature T is given by (7). Using (7) and (9), the above equation can be put in dimensionless form, after dropping the stars, as

$$\frac{d^2\theta}{dy^2} = P_r\, u - K_3\left[\frac{dU}{dy}\cdot\frac{d\bar{U}}{dy} + M^2\frac{dh}{dy}\cdot\frac{d\bar{h}}{dy}\right], \quad (21)$$

where P_r is the Prandtl number $\frac{\nu}{K_2}$ and

$$K_3 = \frac{\nu^3 P_x}{c_p K_2 N L^3}, \quad \theta = \frac{\Phi}{(NLP_x)} \quad (22)$$

and the over bar denotes a complex conjugate. As for the temperature boundary conditions we take the reference temperature T_0^- in (7) in such a manner that the temperature of the lower wall $y = -1$ is $T_0^- + Nx$ and this implies that $\phi(-1)=0$. Hence using (22), the boundary conditions for $\theta(y)$ are:

$$\theta(-1) = 0,\ \theta(1) = \frac{\Phi(1)}{NLP_x} = N_1 \quad (23)$$

where N_1 is taken as the wall temperature parameter. The temperature distribution $\theta(y)$ can be obtained by solving the ordinary linear differential equation (21) of second order with constant coefficients after substituting the expressions for $U(y)$ and $h(y)$ from (17) and (18) and making use of the boundary conditions (23). We omit the details of calculation as they are quite cumbersome.

Particular Case

When K (permeability of a porous medium) $\to\infty$, the problem reduces to that considered by Mazumder *et al.*

Results and Discussion

We have plotted $u(y)$, $-w(y)$, $H_x(y)$ and $H_z(y)$ for different values of Hall parameter (m) and permeability of the porous medium (K) with $G=1$ and $M=5$.

In figures 1 and 2, the profiles of the primary and secondary flows respectively have been plotted for different values of K ($=1, 5, \infty$) and $m(=1,2)$. From figures it is clear that the primary velocity increases with increase in K while secondary velocity decreases numerically. It is also evident that an increase in m increase the velocity of primary and secondary flows which is consistant with the findings of Mazumder *et al*[3].

