MHD FLUCTUATING FLOW OF A NON-NEWTONIAN FLUID PAST A POROUS PLATE

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ABSTRACT

In this paper, we have studied the two-dimesional flow of an incompressible, electrically conducting non-Newtonian fluid (Rivlin-Ericksen[6]) in the presence of a transverse magnetic field in slip-flow regime. Expressions for velocity field have been obtained and discussed graphically.

1. INTRODUCTION

IN the present era of high altitude flights the study of slip flow regime has been recognised to be of immense importance. Many researchers have paid their attention towards the fluctuating flows of viscous and viscoelastic incompressible fluids past an infinite plate (Lighthill[3], Stuart[11], Suryaprakash Rao[12,13], Reddy[5], Messiha[4], Soundalgeker[8,9], Dubey and Battacharya[2], Siddappa and Chetty[7]). In fact, the increasing emergence of non-Newtonian fluids such as molton plastics, pulps, emulsions, aqueous solutions of polyacrylamide and poly-isobutylene, etc., as important raw materials and chemical products in a large variety of industrial processes, has stimulated a considerable amount of interest in the behaviour of such fluids when in motion. Considerable attention has been given in recent years to the steady of non-Newtonian fluids and their related transport processes. Recently, Debler[1] carried out a mathematical analysis connected with the shaking table of a four-drinier paper machine. Such a machine can be described as a porous conveyer belt. A dilute mixture of water wood pulp is deposited at one end of the belt, water continuously drains from the mixture and through the belt. If the belt oscillates laterally the quality of the paper is improved. If in such a device, a uniform and strong magnetic field is applied transverse to the direction of belt's motion it will cause some changes in favour of the improvement of the paper.

In the present paper an attempt has been made to study the fluctuating flow of an incompressible, electrically conducting non-Newtonian fluid (Rivlin-Ericksen[6]) past an infinite flat plate subjected to time dependent suction in slip flow regime in the presence of a transverse magnetic field.

2. MATHEMATICAL ANALYSIS

The equations describing the flow of an incompressible, electrically conducting non-Newtonian fluid (Rivlin-Ericksen visco-elastic model) in the presence of a transverse magnetic field of strength H_0 in slip flow regime are:

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{dU}{dt} + a\frac{\partial^2 u}{\partial y^2} + \beta\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right) + \frac{\sigma}{\rho}B_0^2(U-u), \tag{1}$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + 2(2\beta + r)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial v}{\partial y} = 0. \tag{3}$$

Let the fluctuating free-stream and suction velocities be

$$U = U_0(1 + \epsilon e^{int}), \quad v = -v_0(1 + A\epsilon e^{int}), \tag{4}$$

where U_0 and v_0 are the mean free-stream and mean suction velocity respectively; $\epsilon \ll 1$ and A is such that $A\epsilon \ll 1$; n is the frequency of the fluctuations.

First order velocity slip-boundary condition is given, Street[10], by

$$\left.\begin{aligned} u &= L_1\frac{\partial u}{\partial y} \quad \text{at} \quad y = 0 \\ u &\to U(t) \quad \text{as} \quad y \to \infty \end{aligned}\right\}, \tag{5}$$

where

$$L_1 = (2 - m_1)L/m_1;$$

$$L = \mu\left(\frac{\pi}{2p\rho}\right)^{1/2}$$

is the mean free path and m_1 the Maxwell's reflexion coefficient.

Let us introduce the following non-dimensional quantities:

$$(u^*, v^*, U^*, y^*, n^*, t^*) = \left(\frac{u}{U_0}, \frac{v}{v_0}, \frac{U}{U_0}, \frac{v_0 y}{a}, \frac{na}{v_0^2}, \frac{tv_0^2}{a}\right).$$

By virtue of these non-dimensional quantities equation (1), after dropping the stars, reduces to

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{dU}{dt} + \frac{\partial^2 u}{\partial y^2} + E\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right) + M(U-u), \tag{6}$$

where

$$M\left(=\frac{\sigma B_0^2 a}{\rho v_0^2}\right)$$

is the magnetic field parameter and

$$E\left(=\frac{\beta v_0^2}{a^2}\right)$$

the viscoelastic parameter,

The non-dimensional form of (4) is

$$U = (1 + \epsilon e^{int}), \quad v = -(1 + A\epsilon e^{int}). \tag{7}$$

The boundary conditions become

$$u = R\frac{\partial u}{\partial y} \text{ at } y = 0, \quad u \to U(t) \text{ as } y \to \infty \tag{8}$$

