THE RELATIONSHIP BETWEEN DENSITY AND POTENTIAL FLUCTUATIONS IN LOW FREQUENCY MAGNETOPLASMA TURBULENCE

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ABSTRACT

It is conventional to assume that the relative plasma density fluctuations are equal in magnitude to the potential fluctuations scaled by the electron plasma temperature. We derive corrections to this equality.

1. INTRODUCTION

It is a standard practice in both theoretical[2] and experimental plasma physics[3] to assume that the relative level of plasma density fluctuations, n'_e/n_0, is equal in magnitude to the plasma potential fluctuations, $e\varphi$, scaled by the electron kinetic temperature, KT:

$$\frac{n'_e}{n_0} = \frac{e\varphi}{KT} \tag{1}$$

This assumption is widely used in instability studies, diagnostic work[7], and transport experiments[8] to name a few applications.

2. ION WAVES IN MAGNETIZED PLASMAS

The situation becomes rather more complicated, however, when we consider waves propagating in the presence of magnetic fields. If $\vec{B}$ is taken to define the z direction then the wave vector, $\vec{k}$, can be resolved into a parallel component, k_z, and a perpendicular component which we will denote by k_x. Such waves might, for instance, be driven by a relative drift between electrons and ions $\vec{v}_0 \parallel \vec{k}$. As is conventional[6] and anticipating experimental conditions[4], we will treat the electrons as magnetized but the ions as unmagnetized.

The equation of continuity is:

$$\frac{\partial n'_e}{\partial t} + n_0 \frac{\partial v_x}{\partial x} + n_0 \frac{\partial v_z}{\partial_z} = 0 \tag{2}$$

while the conservation of momentum can be written as:

$$m_e \frac{\partial \vec{v}_e}{\partial_t} + e(\vec{E} + \vec{v} x \vec{B}) + \frac{KT\nabla n'_e}{n_0} = 0 \tag{3}$$

where the notation is standard.

By assuming solutions of the form:

$$e^{i(\omega t - k_x x - k_z z)}$$

equations (2) and (3) can be linearized (for $\omega \ll \omega_c = eB/m_e$) in the electrostatic approximation

$$\vec{E} = i\vec{k}\varphi \tag{4}$$

yielding:

$$\omega n'_e - n_0 k_x v_x - n_0 k_z v_z = 0 \tag{5}$$

$$n_0 \omega v_x + n_0 e k_x \varphi / m_e - i n_0 e B v_y / m_e - KT k_x n'_e / m_e = 0 \tag{6}$$

$$n_0 \omega v_y + i n_0 e B v_x / m_e = 0 \tag{7}$$

$$n_0 \omega v_z + n_0 e k_z \varphi / m_e - KT k_z n'_e / m_e = 0. \tag{8}$$

Equations (6) and (7) are then combined in order to eliminate v_y and the resulting expression

$$v_x = \frac{k_x}{\omega\left(1 - \frac{\omega_c^2}{\omega^2}\right)} \left(\frac{KT}{m_e}\frac{n'_e}{n_0} - \frac{e\varphi}{m_e}\right) \tag{9}$$

along with equation (8), is substituted into (5) in order to obtain

$$\frac{n'_e}{n_0} = \frac{e\varphi}{KT} \left(\frac{1 - \frac{\omega^2}{\omega_c^2}\frac{k_z^2}{k_x^2}}{1 - \frac{\omega^2}{\omega_c^2}\frac{k_z^2}{k_x^2} - \frac{\omega^2 m_e}{k_z^2 KT}} \right) \tag{10}$$

In order to focus our attention on modestly damped waves we demand that:

$$k \ll k_{\text{Debye}}. \tag{11}$$

We can further simplify equation (10) if we restrict our attention to bounded laboratory plasmas and cross-field wave propagation for which $k_z \neq 0$ and:

$$k_x \omega \ll k_z \omega_c. \tag{12}$$

Under these conditions equation (10) further reduces to:

$$\frac{n'_e}{n_0} = \frac{e\varphi}{KT} \left(\frac{1}{1 - \frac{\omega^2 m_e}{k_z^2 KT}} \right) \tag{13}$$

and we find that the relative electron density fluctuations are greater than the scaled potential fluctuations.