FIG. 1 PROFILES OF NON-DIMENSIONAL PRIMARY VELOCITY u(y) FOR G = 1 AND M = 5

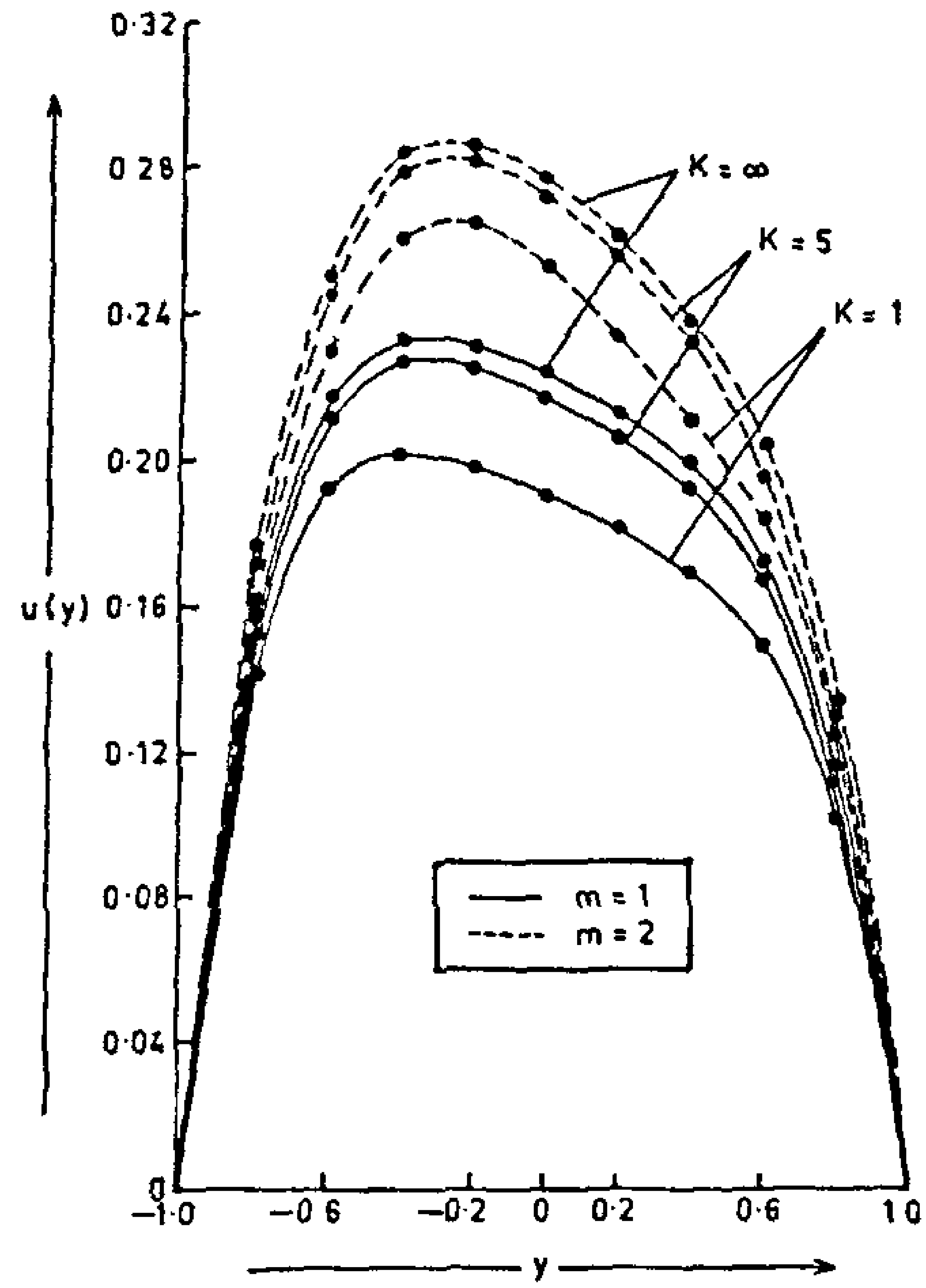


FIG. 2 PROFILES OF