where

$$R = \frac{L_1 v_0}{a},$$

the rarefaction parameter. To solve (6) with the help of (7) and (8), let us follow Lighthill's method and assume the velocity in the neighbourhood of the plate be

$$u = f_0(y) + \epsilon e^{int} f_1(y). \tag{9}$$

Substituting (7) and (9) in (6), neglecting square and higher powers of ϵ and separating harmonic and non-harmonic terms, then in the resulting differential equations, again putting (Soundalgeker[8])

$$f_0 = f_{00} + Ef_{01} + 0(E^2),$$
$$f_1 = f_{10} + Ef_{11} + 0(E^2), \tag{10}$$

and then equating the like powers of E and solving under the corresponding boundary conditions, the velocity field in the boundary layer is obtained as

$$u(y, t) = \left(1 - \frac{1}{1 + r_1 R} e^{-r_1 y}\right) - E\left(y + \frac{R}{1 + r_1 R}\right) a_1 e^{-r_1 y} + \epsilon\,(M_r \cos nt - M_i \sin nt), \tag{11}$$

where

$$r_1 = \frac{1}{2} + \left(M + \frac{1}{4}\right)^{1/2}, \quad a_1 = \frac{r_1^2 a_2}{(1 + 4M)^{1/2}},$$

$$a_2 = \frac{r_1}{1 + r_1 R}, \quad a_3 = \frac{A r_1 a_1}{n},$$

$$h = \frac{1}{2} + \left(M + \frac{1}{4} + in\right)^{1/2}$$

$$h_1 = \frac{-n + iAr_1}{n(1 + hR)},$$

$$H = \frac{a_1 a_2 A}{r_1} + i\frac{A a_2 r_1^3}{n}$$

$$L = \frac{a_1 r_1 A}{n^2}(2r_1 - 1) + i\frac{H}{n},$$

$$K = \frac{1}{2}\frac{h^2 h_1 (h + in)}{\left(M + \frac{1}{4} + in\right)^{1/2}},$$

$$N = \frac{R}{1 + hR}(K - r_1 L - ia_3) - \frac{L}{1 + hR}$$

and M_r and M_i are the fluctuating parts of the velocity profiles and are obtained from

$$M_r + iM_i = \left(1 + h_1 e^{-hy} - i\frac{A}{n} a_2 e^{-r_1 y}\right) + E\,(N e^{-hy} + L e^{-r_1 y} - a_3 y e^{-r_1 y} + Ky e^{-hy}). \tag{12}$$

Shear stress at the wall, τ_w, can be easily obtained from

$$\tau_w = \left\{a\frac{\partial u}{\partial y} + \beta\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial t}\right)\right\}_{y=0} \tag{13}$$

and the expression for pressure (p) can be obtained by (2).

3. Particular Cases

When $A \to 0$, the problem reduces to the case of constant suction velocity in slip flow regime. Again, when viscoelastic parameter $E \to 0$, the problem reduces to the flow of Newtonian fluid in slip flow regime.

4. Discussion

We have studied the effect of the elasticity, rarefaction parameter and magnetic field parameter on the velocity distribution and its fluctuating parts in the boundary layer. We have plotted velocity (u) and its fluctuating parts (M_r, M_i) versus y. In Figs. 1, 2 and 3, we have shown velocity profiles in the boundary

