# ANALYSIS OF FIDELITY IMAGE REPRODUCTION BY HALFTONE SCREENS OF SPECIFIC DESIGNS*

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ABSTRACT

A new and reliable approach for the analysis of various cell patterns of halftone contact screens and the reproduced halftone images is presented. Optimization parameters for highest fidelity image reproduction by screens of line, square, circular and concentric-ring cell patterns of linear cell transmittance have been determined. These parameters can be utilized to design and fabricate the highest fidelity halftone contact screen.

## INTRODUCTION

IN printing and graphic arts industry, the reproduction of a continuous tone photograph is achieved by a halftone process[1] that involves the transformation of the original photograph into binary halftone photograph consisting of array of opaque dots with sizes varying according to the optical transmittance of the original.

To date, very few research results have been reported on the differentiation of the various screen designs and their effects in the quality of the resulting image reproduction. Bryngdahl[2] has shown pictorially that halftone screens of concentric-ring cell patterns can produce more details in case the original image is undersampled. He made a qualitative deduction that this probably is due to the dots produced by the screens having higher circumference-to-area ratio. Pappu *et al.*[3] have experimentally demonstrated that circular halftone dots get distorted in under-sampled images, whereas it is preserved to a great extent in sufficiently sampled images. In any circumstances, it can clearly be seen that the cell pattern of a screen has tremendous influence on the outcome of the halftone photograph which, in turn, is extremely important to both printing and image processing applications.

A very convenient and general approach is to characterize the whole halftoning process by means of transmittance parameters[4]—thes creen-trnsmittances, the threshold transmittance, the input transmittance and the transmittance of the halftone photograph. Based on the method, the optimum parameters for high fidelity tone reproduction by screens of line, square, circular and concentric-ring cell patterns possessing linear optical transmittance have been calculated and the results are presented.

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## ANALYSIS

For simplicity, the analysis will be based on the following fundamental assumptions: (i) the transmittance of the unit cell of the screen is linear, (ii) the continuous tone original is sufficiently sampled by the screen so that the transmittance of the original over any one unit cell remains constant, (iii) the recording is made on a positive high contrast film of infinite (or very high) gamma[5,6], and (iv) the scattering of light under the opaque area of the recording medium[7,8], is neglected.

The halftone processes with the use of screens of periodic line, square, circular and concentric-ring shaped cell patterns, are illustrated respectively in Figs. 1–4. The linear transmittance, $T^S$, in both one-dimensional (line) and two-dimensional (dot) screens are shown in part (*a*) of these figures, where $T_M{}^S$ and $T_m{}^S$ represent respectively the maximum and minimum screen transmittances. The screens have a period W along the $x$-axis, and also the $y$-axis if they are two-dimensional screens. Hence for the line screen, the transmittance can be written as,

$$T^S(x) = T^S(x + W) \tag{1}$$

and for the dot screens,

$$T^S(x, y) = T^S(x + W, y + W) \tag{2}$$

without loss of accuracy and again for simplicity[0] we further assume that for the screens of circular and concentric-ring cells, $T_m{}^S$ is constant in the region of the cell bounded between the curve $x^2 + y^2 - (x + y)W + W^2/4 = 0$ and the straight lines $x = 0$, $y = 0$, $x = W$ and $y = W$. Without this assumption, dots produced by circular and ring-shaped cells (and also for elliptical cells) get distorted at low threshold[3,9] of exposure. The screens in contact with the original image of transmittance $T^P$ has a combined transmittance $T^ST^P$ with a maximum $T_M{}^ST^P$ and a minimum $T_m{}^ST^P$. In the photographic contact-printing process, when the transmittance $T^ST^P$ exceeds or equals to the