NON-DIMENSIONAL SECONDARY VELOCITY w(y) FOR G = 1 AND M = 5

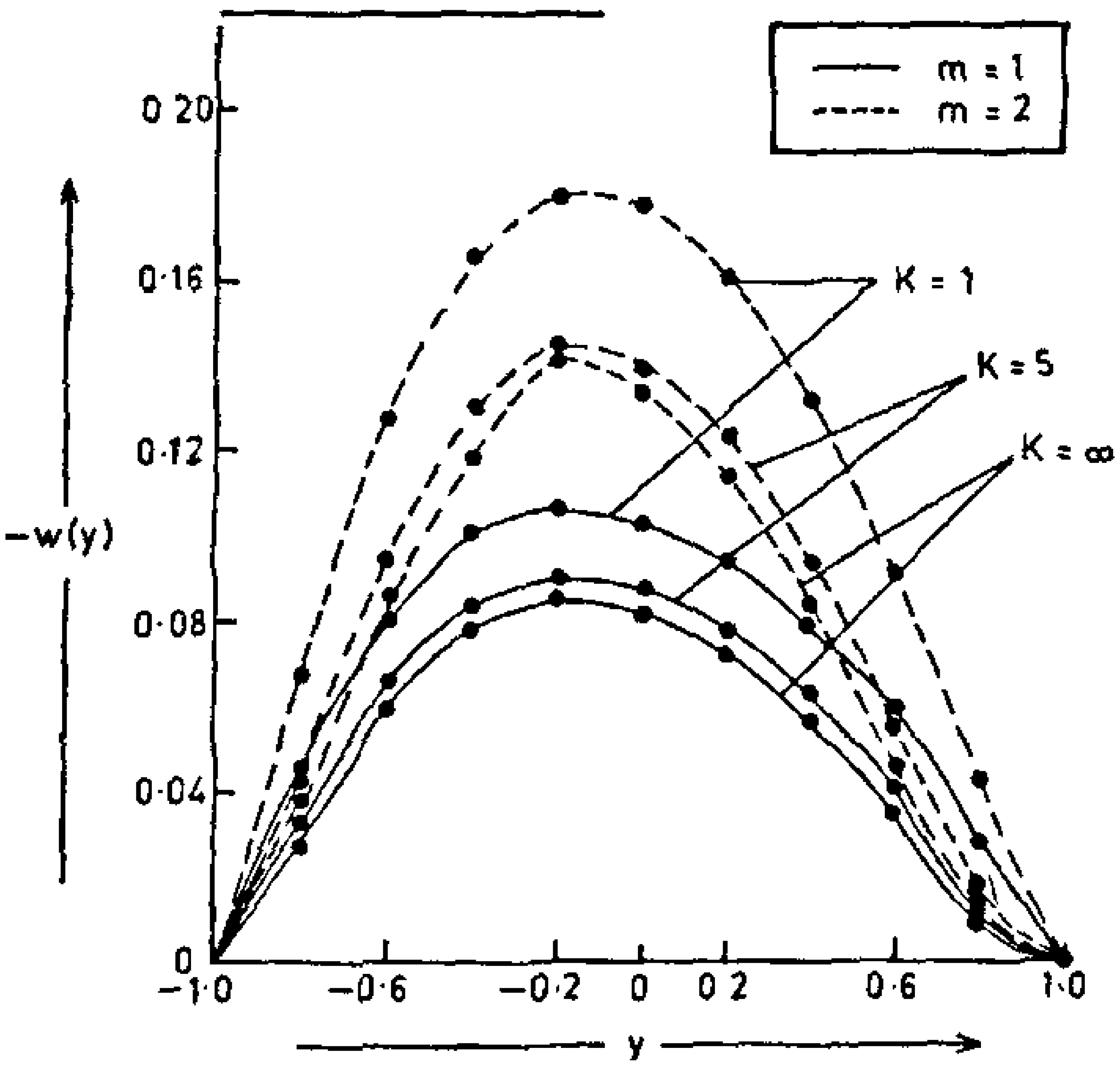
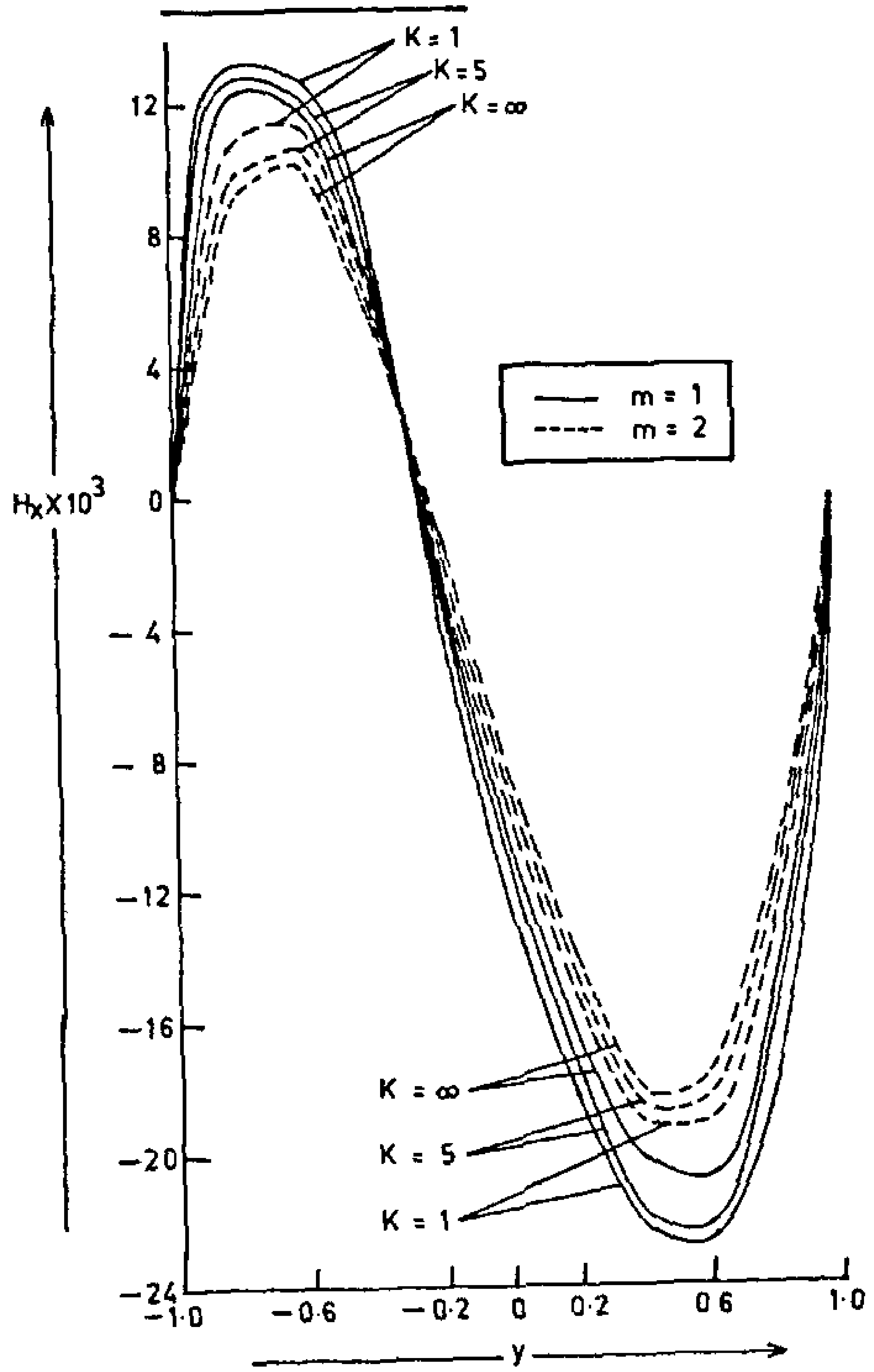