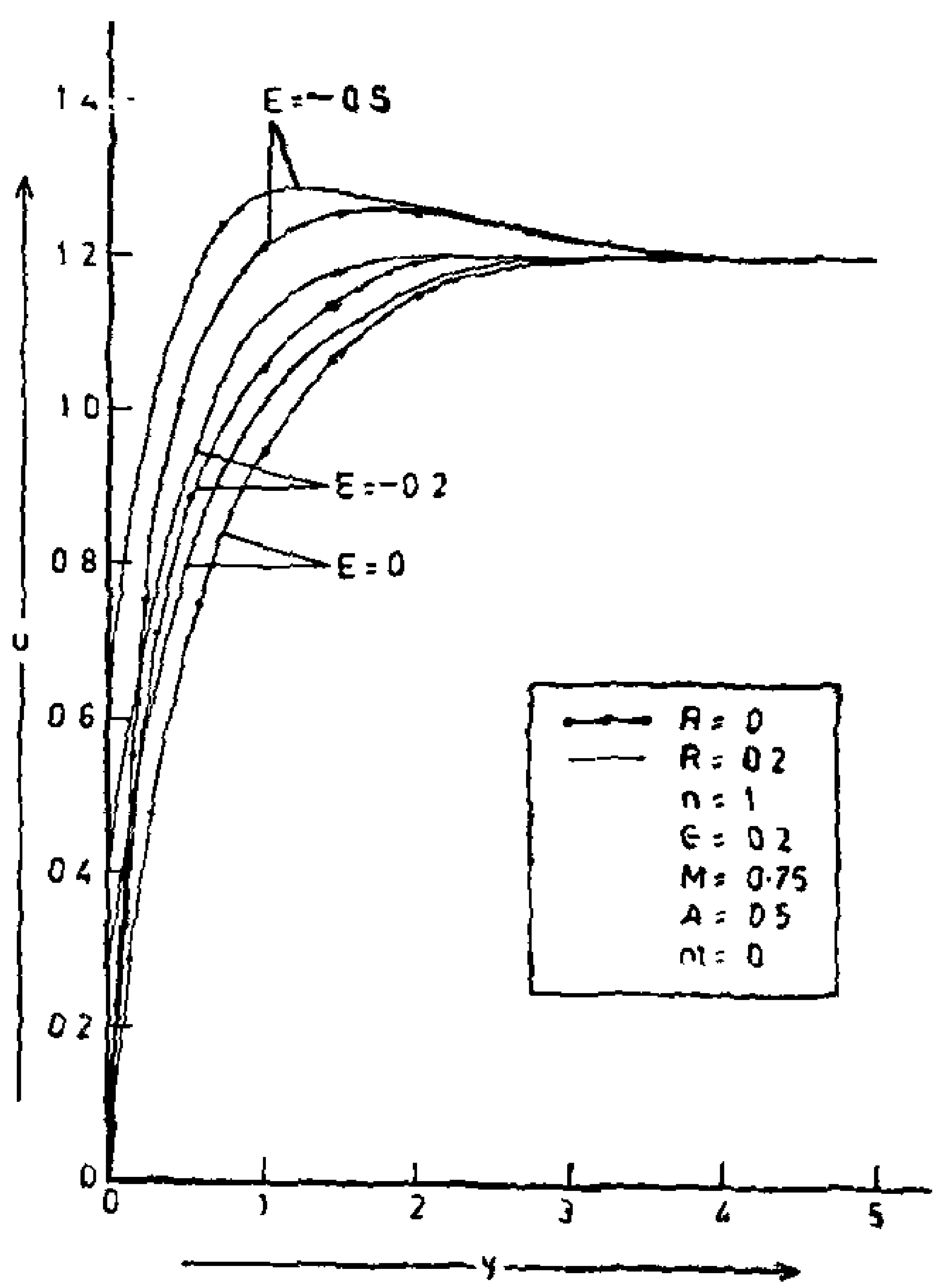

FIG. 1

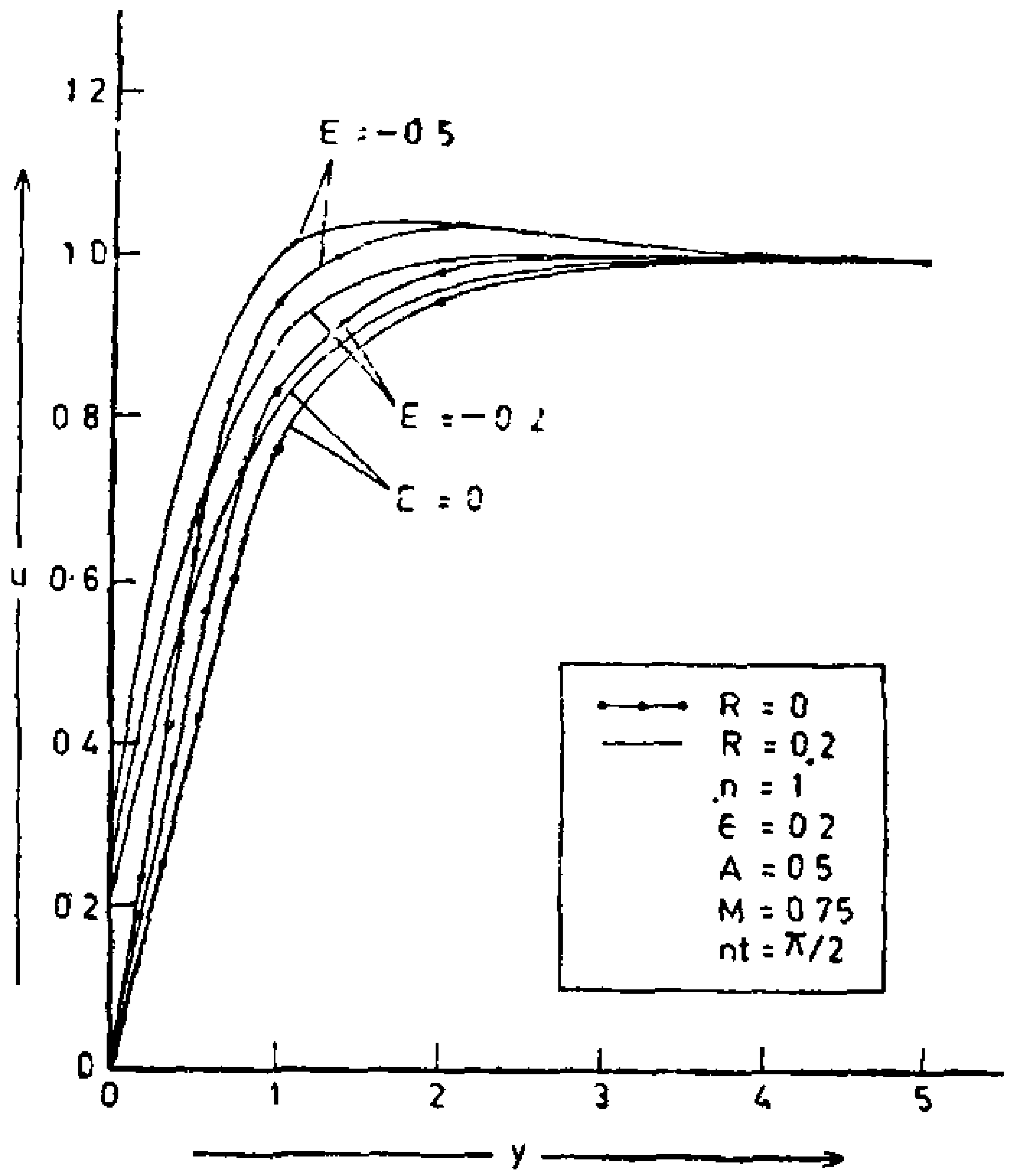

FIG. 2

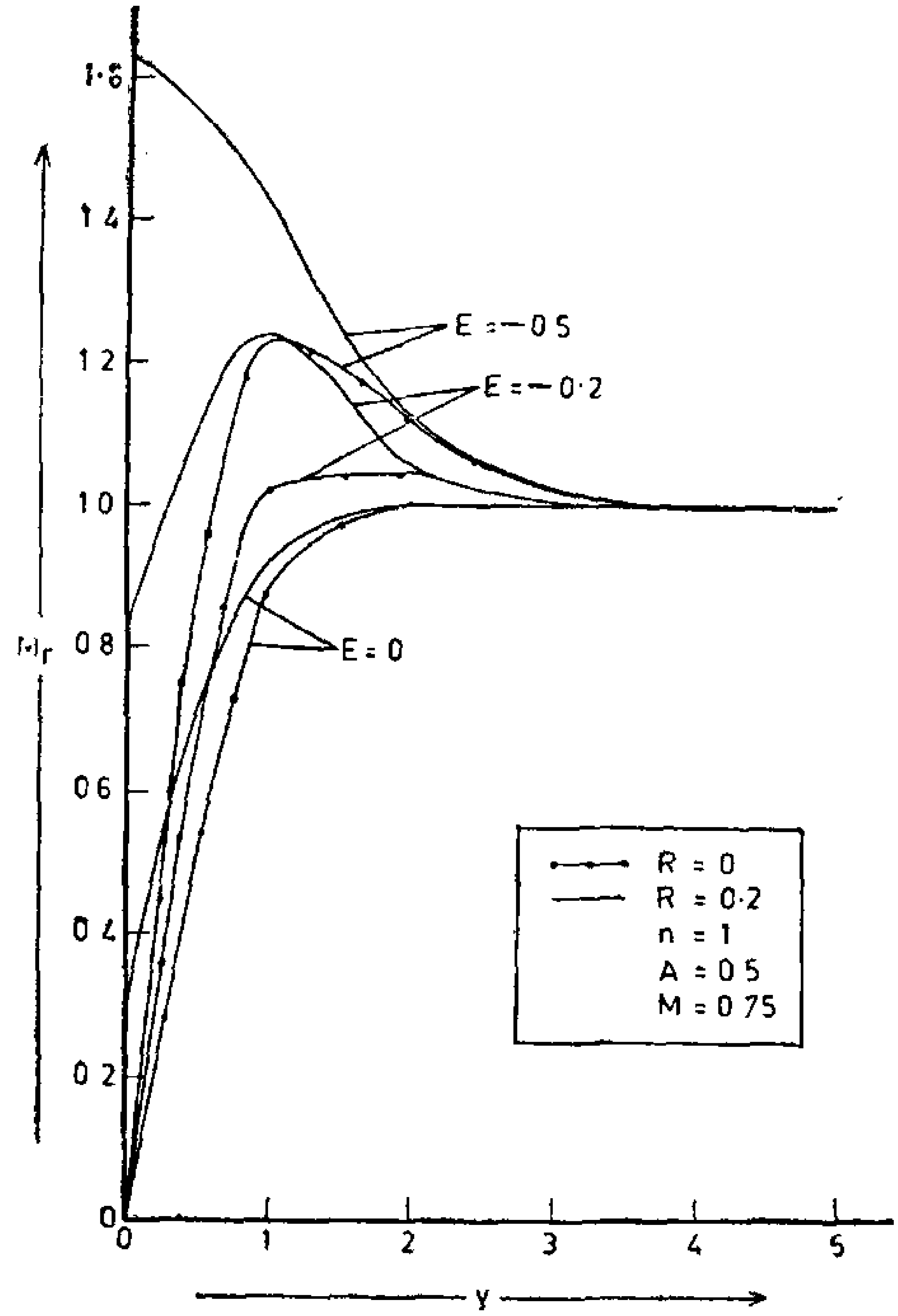

FIG. 4

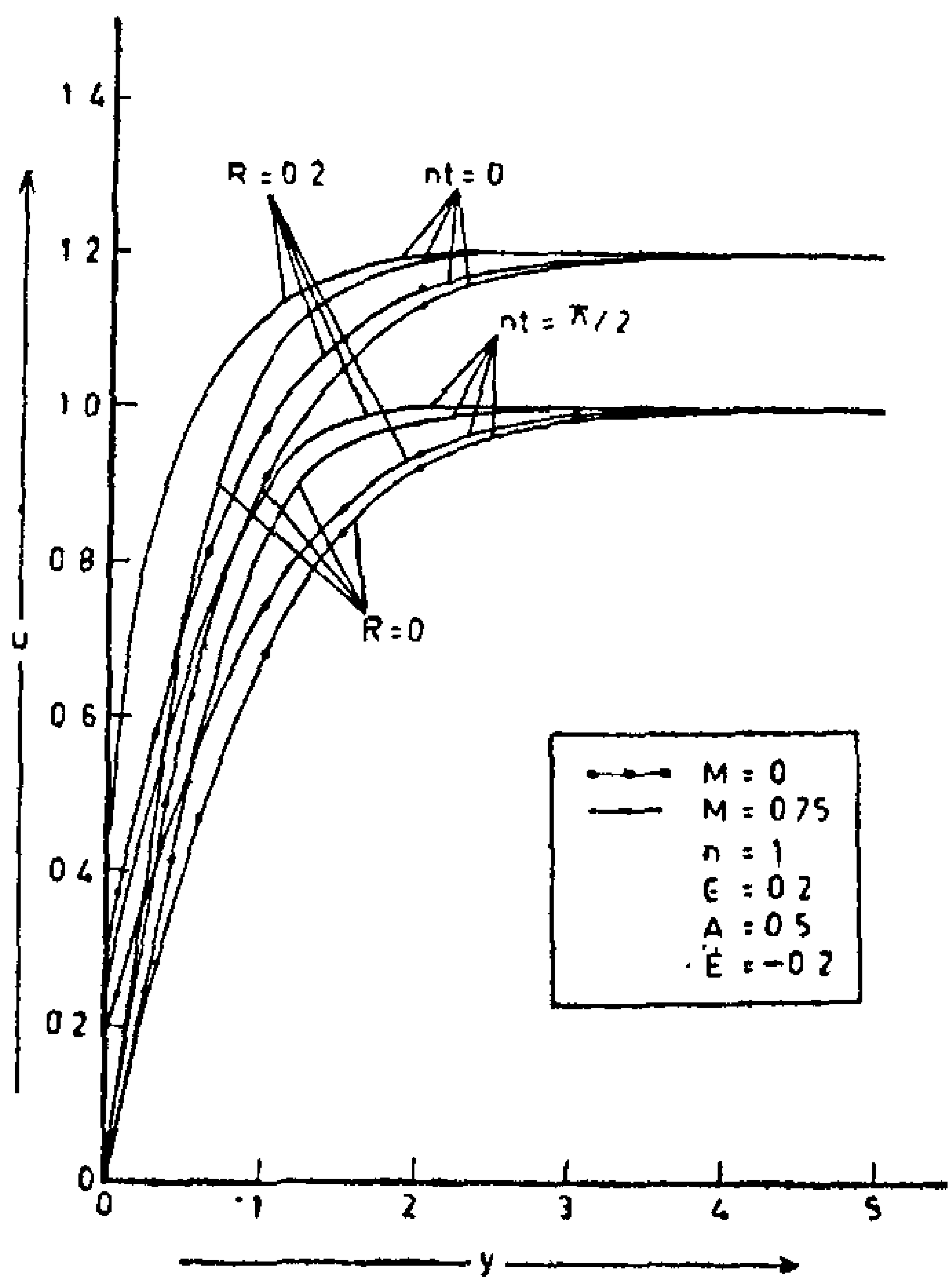

FIG. 3