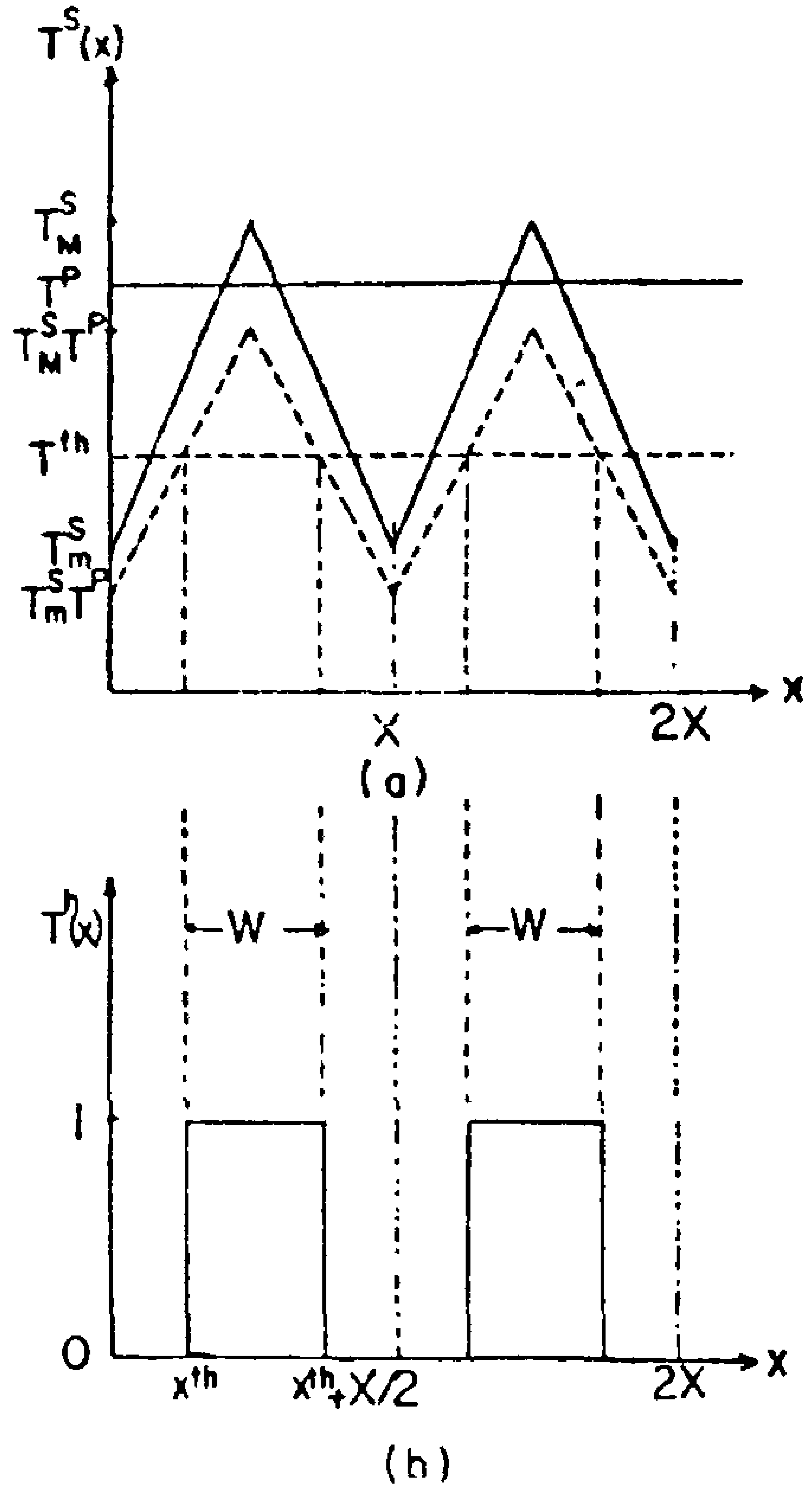


FIG. 1. Halftone process with screens of line cells. (*a*) Transmittance of two neighbouring cells, the image, the threshold, and the screen in contact with the image. (*b*) Transmittance of resulting halftone photograph.

threshold transmittance, $T^{th}$, as defined by $I^{th}/I^{in}$ of the positive high-gamma film, the film will develop to have a transmittance of 1 in reality, the fog level; otherwise, 0. The notations $I^{th}$ and $I^{in}$ denote the threshold energy and the energy of the input illumination respectively.

The overall transmittance of a unit cell, $T^H$, may be defined as the ratio of the transparent region (if the fog-level transmittance is taken as 1) to the total area of the unit cell. In the present case while the original is assumed to be sufficiently sampled, $T^H$ also represents the regional average transmittance of the halftone photograph. Based on the above definition, $T^H$ of screens of line (line), square (sq), circular (cir) and concentric-ring (ring) patterned cells are found respectively as functions of $T_M^S$, $T_m^S$, $T^P$ and $T^{th}$ as follows:

$$T^H_{line} = (T_M^S T^P - T^{th})/(T^P \Delta T^S), \tag{3}$$

$$T^H_{sq} = [(T_M^S T^P - T^{th})/(T^P \Delta T^S)]^2, \tag{4}$$

$$T^H_{cir} = \frac{\pi}{4}[(T_M^S T^P - T^{th})/(T^P \Delta T^S)]^2, \tag{5}$$

$$T^P_{ring} = \frac{\pi}{4}[(T_M^S T^P - T^{th})/(T^P \Delta T^S)], \tag{6}$$

where $\Delta T^S = T_M^S - T_m^S$. (7)