Similar relationships to (10) and (13) can be obtained for ions starting with the equations:

$$\frac{\partial n'_i}{\partial t} + \vec{v}_0 \cdot \nabla n'_i + n_0 \nabla \cdot \vec{v}_i = 0, \tag{14}$$

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and:

$$m_i \frac{\partial \vec{v}_i}{\partial t} + n_i \vec{v}_o \cdot \nabla \vec{v}_i - e\vec{E} = 0, \tag{15}$$

in the limit where the ion drift velocity, $\vec{v}_o$, is large enough and the ion temperature is small.

Linearizing we obtain (for $\vec{v}_i \parallel \vec{v}_o$):

$$\omega n_i' - k v_o n_i' - n_o k v_i' = 0 \tag{16}$$

$$\omega v_i' - k v_o v_i' - e\varphi k/m_i = 0. \tag{17}$$

from which we derive the relationship:

$$\frac{n_i'}{n_o} = \frac{e\varphi}{m_i}\left[\frac{k^2}{(\omega - k v_o)^2}\right] \tag{18}$$

Equations (10) and (18) can be solved along with the Poisson equation:

$$\epsilon_o k^2 \varphi = -e(n_e' - n_i') \tag{19}$$

to obtain the dispersion equation:

$$\frac{k^2}{k_D^2} = \frac{KT k^2}{m_i(\omega - k v_o)^2} - \frac{1 - \left(\frac{k_z \omega}{k_z \omega_c}\right)^2}{1 - \left(\frac{k_z \omega}{k_z \omega}\right)^2 - \frac{\omega^2 n_e}{k_z^2 KT}}. \tag{20}$$

Under restrictions (11) and (12) equation (20) simplifies to:

$$\frac{KT k^2}{m_i(\omega - k v_o)^2} = \frac{1}{1 - \frac{\omega^2 m_e}{k_z^2 KT}} \tag{21}$$

which has the analytic solution

$$\frac{\omega}{k} = \frac{1}{1 + \frac{k^2 m_e}{k_z^2 m_i}} \times \left[v_o \pm \sqrt{\frac{KT}{m_i}} \sqrt{1 - \frac{k^2\left(v_o^2 - \frac{KT}{m_i}\right)}{k_z^2 \frac{KT}{m_e}}}\right] \tag{22}$$

By expansion of (22) in the parameters

$$\frac{k}{k_z}\sqrt{\frac{m_e}{m_i}} \text{ and } \frac{k v_o}{k_z}\sqrt{\frac{m_e}{KT}}$$

we find for the fast mode, $v_o + \sqrt{\frac{KT}{m_i}}$:

$$\frac{\omega}{k} = v_o + \sqrt{\frac{KT}{m_i}} - \frac{k^2 v_o \sqrt{\frac{KT}{m_i}}\, m_e}{2 k_z^2 KT}\left(v_o - 2\sqrt{\frac{KT}{m_i}}\right) \tag{23}$$

and for the slow mode, $v_o - \sqrt{\frac{KT}{m_i}}$:

$$\frac{\omega}{k} = v_o - \sqrt{\frac{KT}{m_i}} + \frac{k^2 v_o \sqrt{\frac{KT}{m_i}}\, m_e}{2 k_z^2 KT}\left(v_o + 2\sqrt{\frac{KT}{m_i}}\right) \tag{24}$$

(Studies of the complex solutions of the dispersion relation will show that the fast mode is damped while the slow mode grows[3,6]). Using equation (24) for the unstable ion wave in equation (18) we obtain:

$$\frac{n_i'}{n_o} = \frac{e\varphi}{KT}\left[1 + \frac{k^2 v_o\left(v_o + 2\sqrt{\frac{KT}{m_i}}\right)}{k_z^2 \frac{KT}{m_e}}\right] \tag{25}$$

and show that the ion density fluctuations also exceed the potential fluctuations.

3. Effects on Transport

The foregoing calculations were suggested by recent studies of ion acoustic wave induced plasma transport[3,8]. In these experiments the ion wave amplitude was compared with the observed plasma diffusion and predictions of several turbulent transport models including simple Bohm diffusion:

$$D = \frac{1}{16}\frac{KT_e}{eB} \tag{26}$$

Equation (26) can, for example be derived[7] from ambipolarity in a fully ionized plasma, $D = D_e \approx D_i$, and the condition for optimal (ion) diffusion, $\nu_i \sim \omega_{c_i}$:

$$D = D_i = r_{L_i}^2 \nu_i = r_{L_i}^2 \omega_{c_i} = \frac{T_i}{T_e}\frac{KT_e}{eB} \quad \text{where } \frac{T_i}{T_e} \text{ was } \sim \frac{1}{16}.$$