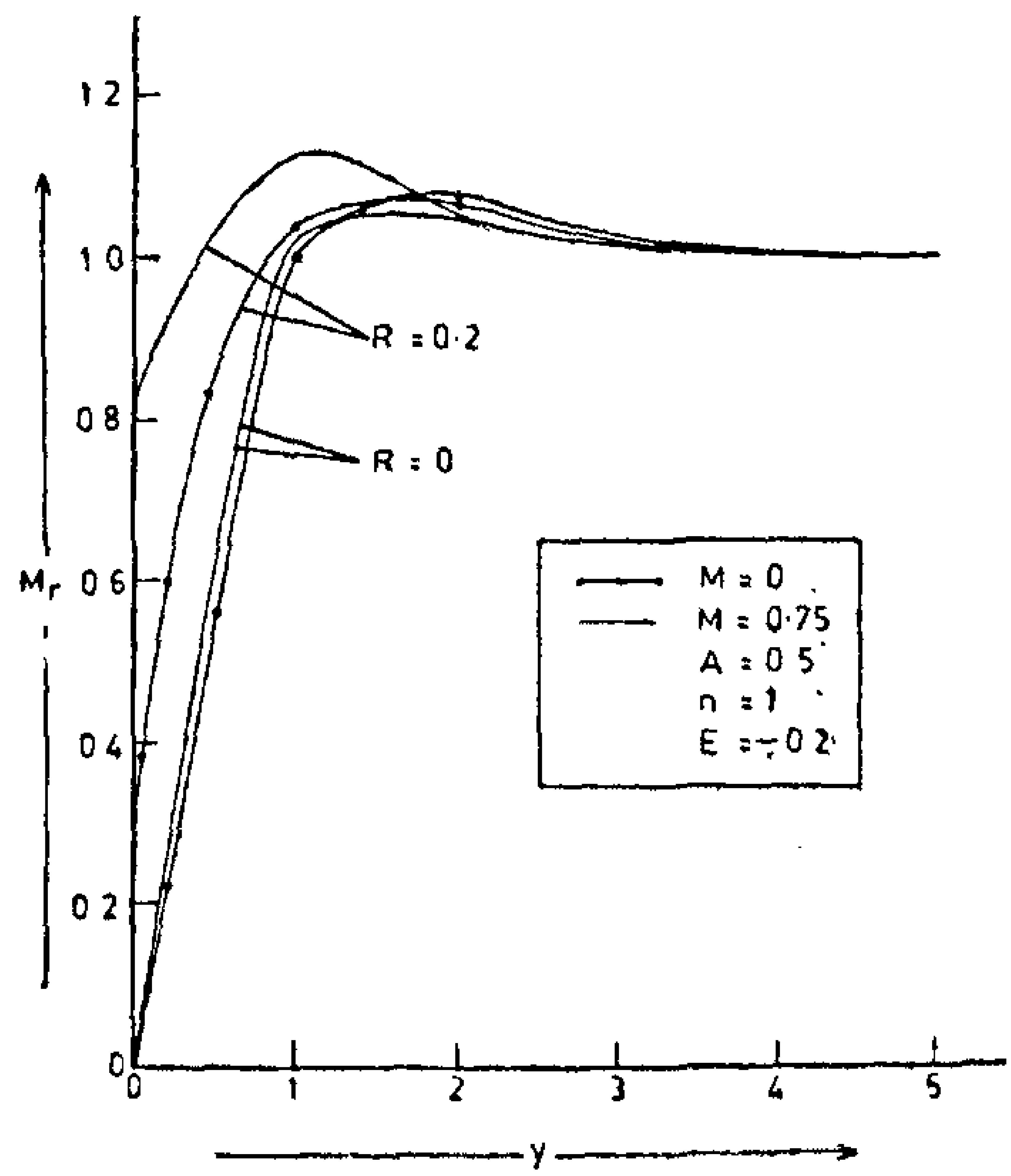

FIG. 5

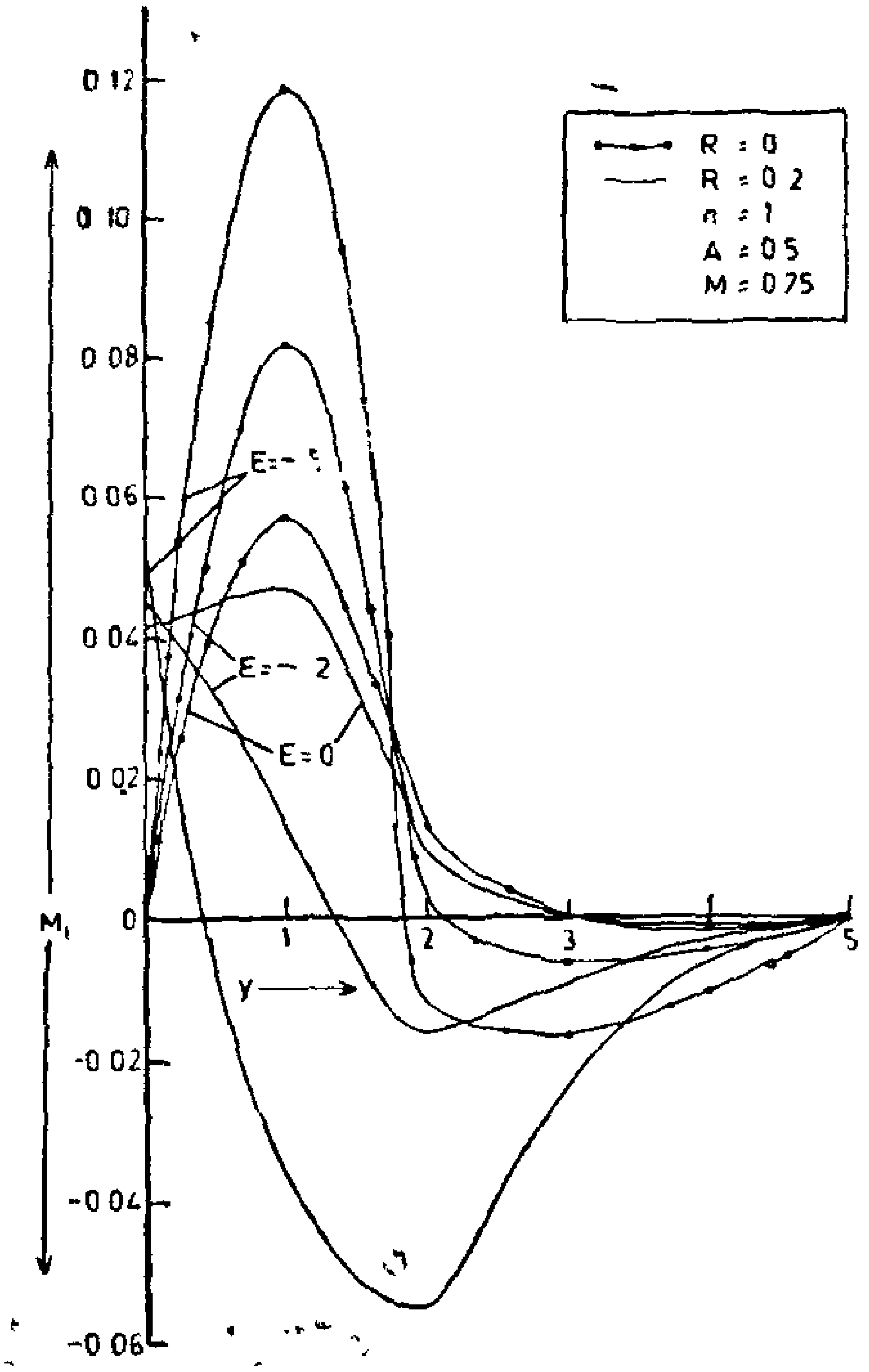

FIG. 6

layer for $\epsilon = 0\cdot 2$, $A = 0\cdot 5$ and different values of M, R, E and nt. From Figs. 1, 2 and 3, we observed that velocity increases sharply as y increases near the plate. From figures it is evident that for the same value of nt an increase in E (numerically) increases the velocity field showing that the velocity field for visco-elastic fluids is more than that of Newtonian fluids. It is clear that an increase in R and M increases the velocity field. This shows that with an increase in the magnetic field parameter (M) increases the velocity. But in the slip flow regime the velocity asymptotically approaches the main stream velocity as per Siddappa and Chetty[7]. It is also evident that the velocity field is larger everywhere for $nt = 0$ than for $nt = \pi/2$.