In addition it is noted that as $T^P$ varies,

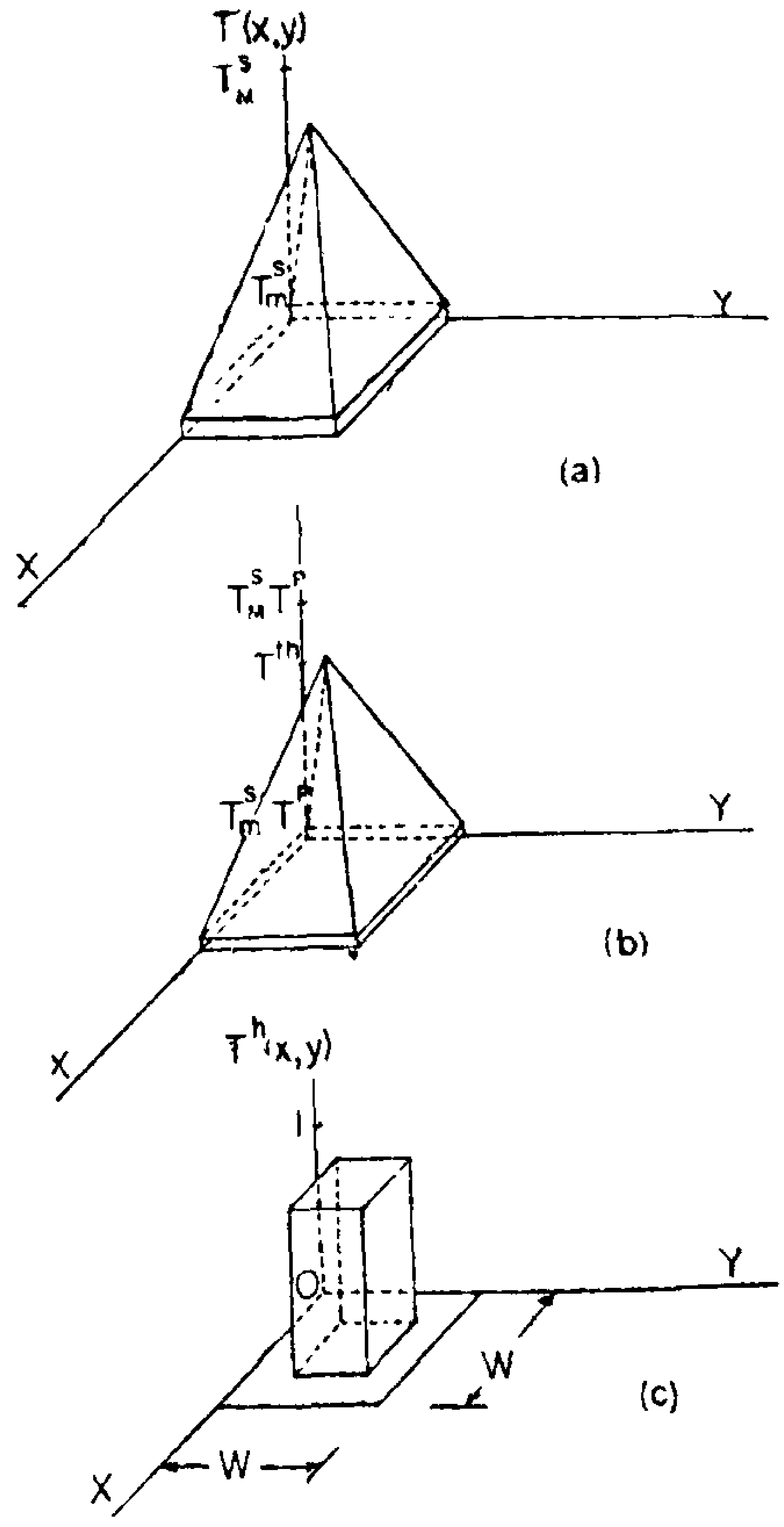


FIG. 2. Halftone process with screens of square cells. (*a*) Screen transmittance of one cell. (*b*) Combined transmittance of the screen in contact with the image. (*c*) Transmittance of the resultant halftone photograph.

