HALFTONE IMAGE ANALYSIS AT NON-ZERO DIFFRACTION ORDERS

MOHAMMAD A. KARIM
School of Engineering Technology, University of Arkansas at Little Rock, 33rd & University, Little Rock, Arkansas 72204, U.S.A.

ABSTRACT

The combination of a nonlinear halftoning step followed by bandpass spatial filtering to yield a nonlinear functional relationship is analyzed. Results of the use of a logarithmic line screen at different diffraction orders are presented.

THE halftone technique[1] has long been used in the reproduction of continuous-tone photographs in the printing and graphic arts industries. Basically a screen is used to transform an original photograph into a high-contrast halftone negative or positive that contains arrays of opaque dots with sizes varying according to the optical density of the original. Recently the screens have also been found to be useful in various nonlinear optical image processing applications[2-7], such as logarithmic transformation, equidensitometry, analog-to-digital conversion, pseudo-colour, level-slice, edge-tailored bandpass and exponential transformation.

The halftone photograph, which is essentially a pulse-width or pulse-area modulated image of the original, is placed at the input plane of a coherent optical processing system[3,4]. A spatial filter is thereafter used at the Fourier plane to select any one or several diffraction orders in the spectrum. Thus the overall nonlinear relationship between the continuous-tone photograph and the output depends on the diffraction orders selected as well as the contact screen characteristics. The importance of the technique in its application to image processing is clear.

Recently we reported[8,9] the analysis of image reproduction of line, square, circular and concentric-ring patterned screens with all their cell transmittances that vary linearly with distance. A later paper[10] has included a graphic analysis of tone reproduction by contact screens that have nonlinear spatial transmittances. However, it need be mentioned that these reports[8-10] have compared only the transmittance of the binary halftone photograph with that of the input photograph. This means that our image processing needs were limited only to those cases where zero-order diffraction was considered. In practice, the spectral dots corresponding to low frequency inputs are too difficult to separate by pin-hole spatial filter of finite aperature. In realistic circumstances, higher order terms cannot be avoided in the reconstructed output image. Besides, there are many image processing applications where higher diffraction orders have to be considered. The purpose of this work is to analyze and predict the general nature of the output transmittance corresponding to non-zero diffraction orders, by using one-dimensional (line) screens, of either linear or nonlinear spatial transmittances.

We consider a continuous-tone image and denote by $T^P(x)$ its intensity transmittance. The line halftone photographs which are produced from the use of the line contact screens and the continuous-tone photograph in the hard-clipping process, consist of opaque bars spaced by transparent regions. The density and the widths of these bars are spatially modulated by the screen characteristics and the threshold transmittance of the infinite (or extremely high) gamma film. It is assumed that the continuous-tone original has a maximum spatial frequency less than the frequency of the contact screen so that T^P over the region of any one unit cell remains constant provided that the edges are not considered. If the sampling rate is not sufficient enough, the spectral islands will not be separable, and aliasing will occur.

It can readily be seen that the amplitude transmittance of the halftone transparency, t^h, is approximately a periodic sequence of pulses, such that

$$t^h(x) = \sum_{m=-\alpha}^{\alpha} t_m \exp(-j2\pi mx/X), \quad (1)$$

$$\text{where } t_m = \frac{1}{X}\int_0^X t^h(x) \exp(j2\pi mx/X)\,dx, \quad (2)$$

and X is the period of the contact screen. Hence the resulting intensity transmittance corresponding to the mth spectral order, T_m^h, at the output plane of a coherent optical system is

$$T_m^h = \left| t_m \right|^2,$$

$$= \left| \frac{1}{X}\int_0^X t^h(x) \exp(j2\pi mx/X)\,dx \right|^2 \quad (3)$$

The diffraction orders of interest are $m = 0$ and $m \neq 0$. Substituting these values in (3) yields

$$T_0^h = T_0^H , \tag{4}$$

$$\text{and } T_q^h = \left\{ \frac{2}{q\pi} \sin \left[\pi q (T_0^H)^{1/2}\right] \cos \left[2\pi q x / X\right] \right\}^2 , \tag{5}$$

where $q = 1, 2, 3, \ldots$ and T_0^H is the average intensity transmittance at the output when only the zero-frequency grating order is considered and is defined as the corresponding ratio of the transparent region to the total area of the unit cell. It is noteworthy that T_q^h 0 when T_0^H is either transparent or opaque. Thus no aliasing occurs at the maximum contrast cases.

It is quite common practice to design contact screen, linear or nonlinear, to achieve

$$T_0^H = f\,(T^P) , \tag{6}$$

where the form of the mapping function f determines the exact nonlinear relationship, *i.e.*, exponential, power, logarithmic etc. While the corresponding half-tone photograph is introduced in the input plane of a coherent optical system, the pin-hole spatial filter in most cases fails to exclude all of the non-zero diffraction orders. Such circumstances would introduce distortions into the expected output image. The exact nature and the quantity of the distortion is, however, determined by (5).