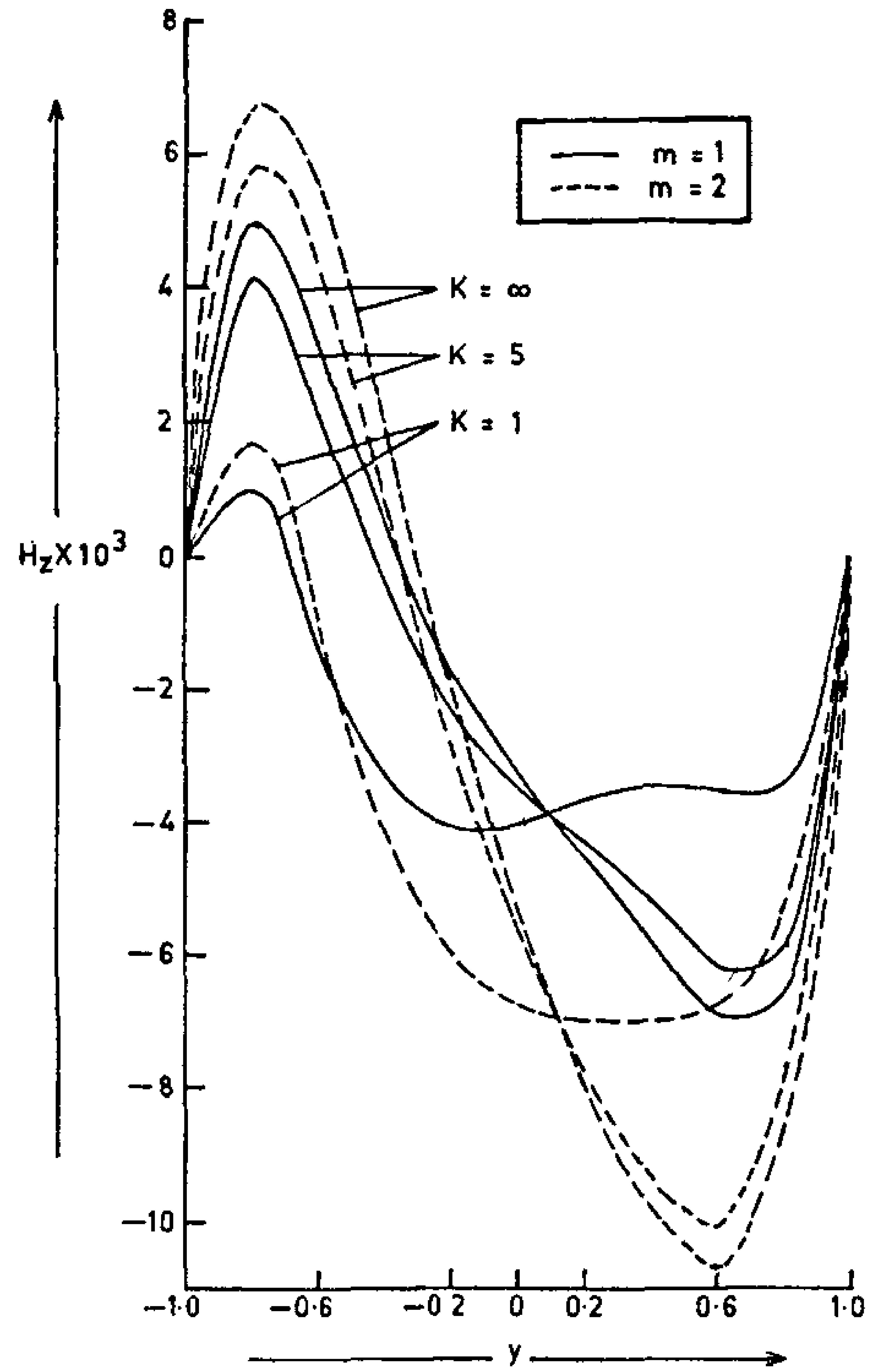
FIG. 3 PROFILES OF THE NON-DIMENSIONAL INDUCED MAGNETIC FIELD Hx FOR M = 5 AND G = 1