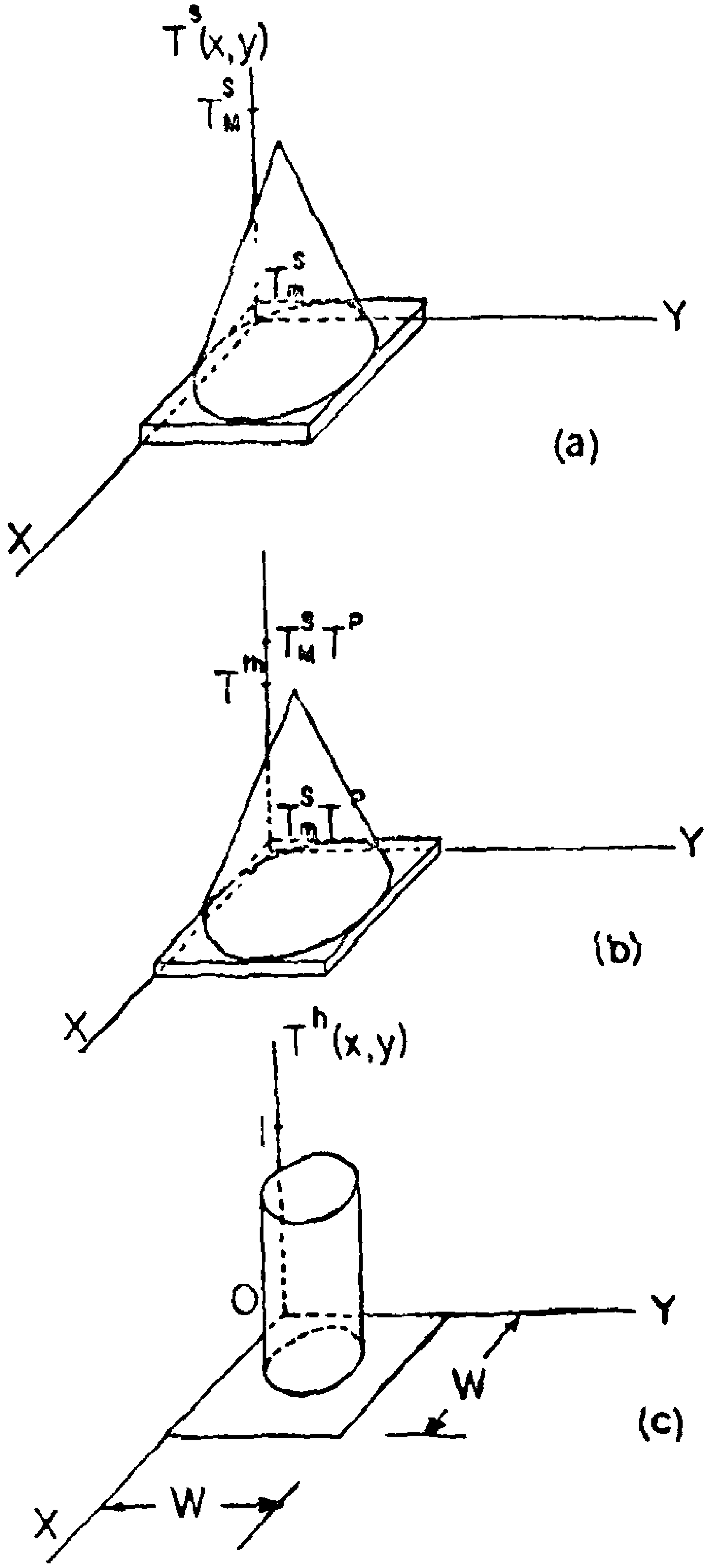


Fig. 3. Halftone process with screens of circular cells. (*a*) Screen transmittance of one cell. (*b*) Combined transmittance of the screen in contact with the image. (*c*) Transmittance of the resultant halftone photograph.

$$T^H = \begin{cases} 1, & T^{th} \leqslant T_m{}^S T^P, \\ 0, & T^{th} \geqslant T_M{}^S T^P. \end{cases} \tag{8}$$

Over any specific region, the difference between $T^H$ and $T^P$ namely, $T^H - T^P$, may be used as a quantitative measure of the fidelity preserved in the halftone process. Since $T^P$ varies from 0 to 1 and $T^H$ varies according to $T_M{}^S$, $T_m{}^S$ and $T^{th}$, consequently $T^H - T^P$ may have both positive and negative values over the range of $T^P$. For example, in case when a line screen is used, a plot of $T^H - T^P$ *versus* $T^P$ for $T_m{}^S = 0$ and $T_M{}^S = 1{\cdot}0$ and $T^{th}$ varying discretely from 0 to 1·0 is shown in Fig. 5, where the '+' and '−' signs indicate the positive or negative values of $T^H - T^P$ over the range of $T^P$ for $T^{th} = 0{\cdot}2$. The difference of $T^H$ and $T^P$ over the whole range of $T^P$ may be defined as

