LETTERS TO THE EDITOR

JUNCTION CONDITIONS IN BIMETRIC RELATIVITY THEORY

In general relativity theory the mathematical model for space-time is (M, g) where M is a connected four dimensional Hausdorff manifold and g is a Lorentz metric with signature -2 defined on M. As a consequence we are able to coordinatize space-time. However, if the topology of the manifold in the large is not equivalent to a single Euclidean four plane, all space-time may not be covered with a single system of allowable coordinates. In such a situation the whole space-time can be built up of pieces of R^4 glued together by homoemorphism. According to Synge[1] all space-time can be covered with a single system of admissible coordinates provided each domain of the admissible coordinates is divided into subdomains by 3-spaces of discontinuity and then the discontinuities will appear in the second derivatives of g_{ij} when one crosses a 3-space of discontinuity. The junction conditions, worked out by Synge[1], were found to be

$$G_i^4 = (c) \tag{1}$$

across the 3-space S of discontinuity

$$x^4 = 0 \tag{2}$$

across which $g_{ij,44}$ will be discontinuous.

Here G_i^j and , stand for the Einstein tensor and the partial derivative respectively. The symbol (c) indicates any quantity which is continuous across S. For any allowable coordinates x^i in which S is expressed as $f(x) = 0$, the junction conditions become

$$G_i^j f_{,j} = (c). \tag{3}$$

In this note the study of junction conditions is extended to bimetric relativity of Rosen[2]. It is seen that the similar conditions as stated in (2) and (3), but replacing Einstein tensor by Rosen tensor, are also the junction conditions for bimetric relativity provided some constraints are imposed on the space-time.

Let a 3-space of discontinuity S be given by (2). The quantities (defined on S) g_{ij}, g^{ij}, $g_{ij,k}$ involve at most one differentiation w.r.t. x^4 and as such they are continuous on S. The discontinuities will appear in $g_{ij,44}$. In the usual conventions of bimetric relativity

$$N_i^j = \tfrac{1}{2}\gamma^{pq}\left(g^{jr} g_{ri|p}\right)_{|q}, \tag{4}$$

where γ^{pq} describes the background flat space-time metric and a bar | stands for γ-differentiation. (For details one may refer to the work of Rosen cited in Rosen[2]).

It is easily seen that

$$N_i^j = (c) + \tfrac{1}{2}\gamma^{44} g^{jr} g_{ri,44}$$

and

$$N = N_i^i = (c) + \tfrac{1}{2}\gamma^{44} g^{ir} g_{ri,44}.$$

We designate $K_i^j = N_i^j - \frac{1}{2} N \delta_i^j$ as the Rosen tensor and find that

$$K_i^j = (c) + \tfrac{1}{2}\gamma^{44}\left(g^{js} g_{si,44} - \tfrac{1}{2}\delta_i^j X\right),$$

with $X = g^{sp} g_{sp,44}$.

Therefore the junction conditions in bimetric relativity will be

$$K_i^j = (c) \tag{5}$$

provided

$$g^{sj} g_{si,44} - \tfrac{1}{2}\delta_i^j X = 0. \tag{6}$$

In general relativity the conditions (1) emerge as a mere matter of direct calculations. But here in bimetric theory we have to impose additional conditions (6). Equations (6) are ten in numbers:

$2g^{si} g_{si,44} = X$, $i = 1, 2, 3, 4$ and no summation on i,

$g^{si} g_{s1,44} = 0$, $i = 2, 3, 4$

$g^{si} g_{s2,44} = 0$, $i = 3, 4$

$g^{s4} g_{s3,44} = 0$. (7)

These equations (7) will be satisfied trivially for $g_{ij,44} = 0$, *i.e.*, for the components g_{ij} have no second derivatives w.r.t. x^4 and in that case

$$g_{ij} = x^4 h(x^1, x^2, x^3) + k(x^1, x^2, x^3). \tag{8}$$

In such a case analogous to (1), the junction conditions can be written as

$$K_i^4 = (c)$$

across $x^4 = 0$ provided the coordinates are admissible. Similarly for any other allowable coordinates x^i, the conditions will read as

$$K_i^j f_{,j} = (c),$$

where equation of S is $f(x) = 0$ and , indicates a γ—covaria .t derivative*. It can easily be seen that all the discussion, not refering to any particular coordinate system of junction conditions given by Synge will be valid in the present case provided (8) holds.

