material beyond its upper critical temperature, supporting the theory of entropy elasticity and the results of Tuzi *et al.*[13] on moderately crosslinked polymers.

The effective fringe order at the hole on the transverse section of symmetry is determined as 4·944, using the analyser as compensator. Hence, the maximum stress at the hole $\sigma_{max} = 4 \cdot 944 \times 1 \cdot 893/0 \cdot 1456 = 64 \cdot 28$ lb/sq. in. The nominal stress $\sigma_{nom} = 2 \cdot 977/1 \cdot 045 \times 0 \cdot 1456 = 19 \cdot 56$ lb/sq. in. So, the stress concentration factor, $\sigma_{max}/\sigma_{nom} = 3 \cdot 286$. This value is very close to the Howland's[14] theoretical value 3·33, when compared with a similar result 3·15 obtained by Leven[15] on Fosterite. This ascribes reliability to the material for more accurate stress analysis by frozen stress technique.

Considering the high photoelastic nature of this resin, besides the ease of casting to any shape or size, negligible heat edge effects, availability in bulk, etc., priority may be given to this resin for photoelastic studies.

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A SHORT NOTE ON SECOND ORDER ROTATABLE DESIGNS

IN statistical designs of experiments in the field of industry, numerous problems involve the fitting of a response surface in which the response depends upon one or more controllable variables or factors. Box and Hunter[1] introduced a new class of designs called rotatable designs in which the variance of the estimated response at a point is a function of the square of the distance of the points from a suitable origin, so that the variances of all estimated responses at points equidistant from the origin are the same. A design involving m factors is called a rotatable design of second order (SORD) if the response function can be expressed as a quadratic in the given variables. The valuable contributions on this subject are due to Box and Behnken[3], Box and Draper[2], Das[4], Das and Narshimham[5], Saha and Das[8], Pal[6], etc. In this paper, necessary and sufficient conditions have been obtained for second order rotatable designs constructed through PB arrays to have minimum number of design points and the result provides the parameters of the array.

For a definition of partially balanced (PB) array, Rafter *et al.*[7] may be consulted. Method of construction of a SORD using PB array is given by Das[4].

We prove the following Theorems.

Theorem 1

The necessary and sufficient condition for a SORD constructed from a PB array $(m, N, 2, 2)$ with $\lambda^2_{\alpha\beta} > \lambda_{\alpha\alpha}\lambda_{\beta\beta}$ to have minimum number of design points is $N = m$.

Proof

Since in this method of construction the number of design points is directly proportional to the number of assemblies N the minimum value of N will provide minimum number of design points. For the existence of a SORD from PB array we should have (Theorem 1, Saha and Das[8]).

$$\lambda_{\alpha\beta} > 2\lambda_{\alpha\alpha} \text{ or } \lambda_{\alpha\beta} > 2\lambda_{\beta\beta}. \qquad (2.1)$$

Necessity. From the linear relation

$$N = \lambda_{\alpha\alpha} + 2\lambda_{\alpha\beta} + \lambda_{\beta\beta},$$

it is evident that for minimum N each of the parameters should be minimum. With this view, let us choose $\lambda_{\alpha\alpha} = 0$ and $\lambda_{\alpha\beta} = 1$ so that (2.1) is satisfied. Now, we have

$$N = \lambda_{\beta\beta} + 2.$$

Under the given condition of the Theorem, we have the following inequality (Theorem 3.2, Rafter and Seiden[7]),

$$N \geqslant m\left(\lambda_{\alpha\beta} - \frac{\lambda_{\alpha\alpha}\lambda_{\beta\beta}}{\lambda_{\alpha\beta}}\right),$$

which reduces to

$$N \geqslant m,$$

therefore, the least value of N is m and then the parameters will be $(0, 1, m-2)$.

Sufficiency. It follows from the inequality $m \leqslant N$ true for all PB arrays (Section 3, Rafter and Seiden[7]). Surprisingly, the greatest lower bound for N does not shift from m. This completes the proof.