$$\delta = \int_0^1 (T^H - T^P)\, dT^P. \tag{9}$$

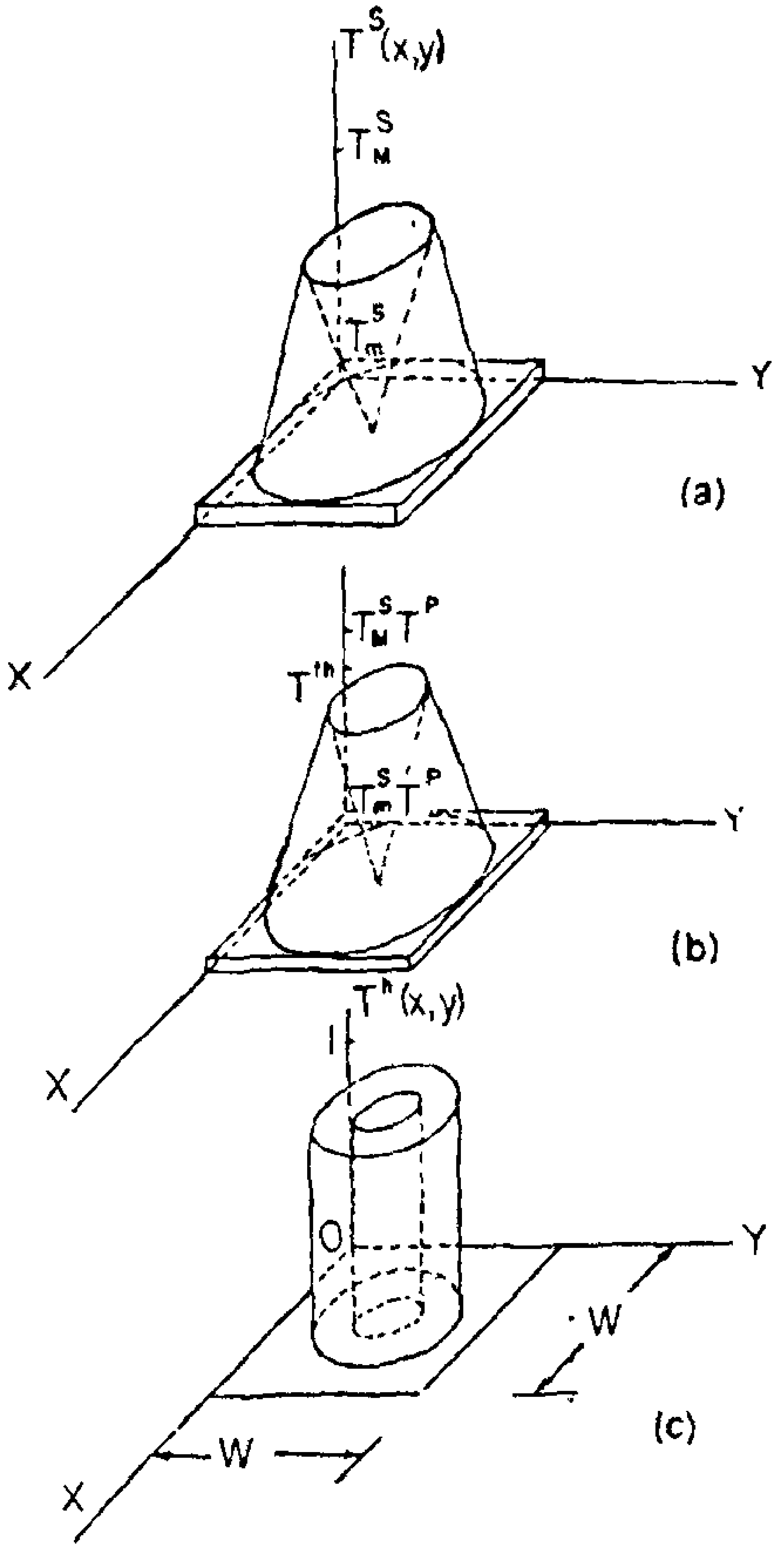


Fig. 4. Halftone process with screens of concentric-ring cells. (*a*) Screen transmittance of one cell. (*b*) Combined transmittance of the screen in contact with the image. (*c*) Transmittance of the resultant halftone photograph.

