

**Indian Mathematical Society:**

(*Journal*, 2, No. 5). E. H. NEVILLE: *Bipolar and Trigeminal Coordinates on a Line*.—If P is a variable point on a line AB, the expressions  $AP^2$ ,  $BP^2$  are called bi-polar co-ordinates. If these are called  $\lambda$ ,  $\mu$ , and if  $h = AB^2$ , there is a relation between  $\lambda$ ,  $\mu$ ,  $h$  which can be expressed by saying that the line  $\lambda x + \mu y + 1 = 0$  touches the conic  $hxy + x + y = 0$ . Thus a correlation is established between points on AB and the tangents to a hyperbola. This sets up a correspondence between point-pairs on the line and points in the plane of the above conic. A systematic study of this correspondence forms the subject-matter of this paper.

MISS. S. PANKAJAM: *On Symmetric Functions of n Elements in a Boolean Algebra*.—If  $A_1, A_2, \dots, A_n$  be elements of a Boolean Algebra, let  $\beta_r(A_1, A_2, \dots, A_n)$  denote the class of elements belonging to exactly  $r$  of the classes A. These functions  $\beta_r$  are considered for different values of  $r$ , and various types of symmetric functions formed from the A's by the Boolean operations  $+$ ,  $\times$ , negation as well as conjunction and disjunction are expressed in terms of the  $\beta$ 's.

D. P. BANERJEE: *A Further Note on the Zero of Bessel Functions*.—It is proved that  $J_n$  and  $J_{n+m}$  have no common zeros except perhaps those at the origin, provided  $m$  and  $n$  are real,

$$|m| < 1, \text{ and } n > \max\left(\frac{1}{2}, \frac{m^2}{2(1-m)}\right). \quad \text{fol-}$$

lows that if  $|m| < 1$ , and  $m, n$  be real,  $Y_n(z)$  and  $Y_{n+m}(z)$  have no common positive zeros except may be those at the origin.

**Mathematics Student:**

(*Journal*, 4, No. 3). This issue is dedicated to the memory of the late Mr. V. Ramaswamy Aiyar, the founder of the Indian Mathematical Society, and contains a portrait of his later years. Life sketches and reminiscences of this remarkable personality are given by Mr. M. T. Naraniengar, by Mr. S. R. Ranganathan, and by several other gentlemen who had the occasion to move or correspond with him frequently. There are also some articles contributed by him, and the substance of one of his lectures on the Fermat point of a three-point system delivered under the auspices of the Madras University. There is also a paper by Mr. A. A. Krishnaswami Ayyangar, entitled "Geometry of the tricusp hypo-cycloid" which was the outcome as well as the development of certain of Mr. V. Ramaswamy Aiyar's results in the subject. Lastly, solutions to several problems

of V. Ramaswamy Aiyar, and some new problems of his in connection with what he has termed the *Durai Rajan point* of a quadrangle are published.

V. RAMASWAMY AIYAR: *The Fermat Point of a Three-Point System*.—If A, B, C be three points, the position (or positions) of P for which the expression  $\lambda PA + \mu PB + \nu PC$  is a minimum is the Fermat point (or points) of the system. The problem is to study the position or positions of the Fermat points for varying values of the constants  $\lambda, \mu, \nu$ . A geometrical study of this problem is explained here, with particular reference to the cases where  $\lambda, \mu, \nu$  are all positive.

V. RAMASWAMY AIYAR: *Note on a Class of Curves*.—Let  $R_{2n}$  be a curve of class  $n+1$  touching the line at infinity  $n$  times, the circular points being two of the points of contact. The curve is determined when  $2n$  tangents are given. We have then the property:

If any  $2n+1$  tangents of an  $R_{2n}$  be taken, their Clifford-Miquel circle is a straight line.

When  $n=2$ , this gives: If any five lines are tangents to a three-cusped hypo-cycloid, their Miquel circle becomes a straight line (given earlier by the author in Question 1250, *J.I.M.S.*).

A. A. KRISHNASWAMI AYYANGAR: *Geometry of the Tri-Cusped Hypo-Cycloid*.—Among the several results, the following may be mentioned here:—

(1) If any transversal cut the sides BC, CA, AB of a triangle ABC at D, E, F, such that  $BC \cdot BD + CA \cdot CE + AB \cdot AF = \Omega$  (a constant) (the segments being taken positively in the direction which makes the area of the triangle positive), then the transversal envelopes a fixed tri-cusp inscribed in the triangle ABC. (Qn. 1458, *J.I.M.S.*).

Conversely, if ABC be any triangle circumscribed to a tri-cusp, and any tangent to the tri-cusp meet BC, CA, AB in D, E, F, then  $BC \cdot BD + CA \cdot CE + AB \cdot AF$  is constant.

(2) If  $P_1, P_2, P_3$  be the points of intersection of any tangent to a tri-cusp with three concurrent tangents  $OT_1, OT_2, OT_3$  whose points of contact are  $T_1, T_2, T_3$  then

$$\sum \frac{OP_1}{OT_1} = 1 \text{ and } \sum \frac{1}{OP_1 \cdot OT_1} = 0$$

(3) If the tangents at  $T_1, T_2, T_3$  to a tri-cusp meet in O, the isogonal conjugate of  $OT_3$  in the angle  $T_1OT_2$  bisects  $T_1T_2$  and is parallel to the other tangent from  $T_3$ .

(4) Tangent pairs from points on any given tangent to a tri-cusp meet the line at infinity in point-pairs of the same involution, of which the circular points are members. The double points of the involution are the points at infinity on the tangents at the extremities of the tangent chord.

**Erratum.**

Vol. 5, No. 11, May 1937, page 595 article entitled "A Note on the Hairiness in the Punjab Cottons".—

In place of "By R. S. Jai Chand Luthra"

read "By R. S. Jai Chand Luthra and Indar Singh Chima".