On the other hand, if only specific non-zero spectral order is allowed to pass (as for example in equidensitometry and pseudo-colour works) the resulting output is no more binary, but undoubtedly periodic. Often, the quantitative measurement of the output transmittance corresponding to non-zero spectral order is extremely desirable. In an analogous manner, regional transmittance over the area of one unit cell T^H corresponding to qth order may be defined as

TABLE 1

Values of T_q^H.

Orders of Diffraction	$[T_0^H]^{1/2}$ → 0 / 1	0.1 / 0.9	0.2 / 0.8	0.3 / 0.7	0.4 / 0.6	0.5
	T^P → 0.200 / 0.800	0.203 / 0.615	0.211 / 0.486	0.227 / 0.395	0.250 / 0.329	0.283
1	.000	.019	.070	.153	.183	.203
2	.000	.018	.046	.046	.046	.000
3	.000	.015	.020	.020	.020	.023
4	.000	.011	.004	.004	.004	.000
5	.000	.008	.000	.008	.000	.008
6	.000	.005	.002	.002	.002	.000
7	.000	.003	.004	.000	.001	.004
8	.000	.001	.003	.003	.001	.000
9	.000	.000	.001	.002	.002	.003
10	.000	.000	.000	.000	.000	.000
11	.000	.000	.001	.001	.002	.002
12	.000	.000	.001	.001	.000	.000
13	.000	.001	.001	.000	.000	.001
14	.000	.001	.000	.000	.001	.000
15	.000	.001	.000	.001	.000	.001
16	.000	.001	.000	.000	.001	.000
17	.000	.000	.001	.000	.000	.001
18	.000	.000	.001	.001	.000	.000
19	.000	.000	.000	.000	.001	.001
20	.000	.000	.000	.000	.000	.000
21	.000	.000	.000	.000	.000	.000
22	.000	.000	.000	.000	.000	.000
23	.000	.000	.000	.000	.000	.000
24	.000	.000	.000	.000	.000	.000
25	.000	.001	.000	.001	.000	.001

$$T_q^H = \frac{1}{X}\int_0^X T_q^h(x)\,dx\,. \tag{7}$$

For the purpose of illustration let us consider a logarithmic contact screen[4] designed so as to achieve

$$T_0^H = f(T^P) = \frac{\ln T^P - \ln T_{min}}{\ln T_{max} - \ln T_{min}} \tag{8}$$

where T_{min} and T_{max} are the minimum and maximum possible values of T^P. Equations (5)–(8) could be used now to determine the exact nature of the regional transmittance corresponding to non-zero spectral order.

A computer program was implemented to determine the values of T_q^H corresponding to various values of q and T_0^H. For this purpose, T_q^h was determined at all values of $x = 0, 0.02X, 0.04X, 0.06X, \ldots, 0.98X$ and X. Accordingly, T_q^H was calculated by using the relationship

$$T_q^H = [T_{q,1}^h + T_{q,2}^h + T_{q,3}^h + \ldots + T_{q,p}^h]/p\,, \tag{9}$$

$$\text{where } T_{q,n}^h = \left[T_q^h([n-1]X/p) + T_q^h(nX/p)\right]/2\,, \tag{10}$$

and $n = 1, 2, 3, \ldots, p$.

For convenience we have chosen the value of p to be 50 which would permit a reasonably sufficient accuracy in the calculation. The variations of T_q^H corresponding to the order of diffraction q and $[T_0^H]^{1/2}$ are shown in table 1. $[T_0^H]^{1/2}$ can be easily identified to be the regional amplitude transmittance of the zeroth order output. The corresponding values of input intensity transmittance T^P for logarithmic processing are also shown, where the minimum and maximum values of T^P were set at 0.2 and 0.8 respectively. The variations of T_q^H with diffraction order q for a specific value of $[T_0^H]^{1/2} = 0.5$ and $T^P = 0.283$ is shown in figure 1.

It can be seen that T_q^H is zero for all values of q when $[T_0^H]^{1/2} = 0$ or 1. Again T_1^H is nonzero for all values of $[T_0^H]^{1/2}$ except when $[T_0^H]^{1/2} = 0$ or 1. In general, $T_0^H > T_1^H$ and T_1^H is greater than all T_s^H, when $S > 1$. Also over the whole range of diffraction orders considered (1 through 25), the number of intensity peaks increases as the absolute value of $[T_0^H]^{1/2} - 0.5$ decreases. It can be concluded further that the values of T_q^H are independent of the nature of the screen, linear or nonlinear, but are dependent on the functional relationship of (6) alone.