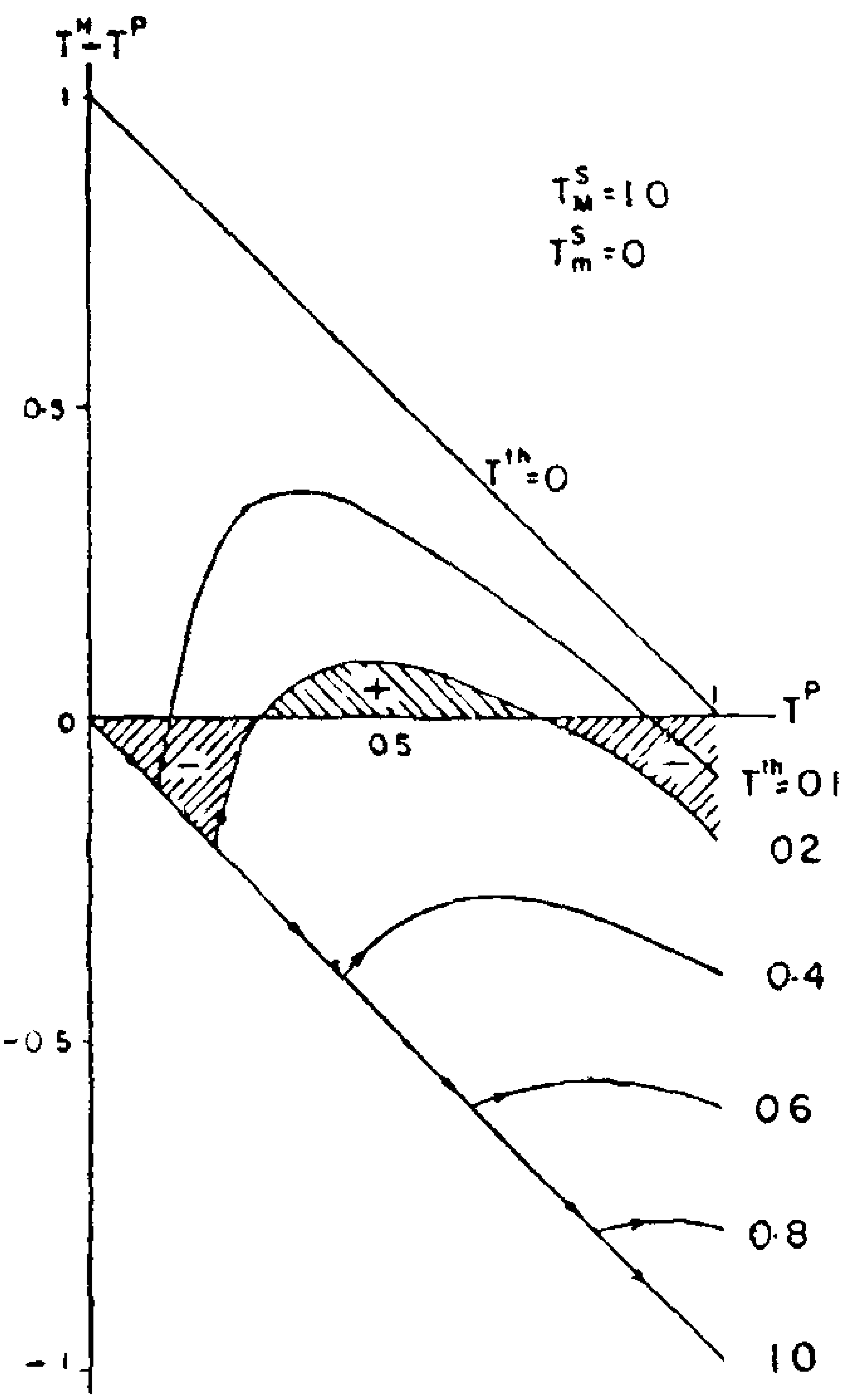


FIG. 5. $T^H - T^P$ *versus* $T^P$ for $T_m{}^S = 0$, $T_M{}^S = 1$, and $T^{th}$ varying discretely from 0 to 1 in case of a screen of line cells of linear transmittance.

With a given linearly transmitting screen of a known set of $(T_M{}^S, T_m{}^S)$, the value of $T^{th}$ with which $\delta = 0$ can be considered as the optimum threshold transmittance denoted by $T_{opt}{}^{th}$ since the net difference of $T^P$ and $T^H$ for the whole range of $T^P$ is minimized. The value of $T_{opt}{}^{th}$ may be used for the selection of the high gamma film and the film development process for achieving optimum tone reproduction. The values of $T_{opt}{}^{th}$ may be calculated by the numerical iteration method.

