LETTERS TO THE EDITOR

A STATIC SOURCE OF THE TAUB SOLUTION

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IN this note a static plane symmetric interior solution of Einstein's equations for perfect fluid with constant density is obtained. It is then matched to the Taub solution[1] which is clearly the exterior solution. The plane of symmetry divides the space into two regions I and II. In the following we have considered the interior and exterior solutions in the region I only. The solutions in the region II can be obtained from them by the transformations $x \to -x$.

We consider the static plane symmetric metric

$$ds^2 = -dx^2 - e^{\beta(x)}(dy^2 + dz^2) + e^{\delta(x)}dt^2 \quad (1)$$

and the Einstein's equations for perfect fluid

$$R^i_j - \tfrac{1}{2} R g^i_j = -8\pi[(p+\rho) V^i V_j - p g^i_j], \quad (2)$$

$$V^1 = V^2 = V^3 = 0, \; V^4 V_4 = 1 \quad (3)$$

where all symbols have their usual meanings.

The equation (2) for the metric (1) is

$$\begin{aligned} 8\pi p &= \tfrac{1}{4}(\beta'^2 + 2\beta'\delta') \\ 8\pi p &= \tfrac{1}{2}(\beta'' + \delta'') + \tfrac{1}{4}(\beta'^2 + \delta'^2 + \beta'\delta') \quad (4) \\ 8\pi q &= -\beta'' - \tfrac{3}{4}\beta'^2 \end{aligned}$$

where a dash denotes the differentiation with respect to x. The solution for constant ρ is obtained as

$$\begin{aligned} ds^2 = &-dx^2 - \left(\pm \frac{3a}{4b}\right)^{4/3} \cos^{4/3}(bx+c) \\ &\times (dy^2 + dz^2) + \left\{\frac{A}{b} - \cos^{-1/3}(bx+c)\right. \\ &\times \sin(bx+c)\left[B - \frac{A}{3}\int \cos^{-2/3}\right. \\ &\left.\left.\times (bx+c)\,dx\right]\right\}^2 dt^2, \end{aligned} \quad (5)$$

where a, b, A are non-zero constants and c, B are arbitrary constants. Choosing $x = 0$ as the plane of symmetry, the solution (5) can be put into the form

$$\begin{aligned} ds^2 = &-dx^2 - \cos^{4/3} bx\,(dy^2 + dz^2) \\ &+ \left\{\frac{A}{b} - \cos^{-1/3} bx\right. \\ &\left.\times \sin bx\left[B - \frac{A}{3}\varphi(x)\right]\right\}^2 dt^2 \end{aligned} \quad (6)$$

where

$$\varphi(x) = \int \cos^{-2/3} bx\,dx \quad (7)$$

by the obvious change of co-ordinates. The density ρ and the pressure p are given by

$$8\pi\rho = 4b^2/3 \quad (8)$$

$$8\pi p = 4b^2/3 \times \frac{b\cos^{-1/3} bx \times \sin bx\left[B - \frac{A}{3}\varphi(x)\right]}{A - b\cos^{-1/3} bx \times \sin bx\left[B - \frac{A}{3}\varphi(x)\right]} \quad (9)$$

The solution (6) is free from singularity if

$$0 \leqslant x < \frac{\pi}{|2b|} \quad (10)$$

The exterior Taub solution[1] may be written in the form

$$ds^2 = -dx^2 - (ax)^{4/3}(dy^2 + dz^2) + (ax)^{-2/3}dt^2. \quad (11)$$

The constant a which may be put one without loss of generality is retained for convenience. Let

$$x = x_1 < \frac{\pi}{|2b|}$$

be the boundary. The vanishing of the pressure and the continuity of g_{ij} at the boundary require

$$A = b\cos^{-1/3} bx_1, \; B = (b/3)\cos^{-1/3} bx_1\,\varphi(x_1),$$

$$a = \frac{1}{x_1}\cos bx_1. \quad (12)$$

Then the solution (6) becomes

$$\begin{aligned} ds^2 = &-dx^2 - \cos^{4/3} bx\,(dy^2 + dz^2) \\ &+ \cos^{-2/3} bx_1 [1 - f(x)]^2 dt^2 \end{aligned} \quad (13)$$

and the pressure p is given by

$$8\pi p = \frac{4b^2}{3}\,\frac{f(x)}{1 - f(x)} \quad (14)$$

where

$$f(x) = (b/3)\cos^{-1/3} bx \sin bx\,[\varphi(x_1) - \varphi(x)]. \quad (15)$$

Clearly p vanishes at $x = x_1$.