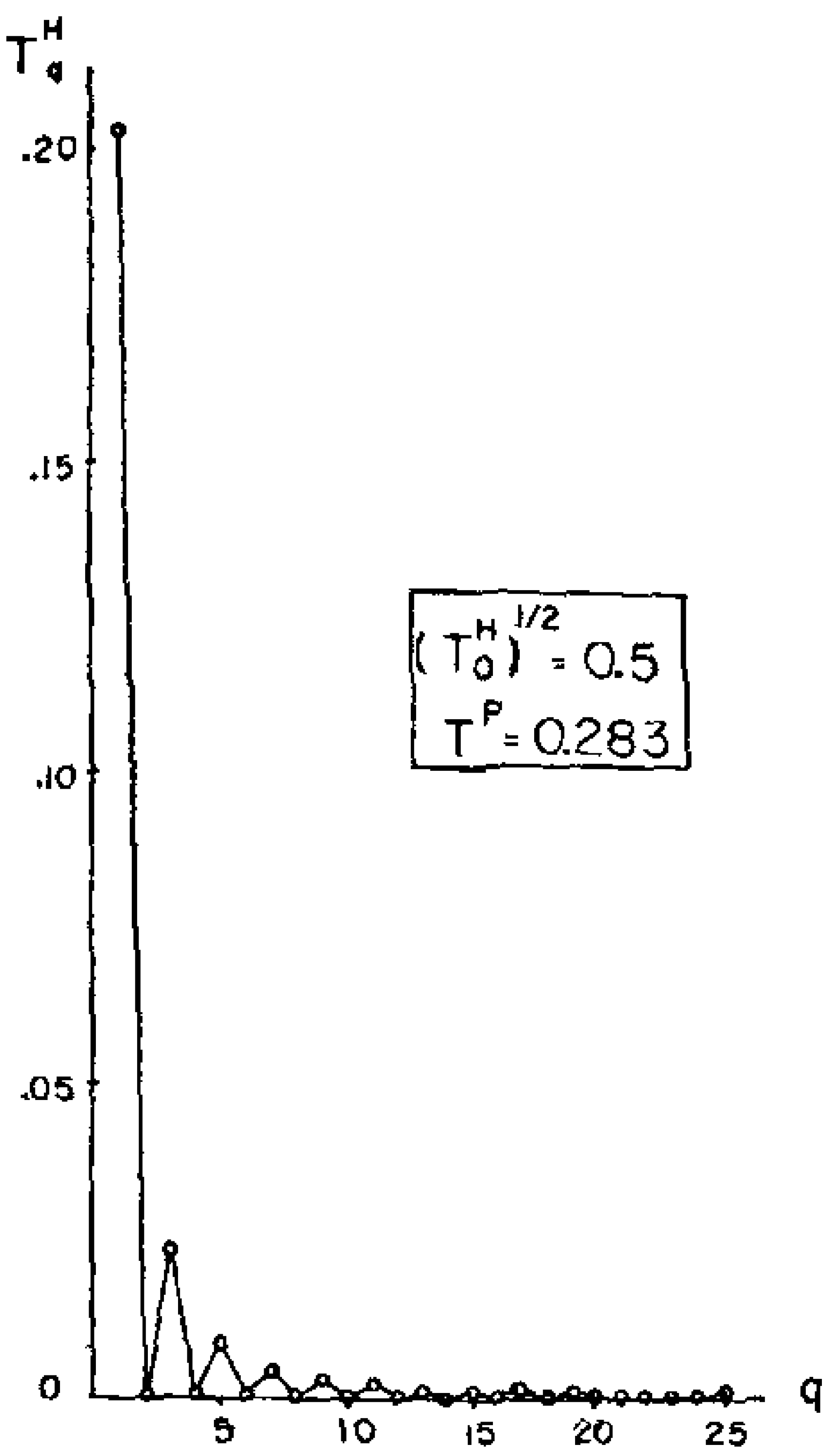

Figure 1. T_q^H values versus q when $[T_0^H]^{1/2} = 0.5$ and $T^P = 0.283$.

1. Wesner, J. W., *Appl. Opt.*, 1974, **13**, 1703.
2. Marquet, M. and Tsujiuchi, J., *Opt. Acta.*, 1961, **8**, 267.
3. Goodman, J. W., *Proc. IEEE.*, 1977, **65**, 29.
4. Karim, M. A. and Liu, H. K., *Opt. Lett.* 1981, **6**, 207.
5. Kato, H. and Goodman, J. W., *Appl. Opt.* 1975, **14**, 1813.
6. Lohmann, A. W. and Strand, T. C., *Proc. Electro-Optics/Int. Laser Conf.* 1975, Anheim, California.
7. Dashiell, S. R. and Sawchuk, A. A., *Appl. Opt.* 1977, **16**, 1009.
8. Liu, H. K. and Karim, M. A., *Opt. Lett.* 1979, **4**, 408.
9. Karim, M. A. and Liu, H. K., *Opt. Lett.* 1980, **5**, 132.
10. Liu, H. K. and Karim, M. A., *Opt. Lett.* 1981, **6**, 319.