After we let $\delta = 0$ and find the corresponding $(T_M{}^S$ $T_m{}^S$, $T_{opt}{}^{th})$, a natural question arises, *i.e.*, which $(T_M{}^S, T_m{}^S, T_{opt}{}^{th})$ of the set will yield the minimum of the absolute value of $T^H - T^P$ over the whole range of $T^P$? The question demands us to look for $(T_M{}^S, T_m{}^S\ T_{opt}{}^{th})$ with which the minimum (denoted by $A_{min}$) of

$$A = \int_0^1 |T^H - T^P| \, dT^P \qquad (10)$$

be reached. Through this secondary and final step of optimization, we can find the optimum values $(T_M{}^S, T_m{}^S, T_{opt}{}^{th})_{opt}$, from which the values of $(T_M{}^S, T_m{}^S)_{opt}$ may be used for the design of screens, and the value of $T_{opt}{}^{th}\,[(T_M{}^S, T_m{}^S)_{opt}]$ for the selection of the high-gamma film such that the highest fidelity tone reproduction can be achieved.

For the screens analysed, the variations of A with $T_m{}^S$ and $T_{opt}{}^{th}$ for a fixed $T_M{}^S = p0{\cdot}912$ are shown in Table I. It can be seen from the table that A decreases as $T_m{}^S$ decreases to a minimum value around the value of $T_m{}^S$ between 0·05 and 0·2 for all the screens. Selection of $T_{opt}{}^{th}$ at the fixed $T_m{}^S$ that makes A to be at the minimum should give the highest fidelity tone reproduction.

TABLE I

*Variations of the area A with $T_M{}^S = 0{\cdot}912$ for the line, square, circular and concentric-ring patterned screens*

| $T_M{}^S$ | Line | | Square | | Circular | | Ring | |
|---|---|---|---|---|---|---|---|---|
| | $T_{opt}{}^{th}$ | A | $T_{opt}{}^{th}$ | A | $T_{opt}{}^{th}$ | A | $T_{opt}{}^{th}$ | A |
| ·9 | ·4450 | ·2540 | ·4520 | ·2470 | ·4515 | ·2520 | ·4523 | ·2423 |
| ·8 | ·4300 | ·2370 | ·4182 | ·2370 | ·3867 | ·2380 | ·4212 | ·2309 |
| ·7 | ·4023 | ·2216 | ·3838 | ·2235 | ·3274 | ·2292 | ·4005 | ·2102 |
| ·6 | ·3725 | ·2030 | ·3483 | ·2075 | ·3167 | ·2041 | ·3542 | ·1880 |
| ·5 | ·3375 | ·1821 | ·3113 | ·1881 | ·2960 | ·1924 | ·3125 | ·1606 |
| ·4 | ·3050 | ·1542 | ·2735 | ·1678 | ·2535 | ·1684 | ·2777 | ·1281 |
| ·3 | ·2750 | ·1142 | ·2329 | ·1356 | ·2082 | ·1323 | ·2334 | ·0912 |
| ·2 | ·2451 | ·0951 | ·1887 | ·0949 | ·1455 | ·0766 | ·1820 | ·0633 |
| ·18 | ·2408 | ·0902 | ·1791 | ·0850 | ·1287 | ·0773 | ·1704 | ·0586 |
| ·15 | ·2300 | ·0885 | ·1728 | ·0891 | ·1018 | ·0820 | ·1535 | ·0580 |
| ·10 | ·2017 | ·0862 | ·1572 | ·0934 | ·0337 | ·0928 | ·1361 | ·0632 |
| ·05 | ·2002 | ·0872 | ·1460 | ·1061 | ·0261 | ·1210 | ·1182 | ·0913 |

To further illustrate the physical meaning of the calculated results, the minimum values of A are plotted *versus* $T_m^s$ for $T_M^s = 0{\cdot}912$ in Fig. 6. From the figure, it can be found that $A_{min}$ equals 0·0862, at $T_{opt}^{th} = 0{\cdot}2017$ and $T_m^s = 0{\cdot}18$ for the square screen; 0·0766 at $T_{opt}^{th} = 0{\cdot}1455$ and $T_m^s = 0{\cdot}20$ for the circualr screen; and 0·05797 at $T_{opt}^{th} = -{\cdot}1535$ and $T_m^s$ $-0{\cdot}15$ for the concentric-ring screen. It is clear from Fig. 6 that for all values of $T_m^s$, except for $T_m^s > 0{\cdot}05$, $A_{min}$ is consistently smaller for screens with cells of concentric-ring patterns.