In order to have $0 \leqslant p \leqslant \rho$, we must have

$$0 \leqslant \frac{f(x)}{1 - f(x)} \leqslant 1 \quad (16)$$

It is found that the condition (16) is satisfied if x_0, where $f(x)$ is maximum, occurs in the interval

$$0 < x_0 \leq \frac{\pi}{|3b|}.$$

Thus the solution is always physically plausible if

$$x_1 \leq \frac{\pi}{|3b|}.$$

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1. Taub, A. H., *Ann. Maths.*, 1951, 53, 472.

CORRECTION FOR THERMALLY AFFECTED FISSION TRACKS IN GLASS (OBSIDIAN) BY AGE PLATEAU METHOD

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A correction of 22·55% for thermally lowered fission track ages of glass (obsidian) from Osham hill, Gujarat State, India has been determined by the age plateau method using fission track technique.

The fading of fission tracks in minerals due to geothermal events results in lowering of the fission track ages and thus needs a correction for the same to be applied. Annealing experiments have been performed by a number of workers[1-5] for correcting tha fission track ages.

In the present investigation, age plateau method developed by Storzer *et al.*[2] and Burchart *et al.*[3] has been applied on the (obsidian) glass of Osham hills, Gujarat State, India. Eight pairs of obsidian samples (each pair consisting of one sample of fossil tracks and the other of freshly induced neutron fission tracks) were annealed at a series of increasing temperature from 50° to 700° C for a period of one hour in each case. The annealed samples were then etched in 48% HF for 30 sec. at 20° C and the tracks were counted using an optical microscope at a magnification of 600 ×.

The f.t. age of obsidian was calculated by using the simplified version of the formula[6]

$$T = 6{\cdot}57 \times 10^9 \ln (1 + 9{\cdot}25 \times 10^{-18} \times \frac{\rho_s}{\rho_i} \times \phi) \quad (1)$$

where :

ρ_s = fossil track density;
ρ_i = induced track density,
ϕ = total thermal neutron dose (5×10^{16} nvt).

The value of ϕ was determined by irradiating a calibrated glass slide along with obsidian samples[7].

The f.t. ages calculated by using equation (1) are summarised in Table I.

From annealing data (Table II) it has been observed that in both the samples, in a pair the track density decreases with the increasing temperature but in the samples containing induced tracks, the rate of decrease is faster than in the sample containing fossil tracks.

F.T. ages for the annealed samples are calculated by using equation (1) and are summarised in Table II.

Table I
Fission track age data for glass (obsidian)
Total thermal neutron dose (ϕ) = 5×10^{16} (*nvt*)

Sample location	Lab. symbol	ρ_s tracks/cm² × 10⁴	ρ_i tracks/cm² × 10⁴	F.T. age (m.y.) T
Obsidian Osham hills,	OGI-1	2·10 (400)*	410·70 (1290)*	15·52±0·89**
Gujarat State, Inoia	OGI-2	2·16 (413)	413 84 (1300)	15·84±0·89
	OGI-3	2·20 (420)	418·60 (1315)	15·95±0·89
				mean 15·77±0·89

* Brackets shows the number of tracks counted, *i.e.*, N_s and N_i.

$$** \ \sigma_s = \frac{100}{\sqrt{N_s}}, \quad \sigma_i = \frac{100}{\sqrt{N_i}}, \quad \sigma = \sqrt{\sigma_s^2 + \sigma_i^2}.$$

Table II
Annealing data for glass (obsidian)
Heating time = 1 hr.

Temperature (° C)	Fossil track density (ρ_s/cm²) × 10⁴	Induced track dansity (ρ_i/cm²) × 10⁴	F.T. age (m.y.) T
30	2·10	410·70	15·52
50	2·10	410·70	15·52
100	1·93	369·63	15·84
200	1·70	313·13	16·57
300	1·36	237·19	17·40
400	0·82	135·39	18·37
500	0·46	73·93	18·88
600	0·25	39·98	18·97
700	·02	3·19	19·02