The effect of m and K on the profiles for the induced magnetic fields H_x and H_z have been shown in figures 3 and 4 respectively. In both the figures, the behaviour of induced magnetic fields are not symmetrical about the axis of the channel. The value of H_x decreases numerically with the increase in K while H_z increases numerically near the lower and upper plates.

The values of shear stress at the upper and lower plates for the primary and cross-flows are given in table 1 in order to show the effect of m. The table shows that the shear stress for the primary flow is negative and positive at the upper and lower plates respectively while a reverse result is observed for the cross-flow. The value of shear stress at both the plates

FIG. 4 PROFILES OF NON-DIMENSIONAL INDUCED MAGNETIC FIELD H_z FOR M = 5 AND G = 1



for the primary and cross-flow increase numerically with the increase in Hall parameter (m). Hence there is no possibility of the flow separation at both the plates when $G = 1$, $K = 1$ and $M = 5$.

TABLE 1

Values of shear stress

m	$\left(\frac{du}{dy}\right)_{y=1}$	$\left(\frac{du}{dy}\right)_{y=-1}$	$\left(\frac{dw}{dy}\right)_{y=1}$	$\left(\frac{dw}{dy}\right)_{y=-1}$
1.0	−0.740408	1.094737	0.101272	−0.197663
2.0	−0.778453	1.197591	0.153687	−0.293120
3.0	−0.800944	1.275771	0.189259	−0.338404

ACKNOWLEDGEMENTS

Authors are thankful to Prof. Har Swarup Sharma, University Professor of Mathematics, Agra University, Agra, for his valuable suggestions. Thanks are also due to the learned referee for his critical comments and suggestions. One of the authors (MG) is also thankful to CSIR for providing financial assistance.

1. Ahmadi, G. and Manvi, R., *Indian J. Tech.*, 1971, **9**, 441.
2. Gupta, A. S., *ZAMP*, 1969, **20**, 506.
3. Mazumder, B. S., Gupta, A. S. and Datta, N., *Int. J. Engg. Sci.*, 1976, **14**, 285.
4. Gupta, M. and Varshney, V. P., *Indian J. Pure Appl. Math.*, 1981, **12**, 753.

EFFECT OF FEEDING HEXACHLOROBENZENE AND ACETYLCHOLINE TO *PHILOSAMIA RICINI* LARVAE DURING DEVELOPMENT

RADHA PANT, SANTOSH KUMAR KATIYAR AND GEETA JAISWAL
Biochemistry Department, Allahabad University, Allahabad 211 002, India.

ABSTRACT

The fungicide, hexachlorobenzene, acts as a nerve poison to *Philosamia ricini* larvae by inhibiting acetylcholine esterase activity and producing toxicity. These lead to the lysis of all nutrients—carbohydrates, glycogen, proteins and lipids as evinced by the enhanced proteolytic, lipolytic, phosphorylase and aminotranferases activities throughout the development of *P. ricini*. It also induce lack of appetite and renders the insects undernourished. Release of total free amino acids, due to the high proteolytic activity could also account for the high mortality rate (47%) of the fungicide-fed insects due to amino acidaemia. Feeding of hexachlorobenzene to acetylcholine fed insects produced more or less the same overall effect as with the fungicide alone.