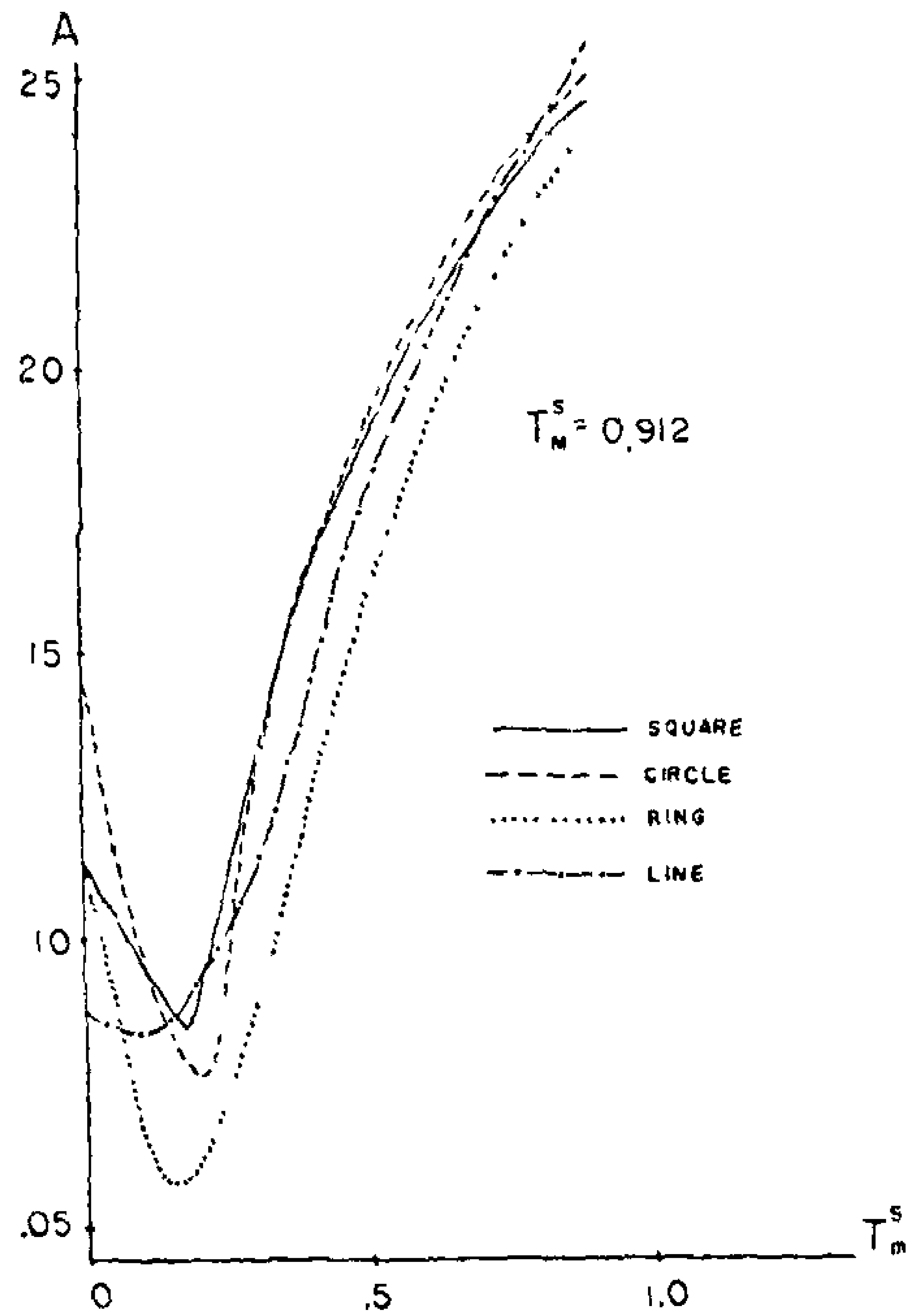


FIG. 6. Area A *versus* $T_m^s$ for $T_M^s = 0{\cdot}912$ for screens of line, square, circular, and concentric-ring cells.

## CONCLUSION

The theoretical analysis of tone reproduction by halftone contact screens with cells of linear optical transmittances, presented herein, shows that the regional average transmittance of the screen can be related to the transmittances of the original image, the threshold of the high-gamma film and the maximum and minimum transmittances of the screens. For each design pattern, corresponding to each pair $(T_M^s, T_m^s)$ there exists an optimum threshold $T_{opt}^{th}$ such that the fidelity of the reproduced image reaches an optimum. This threshold transmittance can be used as a guideline for the selection of the high-gamma film, the exposure, and the film developing process if high fidelity reproduction is desired through the linearly transmitting screens.

Extension of this analysis to design and fabrication of a nonlinear fidelity screen is in progress and the results will be communicated later.

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