"Whereas classical diffusion goes over to Bohm diffusion, as $\nu_{ei} \to \omega_{ce}$, neoclassical diffusion goes over to poloidal Bohm", and a Bohm diffusion coefficient corrected for the appropriate (observed) turbulent amplitude:

$$D = v_\perp \lambda = \frac{E_\perp \lambda}{B} = \frac{\varphi}{B} \tag{27}$$

(Ref. 1)

In the experimental work referred to previously it had been assumed that $n_i'/n_o = e\phi/KT_e$ (eqn. 1) and (27) becomes

$$D = \frac{n_e'}{n_o}\frac{KT_e}{eB} \tag{28}$$

It was thought that the present correction to [eqn. (1) and, therefore] equation (28), *i.e.*,

$$D = \left(1 - \frac{\omega^2}{k_z^2 v_e^2}\right) \frac{n_e'}{n_0} \frac{KT_e}{eB} \tag{29}$$

might bring the experimental observations into better agreement with Bohm theory. (In actual fact, however, for the parameters, of the *cited* experiments $\omega/k_z \ll v_e$ and the correction is small).

Turbulent transport has also been attributed to drift modes (which are closely related to ion acoustic waves) and Kadomtsev[5] p. 83) has derived (for such waves) a result which is similar to eqn. (13)

$$\frac{n_e'}{n_0} = \left(1 - \frac{\omega^2 - \omega^{*2}}{k_Z^2 v_A^2}\right) \frac{e\varphi}{KT} \tag{30}$$

and used this to obtain turbulent diffusion coefficients for drift instabilities (Kadomtsev[5], p. 108). Here ω^* is the diamagnetic frequency and v_A is the Alfven speed.

4. Conclusions

We have used plasma kinetic theory to derive the relationships between electron and ion relative density fluctuations and normalized plasma potential fluctuations. In the more generalized problem of magneto-ion acoustic turbulence we find that the simple equality usually assumed for these quantities does not obtain. This result has various consequences, such as modifications to the magnitude of turbulent diffusion.

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THE REACTIVITY OF α-CYANOCHALCONES AS MICHAEL ACCEPTORS*

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Abstract

Substituted ω-cyanoacetophenone reacts with aromatic aldehydes to give the corresponding chalcones I *a–d*. Compounds I *a–d* reacted with each of malononitrile, ethyl cyanoacetate, ω-cyanoacetophenone and ethyl acetoacetate to give the corresponding cyclic Michael adducts IV *a, b*, V *a, b*, VII and VI respectively. Similarly substituted ω-cyanoacetophenones were readily coupled with a variety of aromatic diazonium chlorides to give the corresponding arylazo derivatives IX *a–d*.

In spite of enormous literature concerning the reactivity of chalcones[1–5] as Michael acceptors, little attention has been paid towards the reactivity of the double bond in α-cyanochalcones as acceptor in the Michael reaction. In continuation of our work on the chemical behaviour of α, β-unsaturated cyano compounds[6–8], it has been found that the arylidene derivatives I *a–d*, prepared *via* condensation of β-ketonitrile derivatives II *a, b* with aromatic aldehydes, react with malononitrile to yield addition products which may be formulated as III or as the enaminopyran IV. Structure IV could be established for the reaction product based on spectral data. Thus, the IR spectra of the reaction products revealed absorption of νNH_2, δNH_2 and two cyano bands for enaminonitrile CN and for conjugated CN. Also the H^1 NMR revealed in addition to aromatic protons a singlet at δ4 for pyran H–4.

Compound I *a, d* reacted with ethyl cyanoacetate to yield the addition products for which structure V was suggested. Similarly I*d* reacted with ethyl acetoacetate to give the cyclic adducts VI. Compound VI was readily acetylated with acetic anhydride to yield the acetyl derivative VII.

Successful attempts have been made to generalize Michael reaction using ω-cyanoketone with α-cyanochalcones. Thus II *a* reacted with *p*-methyl-ω-cyanoacetophenone to give the cyclic Michale adduct VIII.

In continuation of our previous studies in the chemistry of arylazo-β-oxonitriles[9–11], a variety of aryldiazonium salts were coupled with substituted ω-cyanoacetophenone (II) to give the α-arylhydrazon-β-oxo-β-phenylpropionitrile derivatives IX *a–d*. The IR spectra of the coupled products IX *a–d* indicate that they have the hydrazone structure.

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