Now we shall consider an interesting case where $g_{ij,44} \neq 0$. We rewrite each of the equations (7) in order of $g_{11,44}$, $g_{22,44}$, $g_{33,44}$, $g_{44,44}$, $g_{12,44}$, $g_{13,44}$, $g_{14,44}$, $g_{23,44}$, $g_{24,44}$, $g_{34,44}$. Then $g_{ij,44}$ eliminant of (7) yields a 10×10 determinant. This determinant can ultimately be simplified to

$$G_2 G_3 G_4 = 0, \tag{9}$$

where $G_2 = \begin{vmatrix} 33 & 34 \\ 43 & 44 \end{vmatrix}$

$$G_3 = \begin{vmatrix} 22 & 23 & 24 \\ 32 & 33 & 34 \\ 42 & 43 & 44 \end{vmatrix}, \quad G_4 = \text{det.}\,|g^{ij}|,$$

with ij standing for g^{ij}.

As det. $|g_{ij}| = 1\, G_4 \neq 0$, equation (9) implies

$$(a)\ G_2 = 0,\ (b)\ G_3 = 0 \text{ or } (c)\ G_2 = 0,\ G_3 = 0, \text{ etc.} \qquad (10)$$

It is interesting that if any of the three conditions in (10) is satisfied Rosen tensor is continuous across S irrespective of $g_{11,44} \neq 0$.

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* In case of a scalar $f_{ij} = f_{ji}$.

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A NOTE ON THE OCCURRENCE OF TIN-BEARING RARE-METAL PEGMATITES IN THE BENGPAL SERIES (PRECAMBRIAN), GOVINDPAL-CHIURWADA-MUNDVAL AREA, BASTAR DISTRICT, M.P.

THE present note records the occurrence of tin-bearing rare-metal pegmatites from the Bengpal series. Tin-bearing rare-metal pegmatites, cassiterite-quartz, cassiterite-silicate and cassiterite-sulphide are the four main tin-ore associations recognised by Lugov[1].

The tin-ore association of Govindpal (18° 42′ : 81 54′), Chiurwada (18° 44′ : 81° 53′) and Mundval (18° 39′ : 81° 56′) area of Konta tahsil, Bastar district, M.P., is placed in "tin-bearing rare-metal pegmatite association" on the basis of typomorphic trace and ore elements—Li, Rb, Cs, Be, Ta, Nb, Sn; minerals present in the pegmatite-clevalandite, quartz, muscovite, lepidolite, cassiterite, magnetite, tantalite, spodumene, amblygonite, fluorite, beryl; and their occurrence in Precambrian metasediments of the Peninsular shield.

The Bengpal metasediments of Precambrian age with associated basic sills have suffered granite tectonism, as seen in the vicinity of Paliam, and emplacement of pegmatites in the metabasics[2].

The pegmatites are simple as well as complex. They occur in swarms. Of the many pegmatites occurring in the area only a few are tin-bearing.

The pegmatites are very irregularly zoned. The bulk composition of pegmatites is lithophilic as evidenced from the mineralogy. Tantalum and niobium are present. Apparently, fluorine, along with other mineralizers (water), are the main transporting agents in the pegmatite process. Sulphides including stannite are practically absent. Most of the rare-metal mineralisation (especially tantalum and tin) is restricted to the albite and greisen zones.

The tin-bearing rare-metal pegmatites are of hypabyssal high-volatile type and were formed by the pegmatitic-pneumatolytic processes.

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OCCURRENCE OF SPIROPLASMAS OF TWO SEROGROUPS ON FLOWERS OF THE TULIP TREE (*LIRIODENDRON TULIPIFERA* L.) IN MARYLAND

SPIROPLASMAS and mycoplasmas have been reported to occur on the surfaces of flowers of healthy plants in nature[1-8]. Some of the spiroplasmas found on flowers are serologically closely related to a strain pathogenic in honey-bees and may induce disease in bees in nature[3,5,7,8]. Other spiroplasma strains from flowers are unrelated or only distantly related to each other and to the honey-bee pathogen[2,5]. At least three serologically distinct groups have been identified among the flower-inhabiting spiroplasma strains[5]. Spiroplasmas in one of these groups, the serogroup II strains have been found on the flowers of tulip tree (*Liriodendron tulipifera* L.) growing in Maryland[9], whereas serogroup III strains have been found only on flowers of tulip trees growing in Connecticut[9]. The geographical distribution of spiroplasma strains on flowers of *L. tulipifera* has not yet been extensively investigated; but results reported