In Figs. 4, 5, 6 and 7, the trend of flucutating parts are shown for $n = 1$. Figures 4 and 5 illustrate that M_r for viscoelastic fluids is more than that of Newtonian fluids. It is also clear that M_r increases with an increase in R and M. Figures 6 and 7 are particularly interesting because of the sudden rise and fall of M_i near the plate, which is not observed in Newtonian fluids.

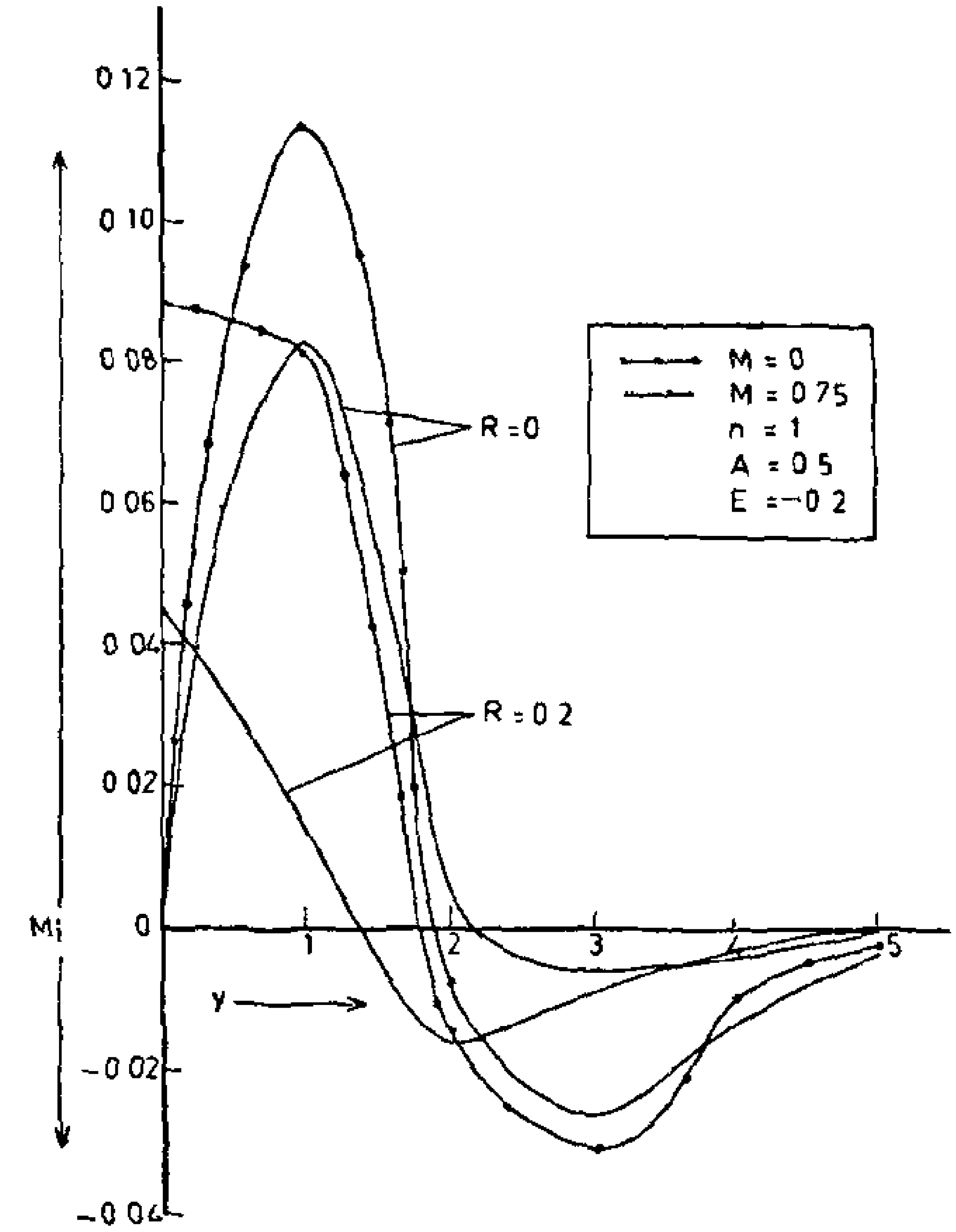

FIG. 7

ACKNOWLEDGEMENT

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