Theorem 2

The necessary and sufficient condition for a SORD constructed through a PB array $(m, N, 2, 2)$ with $\lambda^2_{\alpha\beta} - \lambda_{\alpha\alpha}\lambda_{\beta\beta} = 0$ to have minimum number of design points is $N = m + 1$.

Proof

Under the given conditions of the theorem, we have the following inequality (Theorem 3.4, Rafter and Seiden[7]),

$$N \geqslant m + 1,$$

and necessity part follows immediately. Conversely, if $N = m + 1$, then we have to show that N is minimum. Since for all PB arrays $N \geqslant m$ so the only possible smaller value is $N = m$ which contradicts $N \geqslant m + 1$. Thus $m + 1$ is the minimum value of N.

Now the problem is to find the parameters of the array for a given m to provide least N under the hypothesis of the theorem and can yield SORD, *i.e.*, in other words the set $(\lambda_{\alpha\alpha}, \lambda_{\alpha\beta}, \lambda_{\beta\beta})$ should be such as to satisfy the following:

(i) $\lambda_{\alpha\beta} > 2\lambda_{\alpha\alpha}$ or $\lambda_{\alpha\beta} > 2\lambda_{\beta\beta}$

(ii) $\lambda^2_{\alpha\beta} = \lambda_{\alpha\alpha}\lambda_{\beta\beta}$.

For this we can ascertain the parameters by taking help of two more relations, provided the solution exists,

(iii) $\lambda_{\alpha\alpha} + 2\lambda_{\alpha\beta} + \lambda_{\beta\beta} = m + 1$

and

(iv) $\lambda_{\alpha\beta} \leqslant \lambda_{\alpha\alpha} + \lambda_{\beta\beta}$.

For example when $m = 15$, the set is $\{1, 3, 9\}$.

This completes the proof.

Let us consider a 3-symbol PB array of strength two for m factors in N assemblies with α, β and θ as symbols. Let, then, the pairwise frequencies be $\lambda_{\alpha\alpha}$, $\lambda_{\alpha\beta} = \lambda_{\beta\alpha}$, $\lambda_{\beta\theta} = \lambda_{\theta\beta}$, $\lambda_{\alpha\theta} = \lambda_{\theta\alpha}$, $\lambda_{\beta\beta}$ and $\lambda_{\theta\theta}$.

Here is a result for SORD with 5 levels.

Theorem 3

A three symbol $(\alpha \neq 0, \beta \neq 0, 0)$ PB array of strength two must satisfy

$$\lambda_{\alpha\beta} + \lambda_{\alpha 0} > 2\lambda_{\alpha\alpha} \text{ or } \lambda_{\alpha\beta} + \lambda_{\beta 0} > 2\lambda_{\beta\beta}.$$

to yield a five level SORD.

The proof is straightforward.

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TRACK-ETCHED MICRO-FILTERS

THE technique which led to the development of track-etched micro-filters was discovered in 1962 by Price P. B. and Walker R. M.[1]. They observed fine holes due to fission fragments in 12 μm thick layers of synthetic mica during the course of their studies on chemical etching of charged particle tracks. The existence of these holes was demonstrated by using transmission electron microscopy. The chemical etching thus permits drilling of fine holes of adjustable size in thin sheets of cleavable solids and plastics.

The heavy charged particles passing through any of these solids produce continuous damage along their path and thus leave behind a trail of radiation damaged material. The chemical etching of these irradiated solids leads to the formation of fine hollow channels along the path of the charged particles due to preferential etching of the damage trail. These channels are usually uniform in width along their entire length and maintain the directions of the original tracks[2]. If the thickness of the detector sheet is less than the particle's range in it, the above process leads to the formation of fine holes in the irradiated detector sheet. The micro-filters thus formed have uniform holes and offer certain advantages over conventional filters made of cellulosic plastics and having irregular holes[3]. Fig. 1 is a microphotograph of a micro-filter.

Experimental

The uniformity of the holes is achieved by allowing a collimated beam of charged particles to strike the filter sheet. A simple way[2] of collimating the beam of